量子色动力学简介

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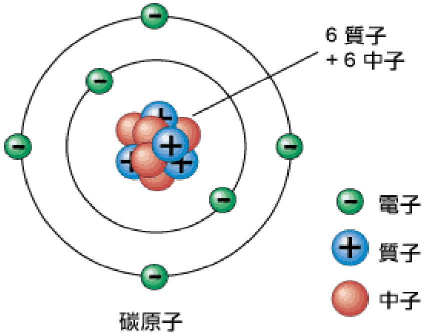
Contents:

Before the advent of QCD

1932年，英国科学家查德威克（James Chadwick，1891-1974）发现了与质子质量相仿的中性粒子

德国科学家海伯森（Werner Karl Heisenberg, 1907-1976）以及前苏联科学家伊凡宁柯各自独立提出，原子核是由质子和中子组成的。



原子核 = 质子 + 中子

问题1：带正电的质子为何被束缚在一起？

问题2：质子和中子是否具有内部结构？

Before the advent of QCD

1935年，日本科学家汤川秀树（Yukawa Hideki，1907-1981）提出了“交换粒子”的概念，作为新相互作用理论的基本概念。

1936年，美国科学家安德森在宇宙线中发现一种比电子约重207倍的粒子，当时误认为就是介子，后来发现这种粒子其实并不参与强相互作用是一种轻子，所以改名为μ子。

1947年，英国物理学家鲍威尔在宇宙射线发现了汤川所预言的介子，被命名为π介子。

1.核力的发现

2.原子核 = 质子 + 中子 + 介子

Before the advent of QCD

Today we know that the strong force between the nucleons in an atomic nucleus is a Van-der-Waals type residual force of a more fundamental strong interaction between quarks. The field theory of this interaction between quarks is called Quantum Chromodynamics (QCD).

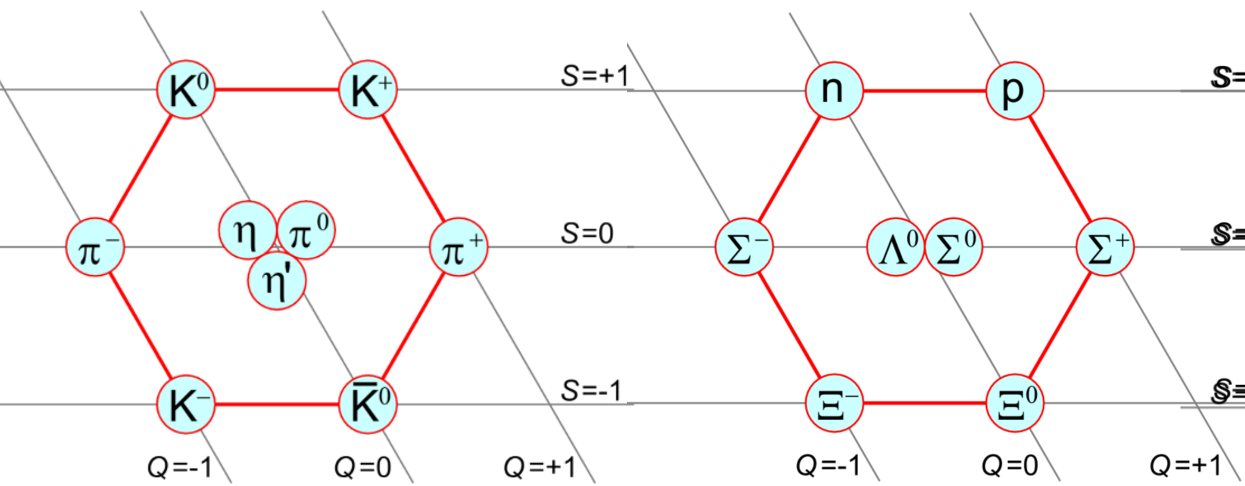
“基本粒子”有内部结构

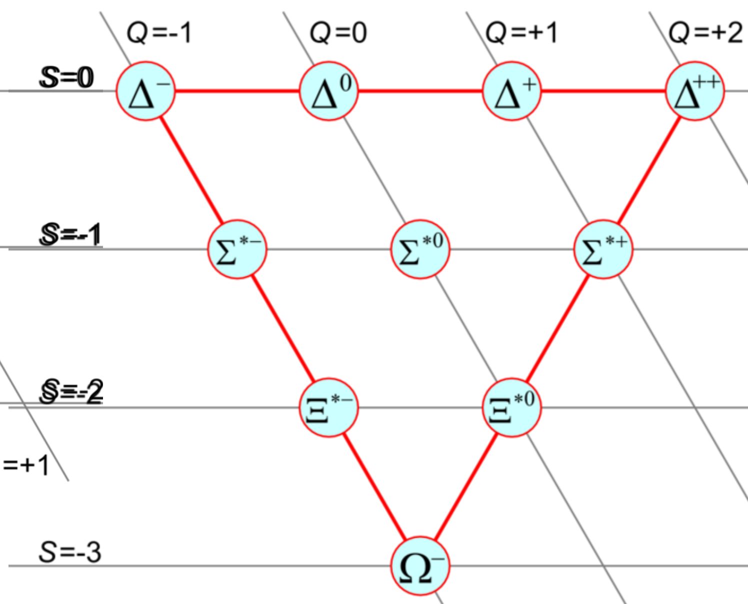
到上世纪五十年代，共发现了100余种粒子，大部分为强子，是否都为基本粒子？

元素周期表中100余种元素都有质子、中子和电子构成。新发现的这些粒子似乎否会有这样的性质？

八重法分类和夸克模型

1961、1962年，内曼和盖尔曼各自独立地提出了基于SU(3)对称性的理论—八重法（eightfold way）对已知的强子进行分类





介子被纳入单态和八重态；重子纳入八重态和十重态，如图所示

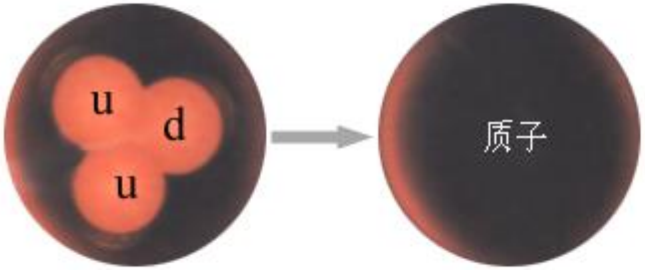
问题：为什么没有三重态？内在规律是什么？

八重法分类和夸克模型

1964年，盖尔曼和兹韦格在强子分类八重法的基础上分别提出了夸克模型，认为中子、质子这一类强子是由更基本的单元——夸克（quark）组成的

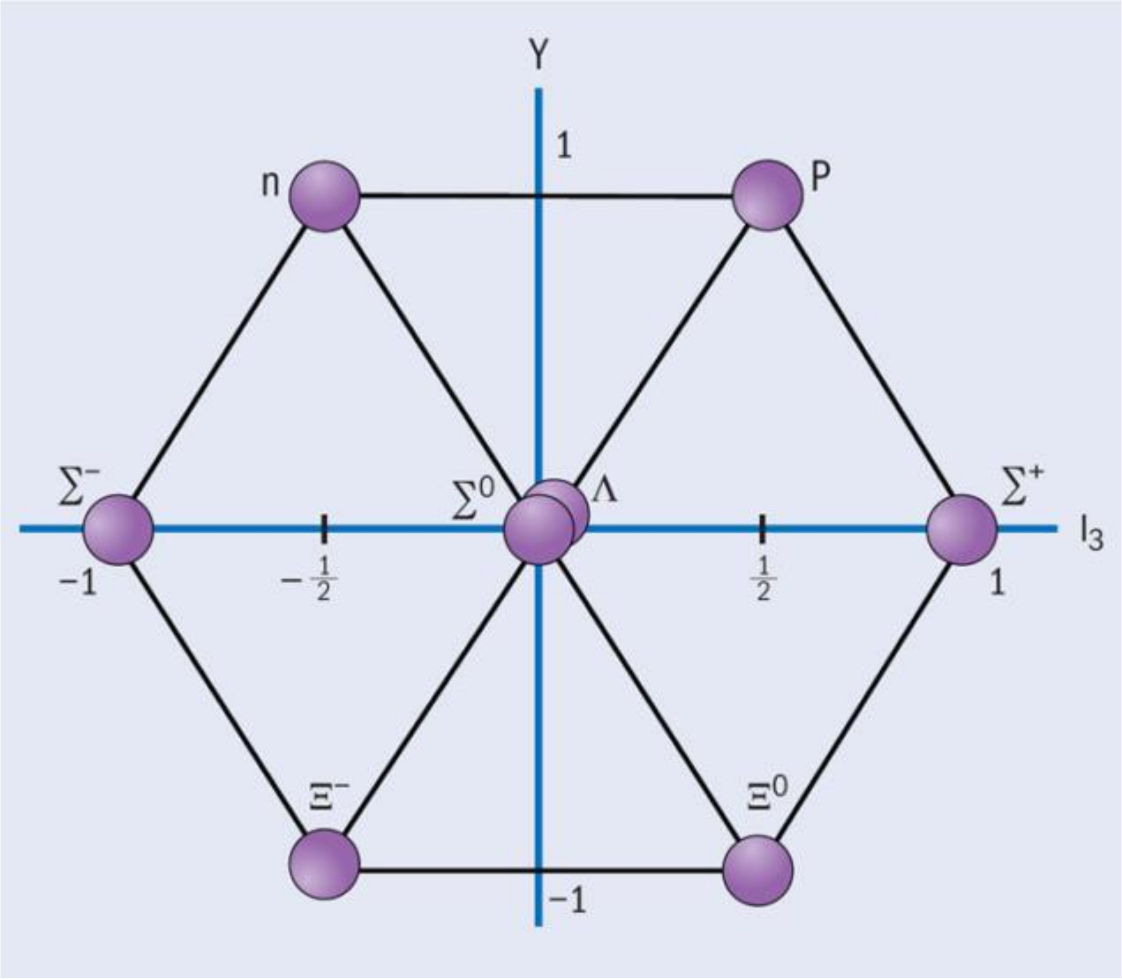
当时的夸克模型：

1. 三种（味道）的夸克，u，d，s；
2. 两个正反夸克组成介子，三个正（反）夸克组成重子；
3. 可以解释当时发现的全部强子；
4. 成功预言了当时并未发现的Omega粒子



质子 = u + u + d
中子 = u + d + d
Ω = s + s + s

八重法分类和夸克模型



练习：按夸克模型，给出如上强子的夸克组分

更多的夸克

1974年，丁肇中与Richter分别在实验中发现了一种新粒子，称为J/ψ粒子。由第四代夸克charm构成。

1977年，莱德曼又发现了一种长寿命的新介子Y，只能引入第五种夸克进行解释，成为bottom（底）或beauty（美）夸克

1994年，美国费米实验室的CDF组在质子-反质子对撞机上发现了一个最重的夸克，质量为176 GeV，取名为顶夸克（top）。

迄今为止，共发现三代，六种味道的夸克

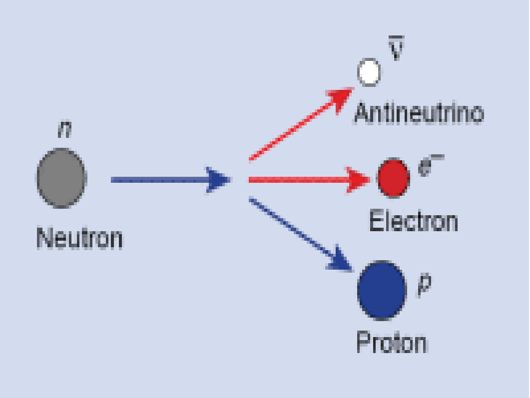
Before the advent of QCD

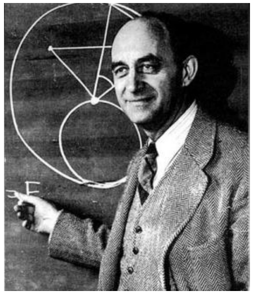
The elementary particles can be classified into leptons (without strong interaction) and quarks (with strong interaction).

轻子家族

◆第一个轻子—电子

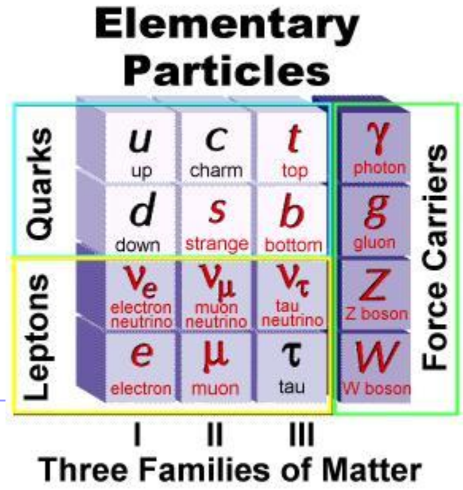
1931年，泡利为了解释β衰变中的能量失踪现象，预言了一种未知的极其微小的中性粒子带走了β衰变中那一部分能量和动量，最终被费米命名为“中微子”（Neutrino）。





1933年，费米指出：β衰变就是核内一个中子通过弱相互作用衰变成一个电子、一个质子和一个 反中微子。中微子只参与弱作用，具有极强的穿透力。由于中微子与物质间的相互作用极其微弱，中微子的检测非常困难。

1. 陆续发现了与电子类似μ子和τ子（带-1电子电荷）
2. 这三种轻子都有各自相伴的轻子中微子（电中性）



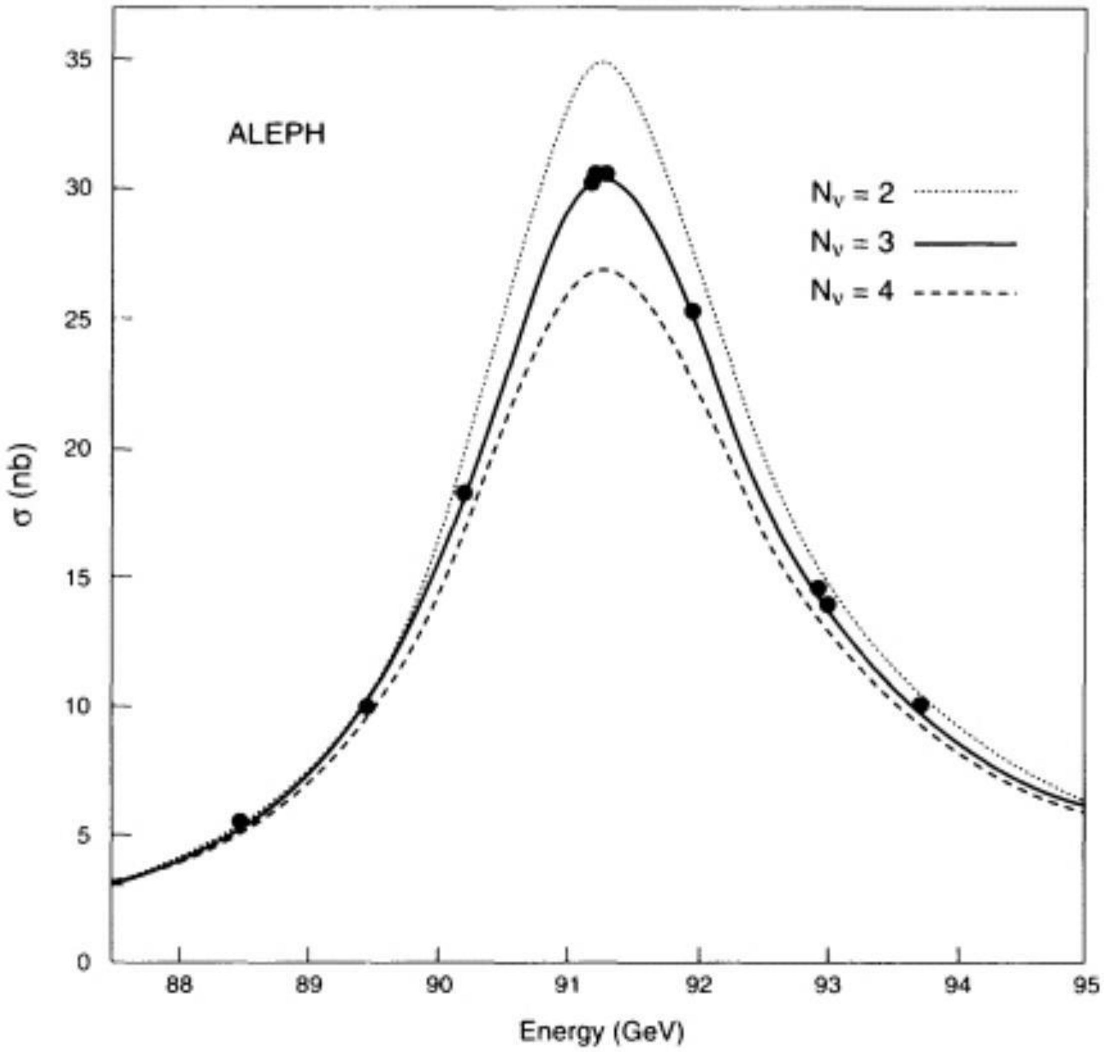
Before the advent of QCD

By examining what fraction of the Z-bosons we create in accelerators decay to:

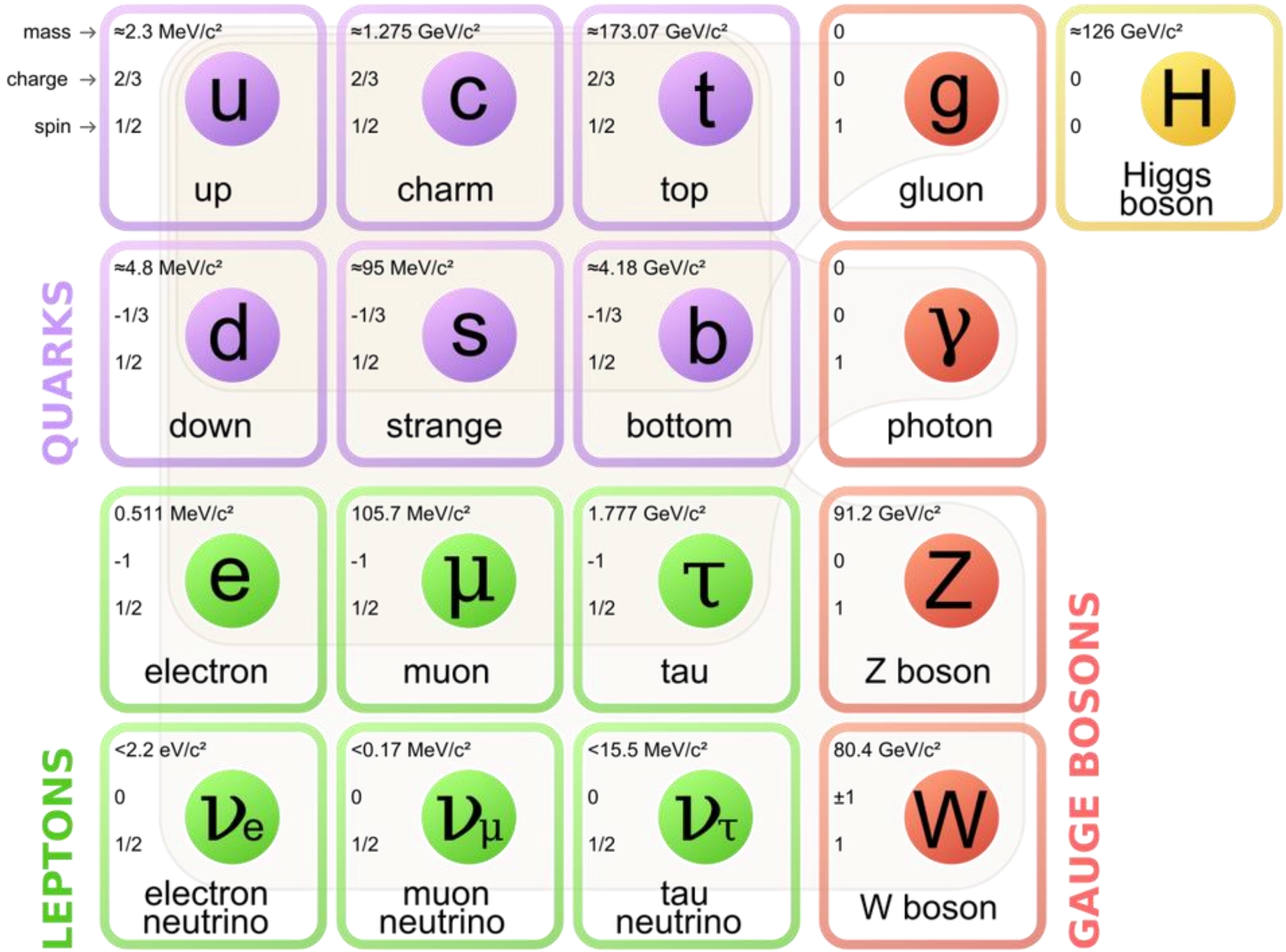
- electron/positron pairs,
- muon/anti-muon pairs,
- tau/anti-tau pairs,
- and "invisible" channels (i.e., neutrinos),

One can determine how many generations of particles there are

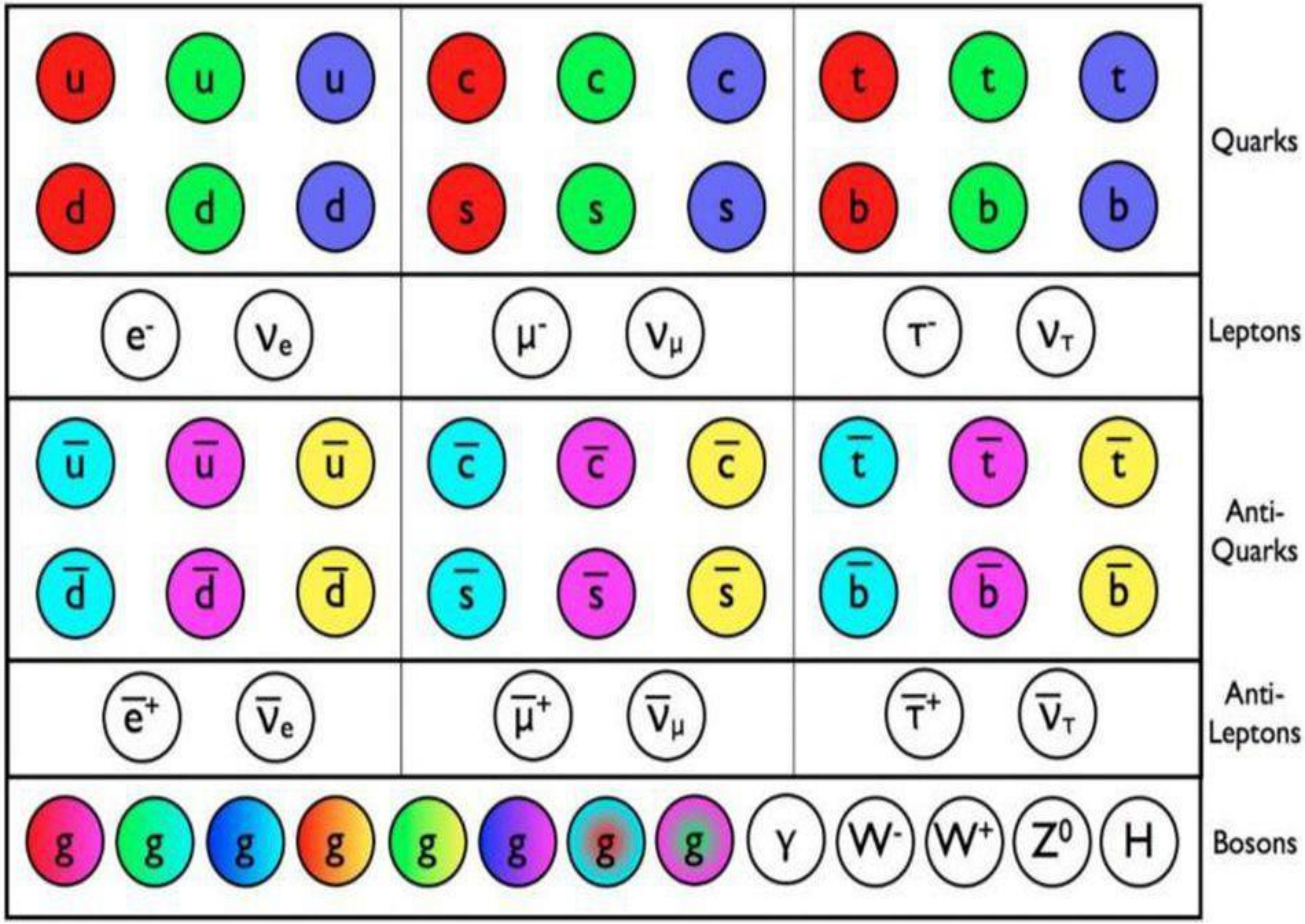
Three generations in SM family



The SM family



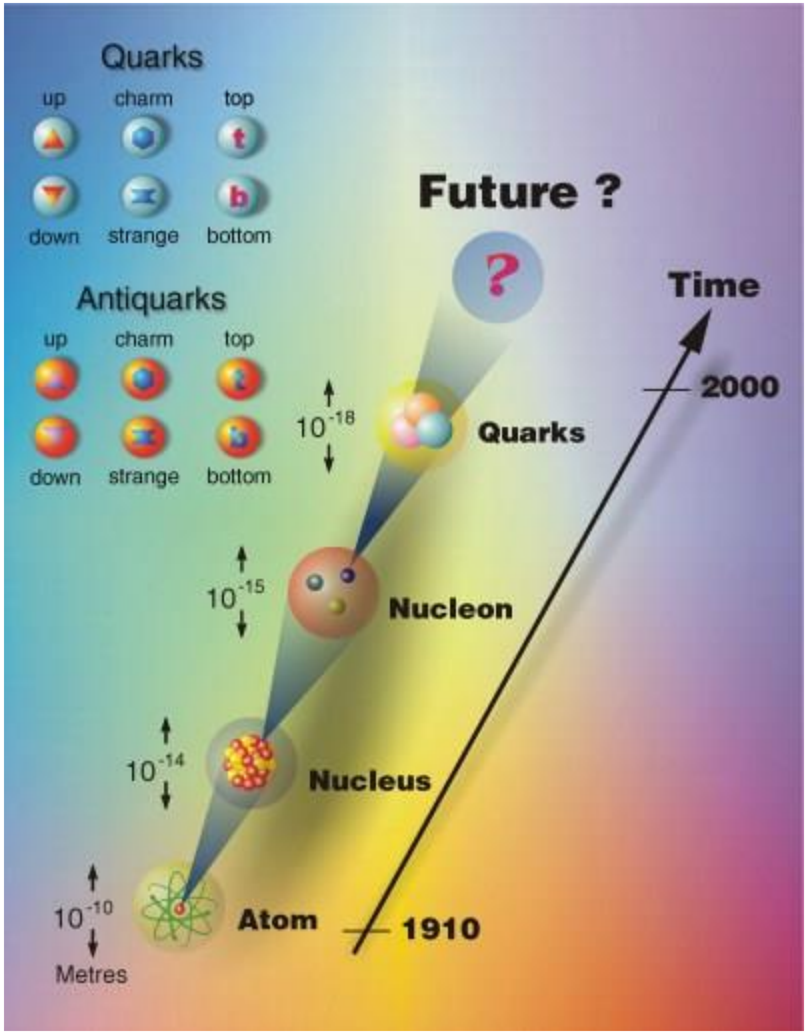
The SM family members



夸克和轻子的未来

◆是否还有更多的夸克和轻子？

◆夸克和轻子有没有再深的内部结构？

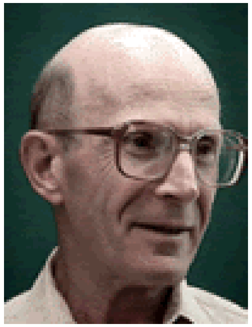


Contents:

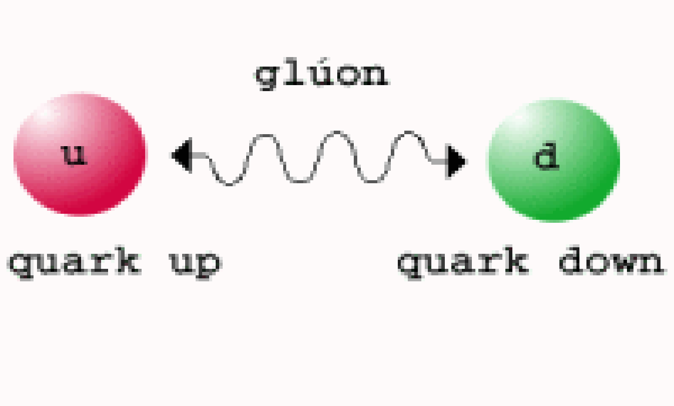
- Before the advent of QCD
- **The establishment of QCD**
- Typical natures of QCD
- Applications of QCD
- Concluding remarks

The establishment of QCD

1964年，Greenberg引入了夸克的一种自由度——“颜色”（color）夸克带颜色荷。每味夸克就有三种颜色分别是红、绿和蓝（RGB）。



1973年，Gross，Politzer和Wilczek证明了量子色动力学（QCD）具有“渐近自由”的性质。QCD成为描述强相互作用的正确理论。



➢ Color structure

- **Quark= fundamental representation 3**
- **Gluon= Adjoint representation 8**
- **Observable particles=color singlet 1**

◆ **Mesons** $$3 \bigotimes 3 = 1 \bigoplus 8$$

◆ **Baryons** $$3 \bigotimes 3 \bigotimes 3 = 1 \bigoplus 8 \bigotimes 8 \bigoplus 10$$

◆ **Glueballs**
$$\begin{cases} 8 \bigotimes 8 = 1 \bigoplus 8 \bigoplus 8 \bigoplus 10 \bigoplus \overline{10} \bigoplus 27 \\ 8 \bigotimes \cdots \bigotimes 8 = 1 \bigoplus 8 \bigoplus \cdots \end{cases}$$

The establishment of QCD

QCD is a so-called gauge theory, like quantum electrodynamics (QED) and the theory of the weak interactions. In such a theory, the constituent fields are described by representations of a symmetry group while the interaction between the fields is described by the exchange of so-called gauge bosons.

K. Dasgupta Lecture

The establishment of QCD

These interactions follow from the requirement that the Lagrangian is invariant under arbitrary local symmetry transformations of the constituent fields.

The establishment of QCD

Note that all particles participate in the weak interaction, all charged particles in the electromagnetic interaction and that only the quarks participate in the strong interaction. Gravity is so weak that it can be neglected at subatomic scales.

The establishment of QCD

Free quarks have never been observed because their coupling is so strong that with increasing separation it becomes easier to produce a quark-antiquark pair than to isolate the quark.

The establishment of QCD

Quarks therefore bind permanently into hadrons which can be classified as mesons ($q\bar{q}$) and baryons (qqq).

The establishment of QCD

A problem with this is that there exist baryons such as the spin 3/2 resonance $\Delta^{++} = u\uparrow u\uparrow u\uparrow$ with a ground state wave function that is fully symmetric under the exchange of two quarks. But for fermions the wave function should be antisymmetric

The establishment of QCD

- A way-out is provided by the color hypothesis which states that each quark comes in one of three colors red(r), green(g) or blue(b). Antiquarks are anti-colored: $\bar{\boldsymbol{r}}$, $\bar{\boldsymbol{g}}$ and $\bar{\boldsymbol{b}}$
- The hypothesis furthermore states that hadrons are color singlets (“white"), that is, they are invariant under rotations in color space

The establishment of QCD

在量子色动力学中，“色”是相互作用荷，胶子是将夸克束缚成强子的色规范场的场量子。与量子电动力学QED中光子不同的是，胶子本身也带色荷，具有自相互作用，存在三胶子和四胶子相互作用顶点。

The establishment of QCD

由于自相互作用的存在，量子色动力学QCD比量子电动力学QED有更丰富和不同的结构和性质，如色禁闭效应。

Puzzles in the development of QCD

Remember, to explain the short range of the nuclear force, Yukawa (1934) proposed that this force is mediated by the exchange of massive field quanta which he called mesons. In his theory, the range of the force is inversely proportional to the mass of the intermediate vector boson. He estimated a mass of about 140 MeV and indeed a candidate, the π meson, was later found in cosmic rays (1937)

Puzzles in the development of QCD

But, as we will see, massive gauge field quanta break the gauge symmetry so that the exchanged boson must necessarily be massless. For instance, the U(1) symmetry of the QED Lagrangian forces the photon to be massless, which indeed is. As a consequence the electromagnetic interaction has an infinite range.

Puzzles in the development of QCD

It follows that the SU(3) gauge symmetry of the QCD Lagrangian forces the gluons to be also massless, like the photon. But if these gluons are massless, how can the strong force then be short-range?

Puzzles in the development of QCD

Another puzzle came with a series of high-energy electron-proton scattering experiments at SLAC(1970) which proved the existence of quarks but also showed that they seemed to behave like free particles, in spite of the fact that they are strongly bound inside the proton

The establishment of QCD

The solution to both these paradoxes was found by Gross, Politzer and Wilczek by their discovery of asymptotic freedom. They could explain why, as Wilczek put it in his Nobel lecture, “Quarks are Born Free, but Everywhere They are in Chains"

The establishment of QCD

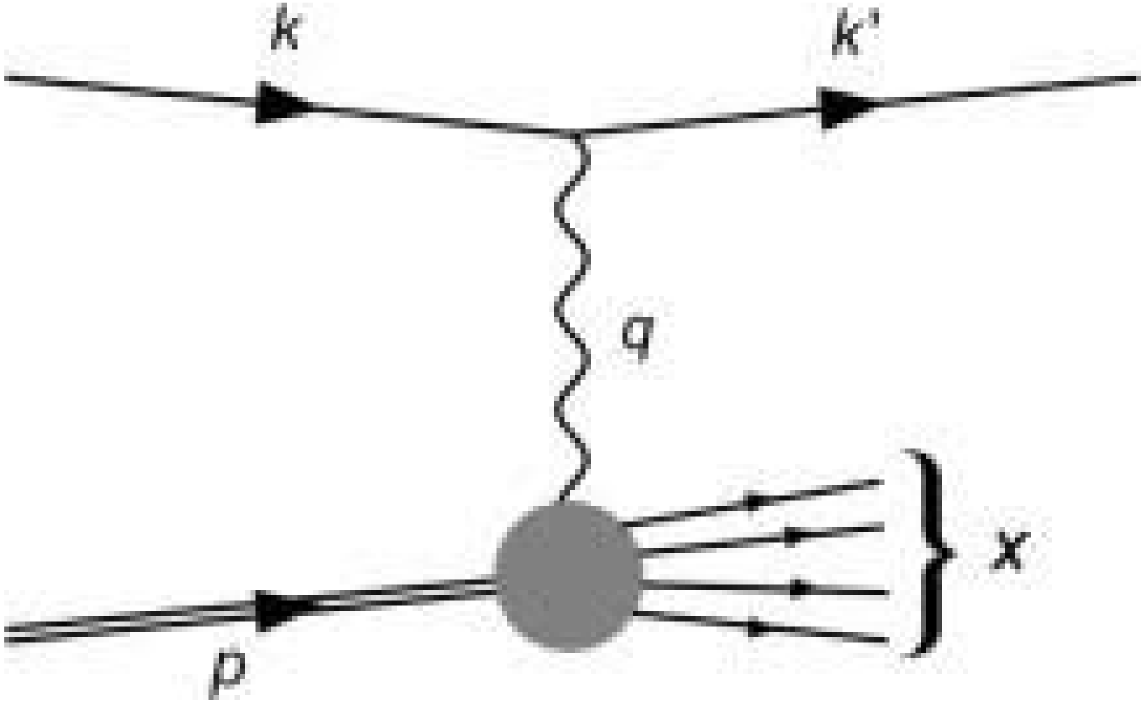
As we will see, the phenomena of asymptotic freedom and confinement are caused by the self-interaction of gluons which, in turn, is a consequence of the non-abelian nature of SU(3)

The establishment of QCD

- ✓ If quarks cannot be observed in isolation, how do we know that they actually exist and are not mere theoretical constructs?
- ✓ One way is to resolve quarks by illuminating protons with photons of large momentum Q and therefore small Compton wavelength 1/Q.

The establishment of QCD

These very short wavelength photons are radiated off highly energetic electrons when they scatter on protons



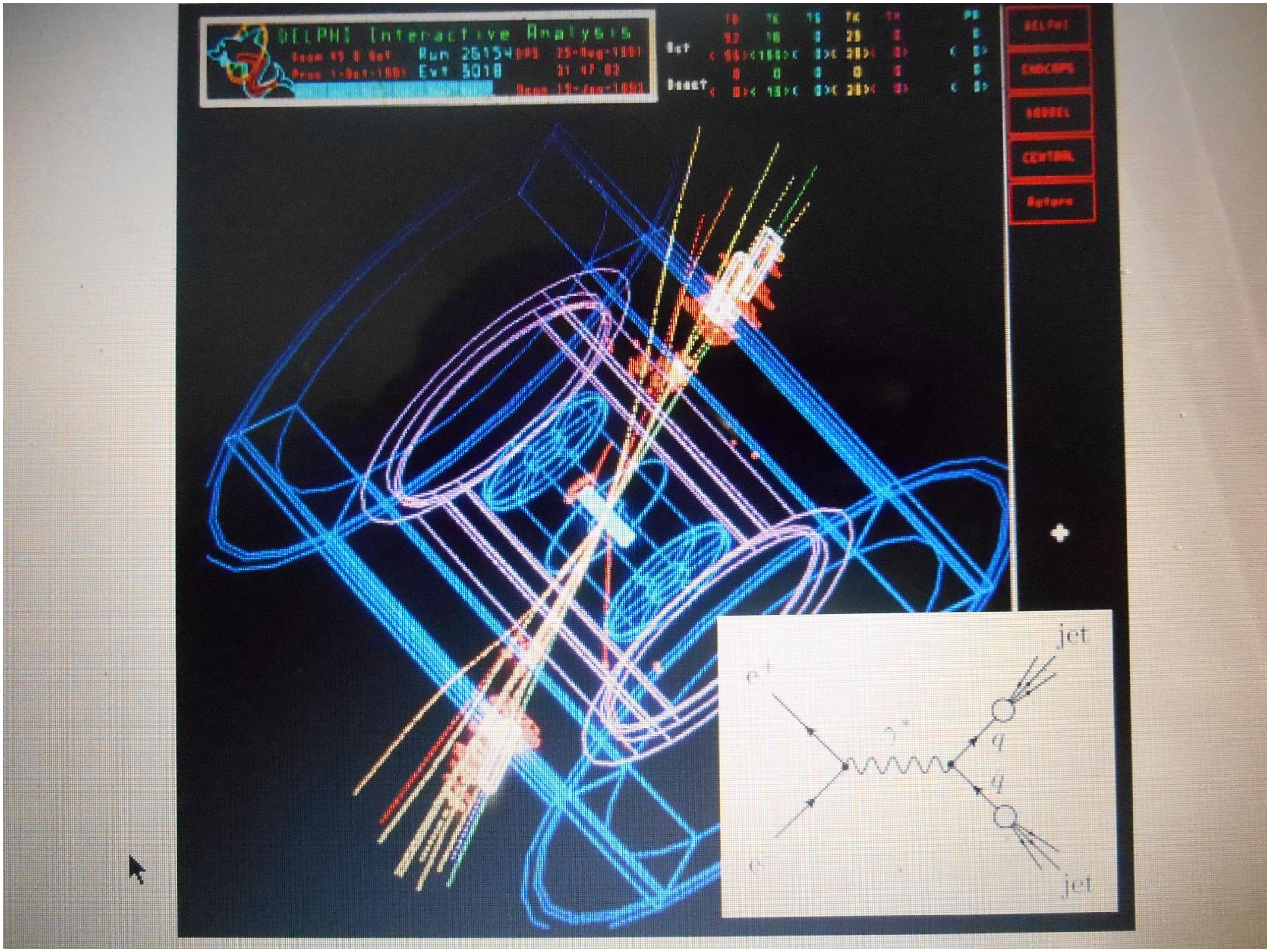
The establishment of QCD

This process is called deep inelastic scattering **which indeed acts as a microscope to reveal the internal quark structure of the proton.**

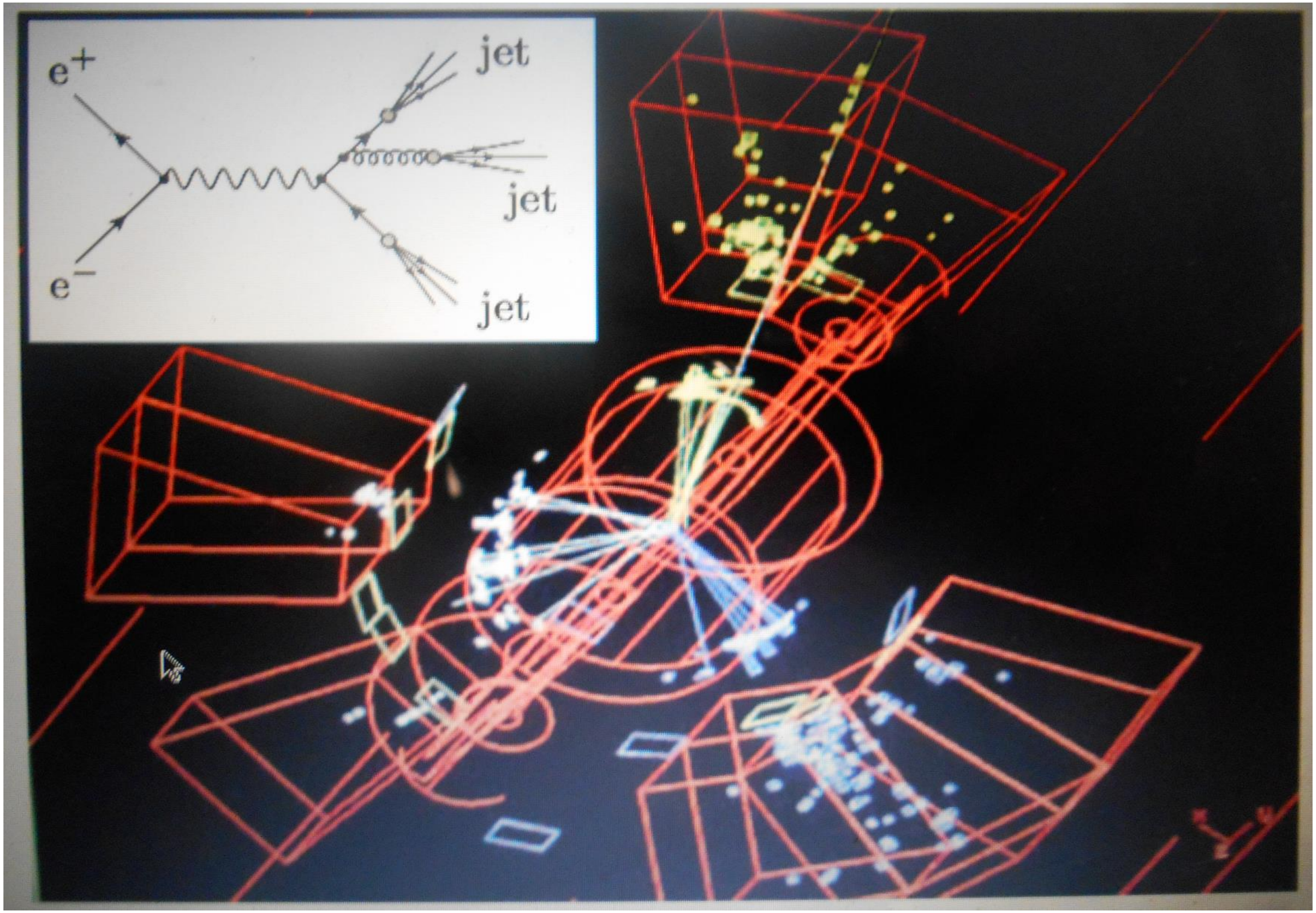
The establishment of QCD

Furthermore it turns out that highly energetic quarks produced in hard e^+e^-, $p\,\bar{p}$ and pp scattering hadronize into collimated sprays of particles (jets!). Thus we can more or less directly probe the dynamics of quarks by measuring jets in experiments at high energy colliders.

Two jets seeing quarks



Three jets seeing gluons



The establishment of QCD

Jet production is clearly a very important tool to confront QCD with experiment and can certainly produce spectacular events at colliders, but, unfortunately, this large field of jet physics can not be covered here.

A hypothetical 2 component Dirac field

Consider two fields ψ_1 and ψ_2 that obey the Dirac equations:

$$(i\not\partial - m_1)\psi_1 = 0 \quad \text{and} \quad (i\not\partial - m_2)\psi_2 = 0$$

The total Lagrangian is then simply the sum:

$$\mathcal{L} = \bar{\psi}_1(i\not\partial - m_1)\psi_1 + \bar{\psi}_2(i\not\partial - m_2)\psi_2$$

We introduce the compact notation:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2), \quad M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

The establishment of QCD

and set $m_1 = m_2$ so that $M = mI$, and write:

$$\mathcal{L} = \bar{\psi}(i \not{\partial} - m)\psi$$

but we have to remember that ψ and $\bar{\psi}$ are now 2-component objects, each component being itself a 4-component spinor.

We immediately see that $\mathcal{L}$ is invariant under a global unitary transformation $\psi' = U\psi$ in our 2-dimensional space because:

$$\bar{\psi}' \not{\partial}\psi' = \bar{\psi} \not{\partial}\psi, \quad \bar{\psi}'\psi' = \bar{\psi}\psi$$

Note that for $m_1 \neq m_2$ the term $\bar{\psi}M\psi$ would not be invariant because then $U^\dagger MU \neq MU^\dagger U$.

Yang-Mills Theory

The Yang-Mills theory describes pairs of spin-1/2 particles of equal mass, and Yang and Mills originally had the proton and neutron in mind as such a pair. A problem, however, is that the quanta of the Yang-Mills field must be massless in order to maintain gauge invariance.

Yang-Mills Theory

The massless quanta should have long-range effects, like the photon, and for this reason the theory was originally abandoned as a candidate theory of the strong interaction, which is short-range.

The establishment of QCD

However, the Yang-Mills theory reappeared because it serves as a prototype of non-Abelian gauge theories, that is, theories for which the generators of the underlying symmetry group do not commute

The establishment of QCD

Indeed, like SU(3) is a generalization of SU(2), we will see that QCD is a generalization of the originally proposed Yang-Mills model

Following we will first present the nuts and bolts of Yang-Mills theory as an important step towards building the QCD Lagrangian

The establishment of QCD

Recap of SU(2)

Unitary SU(2) matrix $U = \exp(i\alpha \cdot \sigma/2)$ with $U^{+}U = UU^{+} = 1$. Here, $\alpha = (\alpha_x, \alpha_y, \alpha_z)$ are parameters and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

With the generator being J_i = $\sigma_i/2$ satisfying $[J_i, J_j] = i\,\varepsilon_{ijk} J_k$

The establishment of QCD

How can we make the following Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not\partial - m)\psi \quad \text{with} \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \text{ and } \bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2)$$

invariant under local SU(2) transformations

$$\mathbf{U}(x) = \exp\left[-ig_w \sigma \cdot \alpha(x)\right]$$

Here g_w is some strength parameter

The establishment of QCD

As in the $U(1)$ case, we replace ∂_μ by a covariant derivative D_μ and require that the Lagrangian is invariant:

$$\mathcal{L}' = \bar{\psi} U^\dagger (i \not{D} - m) U \psi = \mathcal{L} = \bar{\psi} (i \not{D} - m) \psi$$

which is the case if $U^\dagger D'_\mu U = D_\mu$, or $D'_\mu U = U D_\mu$. In analogy with the $U(1)$ case we set:

$$D_\mu = \partial_\mu + i g_W \sigma \cdot \mathbf{A}_\mu$$

which introduces three gauge fields $\mathbf{A}_\mu = [(A^1)_\mu, (A^2)_\mu, (A^3)_\mu]$.

Note: extra gauge fields are required to accommodate higher symmetries in a Lagrangian

The establishment of QCD

Substituting $D_\mu = \partial_\mu + ig_W\sigma \cdot \mathbf{A}_\mu$, we finally get for our $SU(2)$ invariant Lagrangian:

$$\mathcal{L} = \bar{\psi}(i \not{D} - m)\psi = \bar{\psi}(i \not{\partial} - m)\psi - g_W \sum_{a,\mu} \left(\bar{\psi}\gamma^\mu\sigma^a\psi\right) \mathbf{A}_\mu^a$$

where the second term is the interaction term. Note the placement of the Dirac gamma matrices and the Pauli matrices! The story is not over: we still have to add a free term for the gauge fields $\mathbf{A}_\mu$:

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}\sum_{a=1}^{3}(F^a)^{\mu\nu}(F^a)_{\mu\nu} \equiv -\frac{1}{4}\mathbf{F}^{\mu\nu} \cdot \mathbf{F}_{\mu\nu}$$

The establishment of QCD

To finish the story, now we have to look for a definition of $F^a_{\mu\nu}$ that makes $\mathcal{L}_{\text{gauge}}$ invariant under the (infinitesimal) gauge transformation

$$A'^a_\mu \approx A^a_\mu + \partial_\mu \alpha^a + 2g_W \left(\alpha \times \mathbf{A}_\mu\right)^a$$

It can be shown (elaborate, but straight forward algebra) that the sought-after gauge field tensor is:

$$F^a_{\mu\nu} \equiv \partial_{[\mu} A^a_{\nu]} - 2g_W \left(\mathbf{A}_\mu \times \mathbf{A}_\nu\right)^a$$

Giving rise to the final $SU(2)$ invariant Lagrangian:

$$\mathcal{L}_{SU(2)} = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi - g_W \sum_{a=1}^{3} \left(\bar{\psi}\gamma^\mu \sigma^a \psi\right) A^a_\mu - \frac{1}{4}\sum_{a=1}^{3} (F^a)^{\mu\nu}(F^a)_{\mu\nu}$$

练习：证明如上拉氏量的SU(2)规范不变性

Global SU(3)$_c$ invariance

We are now ready to play the same game as before, but now with a difference: we will allow three spin-1/2 Dirac fields:

$$\psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix} \quad \text{and} \quad \bar{\psi} = (\bar{\psi}_r, \bar{\psi}_g, \bar{\psi}_b)$$

The free Lagrangian is, again,

$$\mathcal{L} = \bar{\psi}(i \not{\partial} - m)\psi$$

but we have to remember that ψ and $\bar{\psi}$ represent 3-component objects, with colour index (r, g, b), and that each component is by itself a 4-component Dirac spinor.

The establishment of QCD

For simplicity, we have assumed that quarks of all flavors are identical by having the same mass m. This is not true, of course, and we should introduce a flavor index f = (d, u, s, c, b, t), and different masses m_f

The establishment of QCD

This Lagrangian is manifestly invariant under U(3) = U(1)×SU(3) global transformations. The U(1) phase invariance was already explored so we only investigate here SU(3) invariance:

$$\psi' = U\psi \quad \text{and} \quad \bar{\psi}' = \bar{\psi} U^{\dagger}$$

The establishment of QCD

To make $\mathcal{L}$ invariant under local SU(3) transformations is now a relatively easy task since we can just replace the 2×2 SU(2) matrices in the Yang-Mills theory by 3×3 SU(3) matrices!

Local SU(3)$_c$ invariance

We want to make the Lagrangian invariant under local $SU(3)$ transformations (g_s is the strong coupling constant)

$$U(x) = \exp[ig_s \lambda \cdot \alpha(x)]$$

Here we have eight angles $\alpha = (\alpha_1, ..., \alpha_8)$ and the eight Gell-Mann matrices $\lambda = (\lambda_1, ..., \lambda_8)$ that we already discussed earlier.

We can now simply repeat the steps made in the Yang-Mills theory and define the covariant derivative:

$$D_\mu = \partial_\mu + ig_s \lambda \cdot \mathbf{A}_\mu$$

where we have now 8 gauge fields $\mathbf{A}_\mu = (A^1_\mu, ..., A^8_\mu)$. These are the eight gluons that we alluded to earlier.

The establishment of QCD

For infinitesimal transformations, the gauge fields transform as:

$$A'^{a}_{\mu} \approx A^{a}_{\mu} + \partial_{\mu}\alpha^{a} + 2g_{s}(\alpha \times \mathbf{A}_{\mu})^{a}$$

but here we have to use the general expression for the cross product

$$(\mathbf{a} \times \mathbf{b})^{i} = f^{ijk} a^{j} b^{k}$$

where summation over j and k is implied and f^{ijk} are the structure constants of $SU(3)$

Now following similar logic as before, the gauge field tensor is given by

$$F^{a}_{\mu\nu} = \partial_{[\mu}A^{a}_{\nu]} - 2g_{s}(\mathbf{A}_{\mu} \times \mathbf{A}_{\nu})^{a}$$

where we have to take the $SU(3)$ cross product for the last term.

The establishment of QCD

Now we are ready to write the QCD Lagrangian. This is given by

$$\mathcal{L}_{\rm QCD} = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi - g_s \sum_{a=1}^{8} \left(\bar{\psi}\gamma^{\mu}\lambda^{a}\psi\right) A_{\mu}^{a} - \frac{1}{4}\sum_{a=1}^{8}(F^{a})^{\mu\nu}(F^{a})_{\mu\nu}$$

$$+ \ \text{extra pieces}$$

The extra pieces in the QCD Lagrangian given above are related to the so-called gauge-fixing terms and ghost fields which must be introduced to make the theory consistent.

The establishment of QCD

We have now eight color fields $\mathbf{A}_\mu$ (gluon fields) and eight color currents $\mathbf{j}_\mu = g_s \bar{\psi} \gamma^\mu \lambda \psi$ that act as sources for the color fields, like the electric current is the source for the electromagnetic field.

The first term is the free Dirac Lagrangian, just like in QED. It will give rise to quark propagators.

The establishment of QCD

The second term also looks familiar: it is an interaction term that gives rise to the quark-gluon vertex. Using quark color (i, j), gluon color (a, b), and spinor (α, β) notations, we have:

$$\bar{\psi}^{i}{}_{\alpha}\gamma^{\mu}_{\alpha\beta}\lambda^{a}{}_{ij}\psi^{j}{}_{\beta}A^{a}{}_{\mu}$$

练习：Where are the quark flavor DoFs

The establishment of QCD

The last term is the free Lagrangian of the gluon fields, which also looks familiar from QED, but has a much richer structure. It gives rise to the gluon propagator, but also to 3- and 4-gluon vertices, which is something that does not exist in QED.

Indeed, these gluon self-interactions are responsible for a characteristic feature of QCD interactions: asymptotic freedom.

The establishment of QCD

Since quarks come in three colors i = (r , g, b) so that the wave function can be written as:

$$\psi_i = \begin{cases} c_i u_f^{(s)}(p^\mu) & \text{incoming quark} \\ c_i \bar{u}_f^{(s)}(p^\mu) & \text{outgoing quark} \\ c_i \bar{v}_f^{(s)}(p^\mu) & \text{incoming antiquark} \\ c_i v_f^{(s)}(p^\mu) & \text{outgoing antiquark} \end{cases}$$

In above, the Lorentz index μ=(0, 1, 2, 3), the spin index s = (1, 2) = (up, down) and the flavor index f = (d, u, s, c, b, t)

练习：Find the expressions for the 4-component spinors u and v

The establishment of QCD

The colour index $i = (1, 2, 3) = (r, g, b)$ is taken care of by defining the following basis vectors in colour space

$$c_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad c_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad c_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

for red, green and blue, respectively. The Hermitian conjugates $c^\dagger$ are just the corresponding row vectors.

A colour transition like $\psi_r \rightarrow \psi_g$ can now be described as an $SU(3)$ matrix operation in colour space. For example $\psi_g = (\lambda_1 - i\lambda_2)\psi_r$, i.e

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

The establishment of QCD

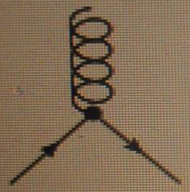
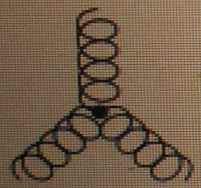
The QCD Lagrangian with color, flavor, spinor and Lorentz indices may be expressed as:

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}^{if}_{\alpha}(i\gamma^{\mu}_{\alpha\beta}\partial_{\mu} - m_f\delta_{\alpha\beta})\psi^{if}_{\beta} - g_s \sum_f \left(\bar{\psi}^{if}_{\alpha}\gamma^{\mu}_{\alpha\beta}\lambda^{a}_{ij}\psi^{jf}_{\beta}\right) A^{a}_{\mu}$$

$$-\frac{1}{4}(F^a)^{\mu\nu}(F^a)_{\mu\nu} + \text{ extra pieces}$$

Leading to the following Feynman vertices that may be read up from the Lagrangian (suppressing the flavor and the spinor indices)

The establishment of QCD

	$\bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i$	quark propagator
	$(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$	gluon propagator
	$g_s\, \bar{\psi}_i \lambda^a_{ij} \psi_j \gamma^\mu A^a_\mu$	quark-gluon verte
	$g_s\, (\partial^\mu A^\nu_a - \partial^\nu A^\mu_a) f_{abc} A^b_\mu A^c_\nu$	3-gluon vertex
	$g_s^2\, f_{abc} A^\mu_b A^\nu_c\, f_{ade} A^d_\mu A^e_\nu$	4-gluon vertex

The SU(3)$_c$ matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

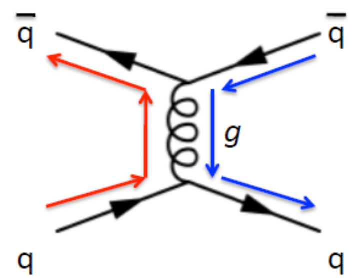
$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

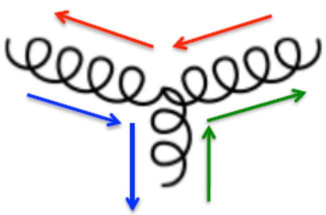
The establishment of QCD

QCD is about the theory of strong interactions between quarks and gluons

Coupling is to three "color" charges r(ed), b(lue) and g(reen)

Gluons carry color---anticolor charges and have self-interactions





The establishment of QCD

Gluon Colour States

Naively there are 9 states: $r\bar{r}, b\bar{b}, g\bar{g}, r\bar{b}, b\bar{r}, r\bar{g}, g\bar{r}, b\bar{g}, g\bar{b}$

In SU(3) these are arranged into a **colour octet** (allowed for gluons):

$G_1 = 1/\sqrt{2}\,[\,r\bar{b} + b\bar{r}\,]$ $G_2 = 1/\sqrt{2}\,[\,r\bar{b} - b\bar{r}\,]$

$G_4 = 1/\sqrt{2}\,[\,r\bar{g} + g\bar{r}\,]$ $G_5 = 1/\sqrt{2}\,[\,r\bar{g} - g\bar{r}\,]$

$G_6 = 1/\sqrt{2}\,[\,b\bar{g} + g\bar{b}\,]$ $G_7 = 1/\sqrt{2}\,[\,b\bar{g} - g\bar{b}\,]$

$G_3 = 1/\sqrt{2}\,[\,r\bar{r} - b\bar{b}\,]$ $G_8 = 1/\sqrt{6}\,[\,r\bar{r} + b\bar{b} - 2g\bar{g}\,]$

and a **colour singlet** which is symmetric (forbidden for gluons) :

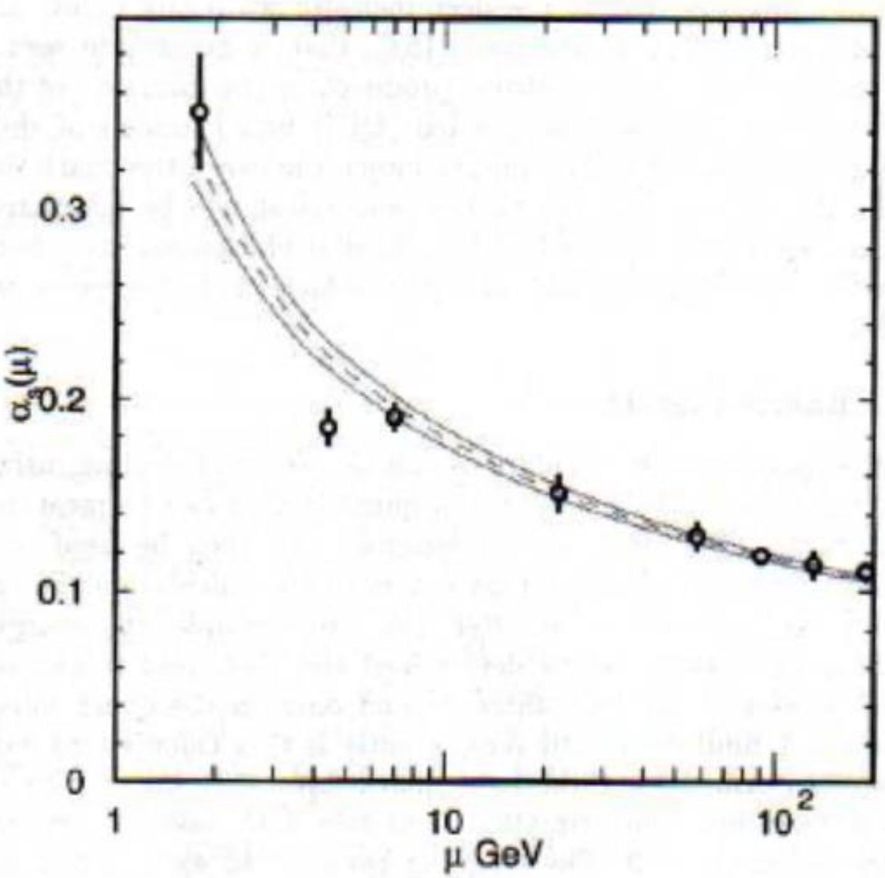
$G_0 = 1/\sqrt{3}\,[\,r\bar{r} + b\bar{b} + g\bar{g}\,]$ 练习：Why?

Contents:

- Before the advent of QCD
- The establishment of QCD
- Typical natures of QCD
- Applications of QCD
- Concluding remarks

Typical natures of QCD

Strong Coupling α_s



$\alpha_s = g_s^2$ is a steep function of energy scale μ (renormalization scale)

$\alpha_s \sim 1$ at low energy

$\alpha_s \sim 0.1$ at $\mu = M_Z$

Typical natures of QCD

Description of Running of α_s

Reminder – running of α_{EM} was attributed to screening of electric charge by fermion-antifermion pairs:

$$\alpha_{EM}(q^2) = \alpha(\mu^2)\left(1 - \alpha(\mu^2)\,\frac{z_f}{3\pi}\ln\left(|q|^2/\mu^2\right)\right)^{-1}$$

Running of α_s is attributed to:

1) Screening of colour charge by quark-antiquark pairs
2) Anti-screening of colour charge by gluons

$$\alpha_s(q^2) = 12\pi\left(\left(33 - 2N_f\right)\ln\left(|q|^2/\Lambda^2\right)\right)^{-1}$$

$\Lambda_{QCD} \sim 220$ MeV

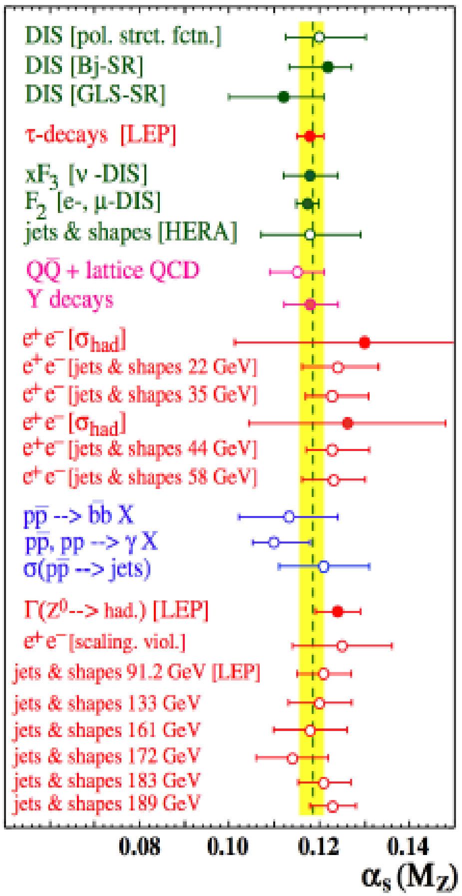
$$\ln(\Lambda^2) = \ln(\mu^2) - 12\pi\left(\left(33 - 2N_f\right)\alpha_s(\mu^2)\right)^{-1}$$

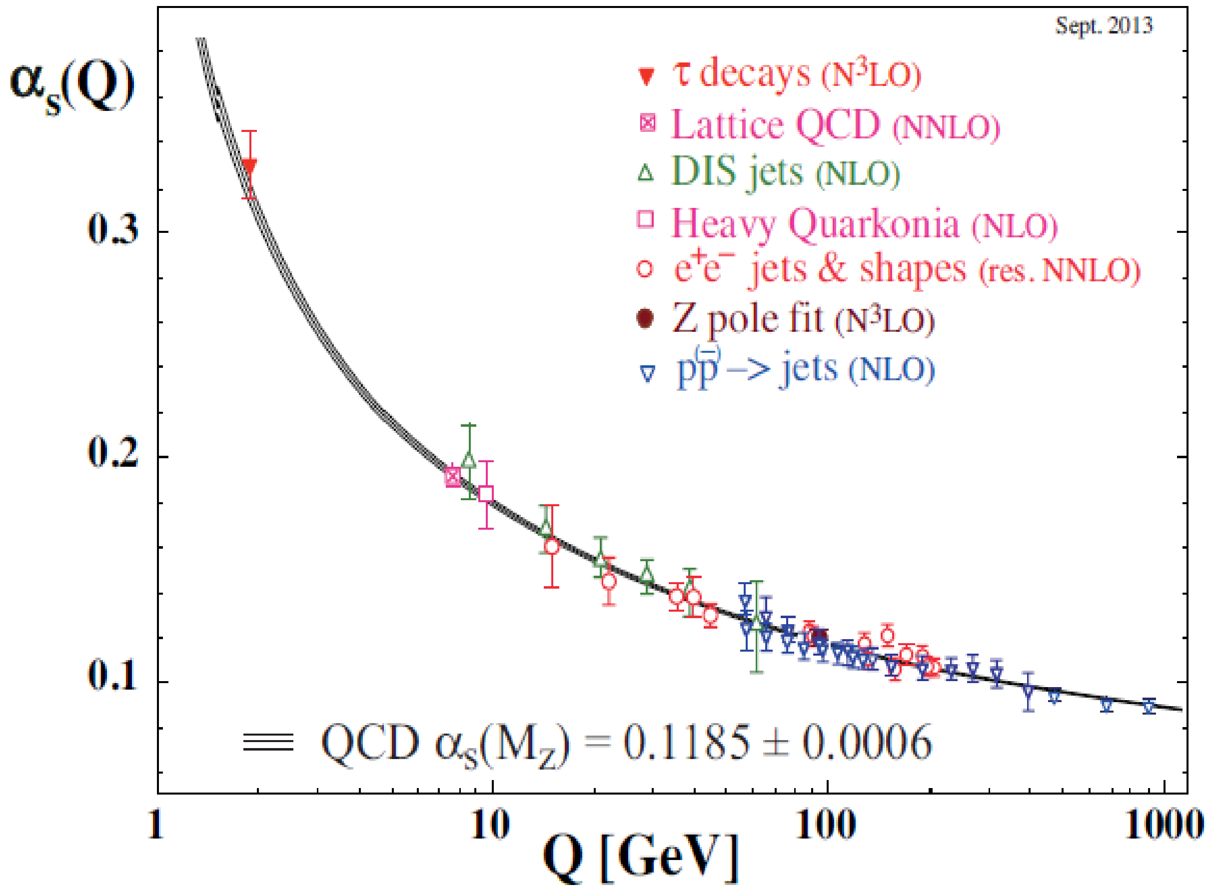
N_f = 2-6 is the number of "active" quark flavours (depends on q^2)

Anti-screening by gluons dominates

Leads to a decrease of α_s as a function of q^2

Typical natures of QCD





arXiv: 1506.05407

Asymptotic freedom of QCD

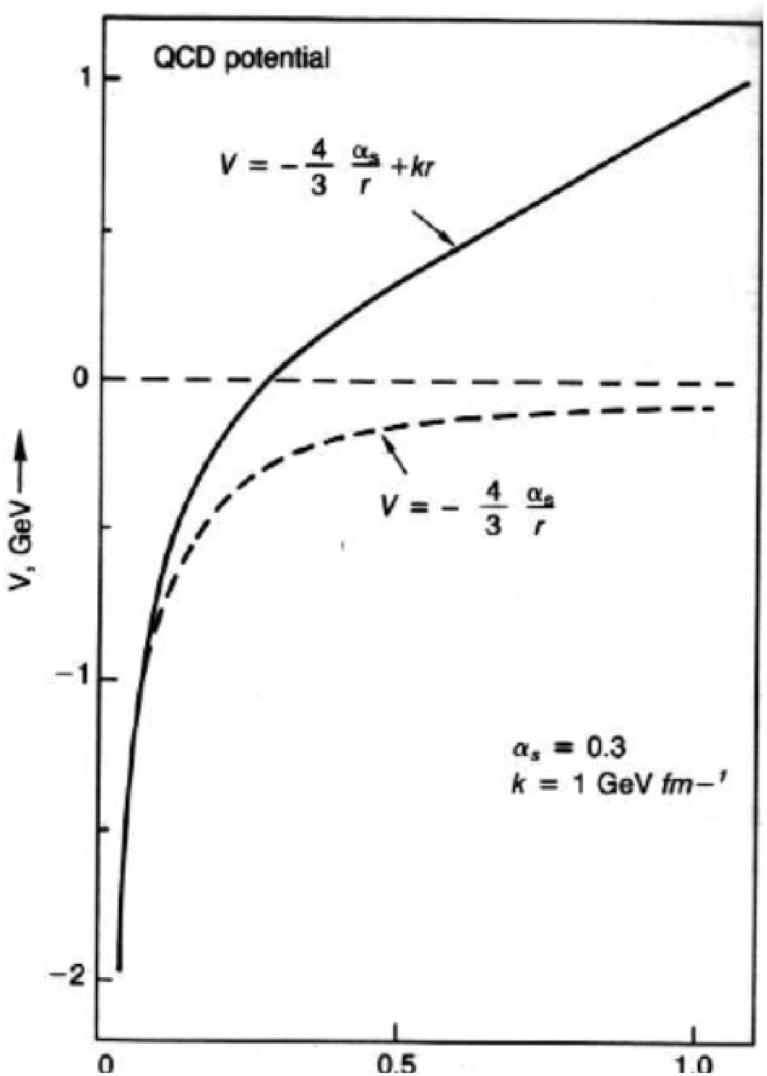
- At large $q^2 >> \Lambda^2$ the strong coupling α_s is significantly < 1
  - In this limit it is possible to calculate strong amplitudes perturbatively
  - Sum over expansion in powers of α_s : leading order, next-to-leading order (NLO), next-to-next-to-leading order (NNLO) …
- Large q^2 corresponds to short distances << 1 fm
  - Cannot describe mesons and baryons perturbatively
  - Can describe high energy collisions perturbatively
  - Heavy quark decays (Lecture 11) are somewhere between these limits
  - Fragmentation of partons to form hadronic jets (Lecture 10) is another example of an intermediate case
- Deep inside hadrons the quarks and gluons do behave like free particles. Hence the validity of the parton model for high energy proton collisions.

Typical natures of QCD

Color Confinement

- At small $q^2 \sim \Lambda^2$ the strong coupling α_s is large (and diverging)
  - In this limit it is not possible to calculate strong amplitudes perturbatively
- Corresponds to distances ~ 1 fm
  - This is the size of mesons and baryons
- Non-perturbative calculations of strong interactions are done using numerical methods (Lattice QCD)
- The quarks and gluons are no longer free particles inside hadrons
  - What is the mechanism that confines them?
  - Why are mesons and baryons the only allowed bound states?
  - Why are strong interactions between hadrons short range (~1fm) even though the gluon is massless?
- Consider a set of models of confinement ...

Typical natures of QCD



QCD Potential

Short distance part (1/r term)
from quark-antiquark gluon exchange

$$V(q\bar{q}) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Long distance part (k r term)
is modelled on an elastic spring

k is known as the string tension

This model provides a good description
of the bound states of heavy quarks:
charmonium ($c\bar{c}$)
bottomonium ($b\bar{b}$)

Contents:

- Before the advent of QCD
- The establishment of QCD
- Typical natures of QCD
- **Applications of QCD**
- Concluding remarks

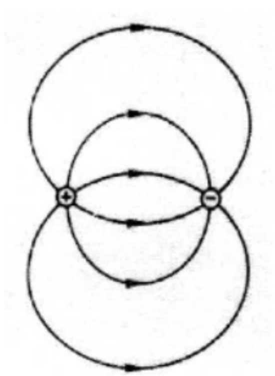
Applications of QCD

Colour Flux-tube Model

QED

Field lines extend out to infinity with strength $1/r^2$

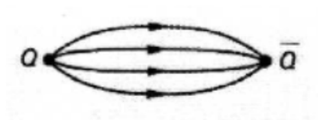
Electromagnetic flux conserved to infinity



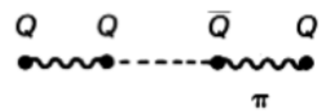
QCD

Field lines are compressed into region between quark and antiquark

Colour flux is confined within a tube. No strong interactions outside the flux-tube .



Breaking a flux tube requires the creation of a quark-antiquark pair



Like breaking a string! Requires energy to overcome string tension

Applications of QCD

Valence Quark Model

- Mesons are quark-antiquark bound states
  - Symmetric colour singlet state:

$$1/\sqrt{3}\ [\, r\bar{r} + b\bar{b} + g\bar{g} \,]$$

  - Colour singlet does not couple to a gluon (no G_0 gluon state)
- Baryons are three quark bound states
  - Antisymmetric colour singlet state:

$$1/\sqrt{6}\ [\, rgb - rbg + brg - bgr + gbr - grb \,]$$

  - Also does not couple to gluon because colour singlet
- Gluon exchanges only occur inside mesons and baryons
- Model ignores sea quarks and gluons (they don't matter at low q^2)
- Are there other types of colour singlet bound states?
  - Some evidence for glueballs (gg, ggg) as predicted by Lattice QCD
  - Hybrid mesons ($q\bar{q}g$), four quark states ($q\bar{q}\,q\bar{q}$), Pentaquarks ($qqq\,q\bar{q}$) ?

5/2/10 Particle Physics Lecture 8 Steve Playfer 16

Applications of QCD

- **A Brief Review of Glueball Studies**
- 0^{--} Oddballs via QCDSR
- Hunting for Oddballs
- Concluding remarks

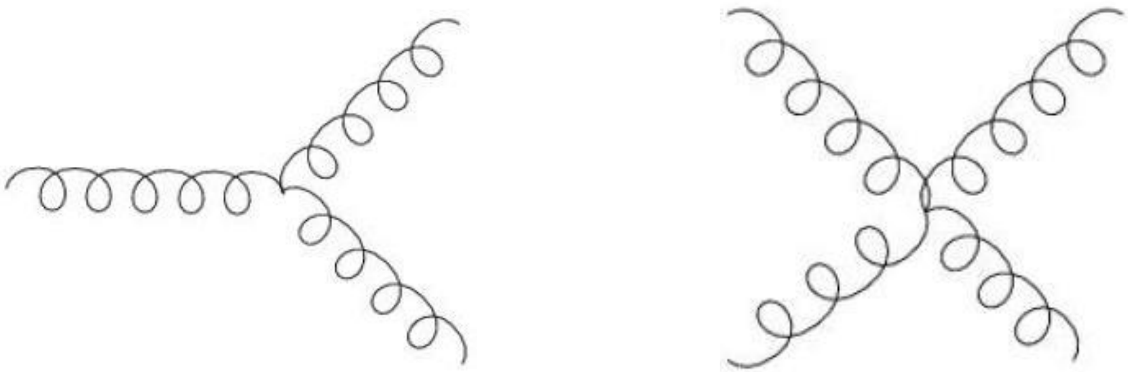
I. A Brief Review of Glueball Studies

The QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\gamma^\mu D_\mu - m_q)\psi_q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

There exist gluon self-interactions



I. A Brief Review of Glueball Studies

- **Color structure**
  - **Quark= fundamental representation 3**
  - **Gluon= Adjoint representation 8**
  - **Observable particles=color singlet 1**
    - **Mesons** $3 \bigotimes 3 = 1 \bigoplus 8$
    - **Baryons** $3 \bigotimes 3 \bigotimes 3 = 1 \bigoplus 8 \bigotimes 8 \bigoplus 10$
    - **Glueballs** $\begin{cases} 8 \bigotimes 8 = 1 \bigoplus 8 \bigoplus 8 \bigoplus 10 \bigoplus \bar{10} \bigoplus 27 \\ 8 \bigotimes \cdots \bigotimes 8 = 1 \bigoplus 8 \bigoplus \cdots \end{cases}$

I. A Brief Review of Glueball Studies

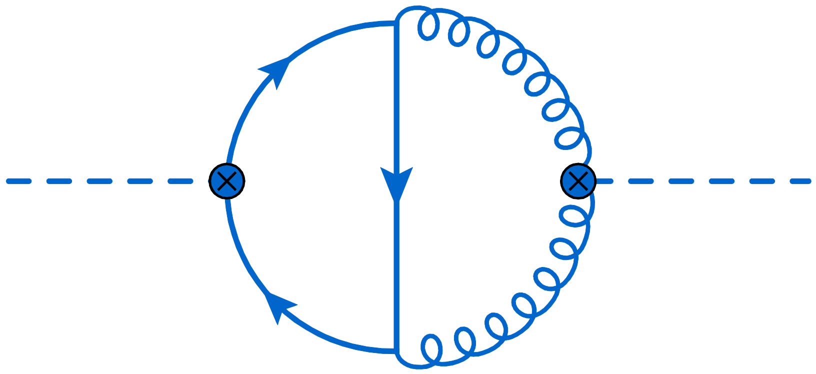
- Glueballs are allowed by QCD
- No definite observation in experiment up to now

The main difficulties in observing glueballs lie in

- lack of the knowledge about their production & decay properties
- mixing with quark states adds difficulty to isolate them.

I. A Brief Review of Glueball Studies

➢ **Typical meson-glueball mixing**

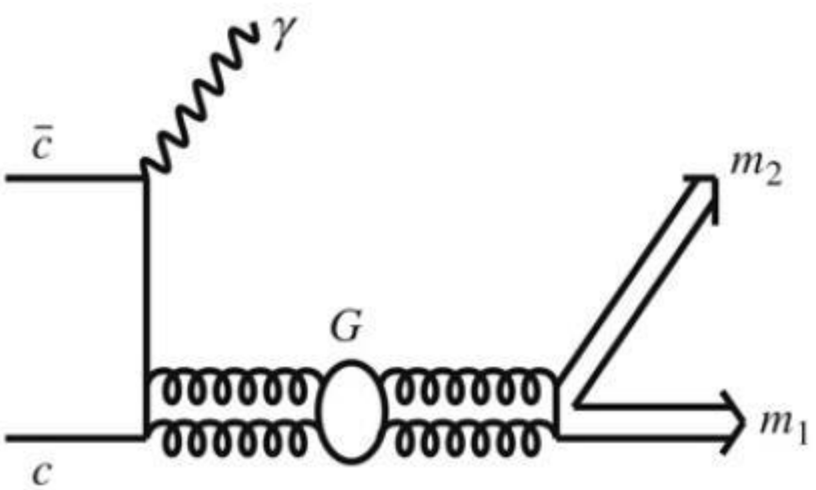


It might be ture: No glueball-dominated states below 2 GeV

I. A Brief Review of Glueball Studies

➢ **Gluon-rich processes (Taking gg as an example)**

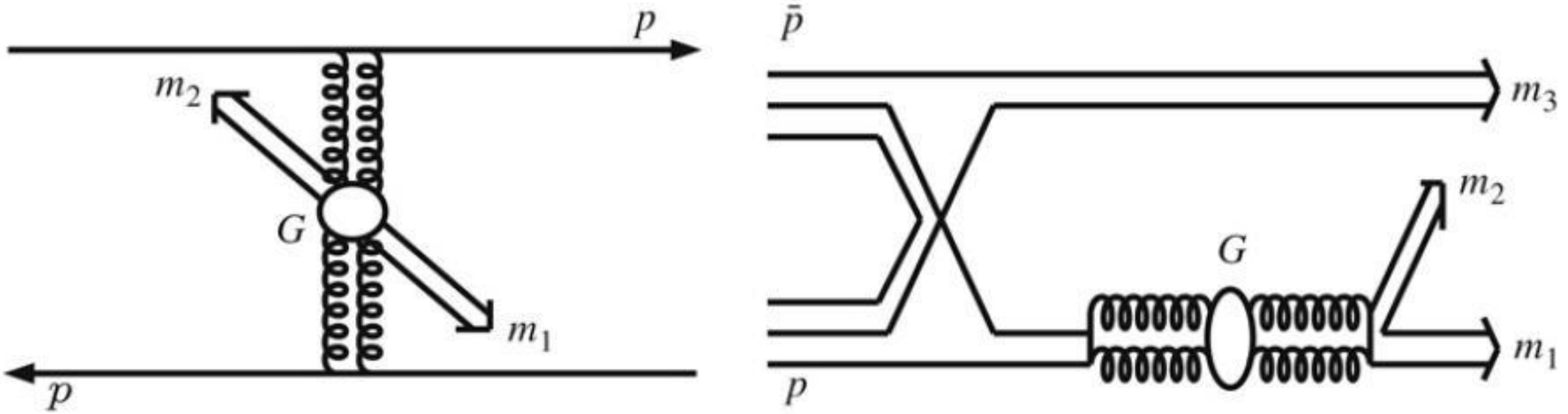
The most prominent example in e^+e^- colliders.



I. A Brief Review of Glueball Studies

- **Gluon-rich processes (Taking gg as an example)**

Very promising examples in hadron colliders.



V.M.Abazov *et al.*[D0 and TOTEM], arXiv:2012.03981

I. Glueballs and Glueball Studies

- **Good evidence exists for the lightest scalar guleball 0^{++} , which however mixes with nearby mesons. There are several candidates, e.g. $f_0(980)$, $f_0(1500)$, $f_0(1710)$, but no definitive conclusions can be drawn concerning the nature of these states.**
- **Evidence for tensor 2^{++} and pseudoscalar 0^{-+} glueballs are weak**
- **The study of the oddballs in experiment is very limited**

To pin down a glueball in experiment is a challenging task

V. Crede and C.A. Meyer, Prog. Part. Nucl. Phys. 63(2009) 74-116, and refs. therein

I. A Brief Review of Glueball Studies

- **Theoretically:**
  - **Lattice QCD**
  - **Flux tube model**
  - **MIT bag model**
  - **Coulomb gauge model**
  - **QCD Sum Rules (QCDSR)**

} **Constituent Models**

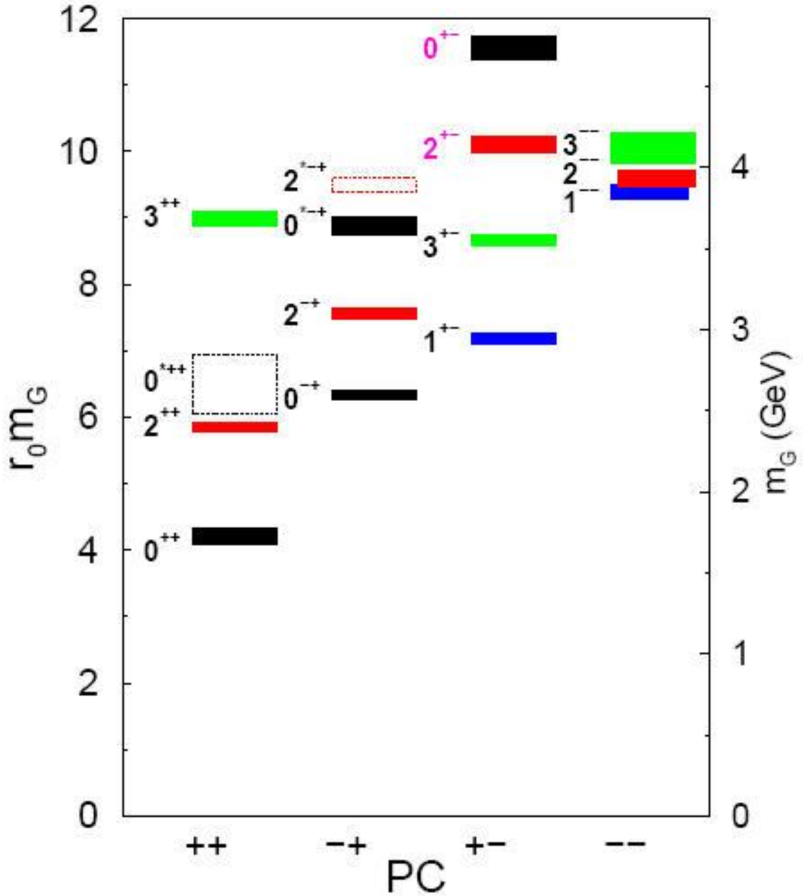
V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009)

Morningstar & Peardon, PRD60 (1999) 034509

- **Results from Lattice QCD**

J^{PC}	Other J	$r_0 m_G$	m_G (MeV)
0^{++}		4.21 (11)(4)	1730 (50)(80)
2^{++}		5.85 (2)(6)	2400 (25)(120)
0^{-+}		6.33 (7)(6)	2590 (40)(130)
0^{*++}		6.50 (44)(7)†	2670 (180)(130)
1^{+-}		7.18 (4)(7)	2940 (30)(140)
2^{-+}		7.55 (3)(8)	3100 (30)(150)
3^{+-}		8.66 (4)(9)	3550 (40)(170)
0^{*-+}		8.88 (11)(9)	3640 (60)(180)
3^{++}	$6,7,9,\ldots$	8.99 (4)(9)	3690 (40)(180)
1^{--}	$3,5,7,\ldots$	9.40 (6)(9)	3850 (50)(190)
2^{*-+}	$4,5,8,\ldots$	9.50 (4)(9)†	3890 (40)(190)
2^{--}	$3,5,7,\ldots$	9.59 (4)(10)	3930 (40)(190)
3^{--}	$6,7,9,\ldots$	10.06 (21)(10)	4130 (90)(200)
2^{+-}	$5,7,11,\ldots$	10.10 (7)(10)	4140 (50)(200)
0^{+-}	$4,6,8,\ldots$	11.57 (12)(12)	4740 (70)(230)

$$r_0^{-1} = 410 \pm 20 \text{ MeV}$$



I. A Brief Review of Glueball Studies

- **Results of Lattice QCD**

R^{PC}	Possible J^{PC}	$r_0 M_G$	$r_0 M_G$
A_1^{++}	0^{++}	4.16(11)	4.21(11)
E^{++}	2^{++}	5.82(5)	5.85(2)
T_2^{++}	2^{++}	5.83(4)	5.85(2)
A_2^{++}	3^{++}	9.00(8)	8.99(4)
T_1^{++}	3^{++}	8.87(8)	8.99(4)
A_1^{-+}	0^{-+}	6.25(6)	6.33(7)
T_1^{+-}	1^{+-}	7.27(4)	7.18(3)
E^{-+}	2^{-+}	7.49(7)	7.55(3)
T_2^{-+}	2^{-+}	7.34(11)	7.55(3)
T_2^{+-}	3^{+-}	8.80(3)	8.66(4)
A_2^{+-}	3^{+-}	8.78(5)	8.66(3)
T_1^{--}	1^{--}	9.34(4)	9.50(4)
E^{--}	2^{--}	9.71(3)	9.59(4)
T_2^{--}	2^{--}	9.83(8)	9.59(4)
A_2^{--}	3^{--}	10.25(4)	10.06(21)
E^{+-}	2^{+-}	10.32(7)	10.10(7)
A_1^{+-}	0^{+-}	11.66(7)	11.57(12)

Chen *et al.*, PRD73(2006) 014516

Morningstar & Peardon, PRD60 (1999) 034509

Mass(0^{--}) = (5166 ± 1000) MeV (Unquenched)

Gregory, *et al.*, JHEP1210 (2012) 170.

I. A Brief Review of Glueball Studies

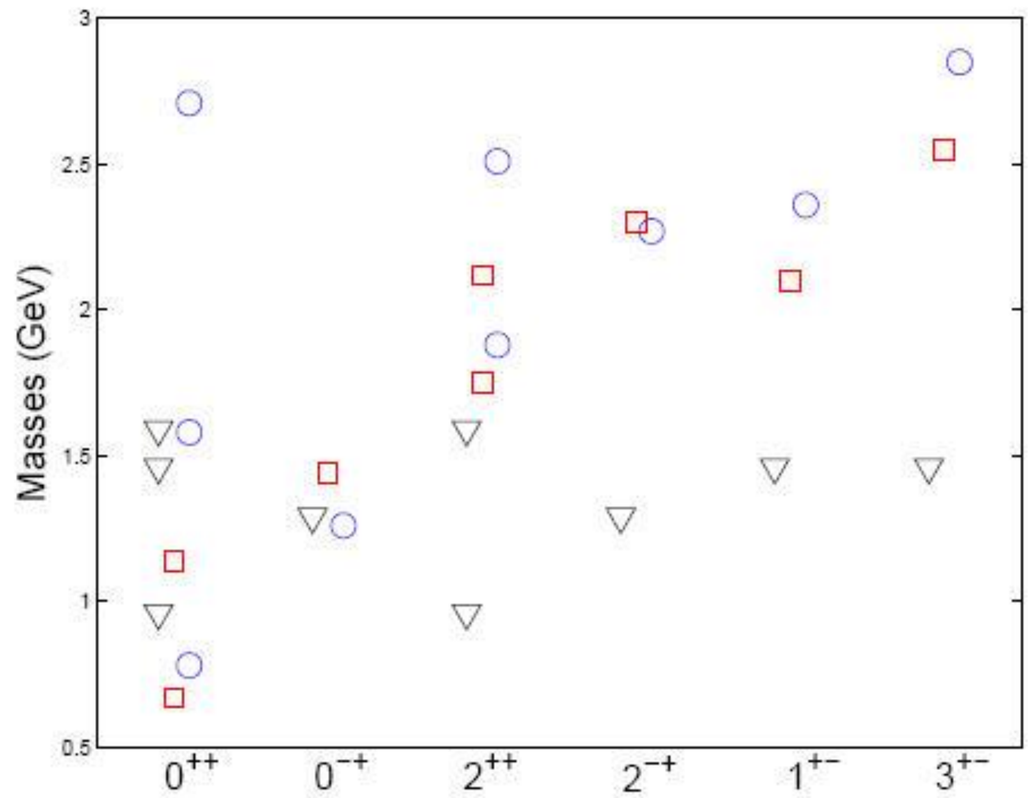
- **Flux tube model**

J^{PC}	Mass (GeV)
0^{++}	1.52
1^{+-}	2.25
0^{++}	2.75
$0^{++}, 0^{+-}, 0^{-+}, 0^{--}$	2.79
2^{++}	2.84
$2^{++}, 2^{++}, 2^{++}, 2^{++}$	2.84
1^{+-}	3.25
3^{+-}	3.35

Isgur & Parton, PRD31(1985)2910.

I. A Brief Review of Glueball Studies

- **MIT bag model**



▽ **=Jaffe &Johnson, PLB60,201(1976).**

○ **=Carlson *et al.*, PRD27 (1983)1556.**

□ **=Chanowitz &Sharpe, NPB222(1983)211.**

I. A Brief Review of Glueball Studies

- **Coulomb Gauge model**

Model	J^{PC}	0^{-+}	1^{--}	2^{--}	3^{--}	5^{--}	7^{--}
	color	f	d	d	d	d	d
	S	0	1	2	3	3	3
	L	0	0	0	0	2	4
H^g_{eff} (this work)		3900	3950	4150	4150	5050	5900
H_M (this work)		3400	3490	3660	3920	5150	6140

Llances-Estrada, Bicudo &Cotanch, PRL96 (2006) 081601

I. A Brief Review of Glueball Studies

- **QCD Sum Rules**

Two-gluon glueballs in QCDSR

	Novikov *et.al.*	Forkel	Bagan *et.al.*	Huang *et.al*
0^{++}	0.7-0.9 GeV	1.25 GeV	1.7 GeV	1.66 GeV
0^{-+}	-	2.2 GeV	-	-

Novikov et al., NPB165 (1980) 67.

Bagan&Steele, PLB243 (1990) 43.

Forkel, PRD64 (2001) 034015.

Huang, Jin&Zhang, PRD59 (1999) 034026.

S. Narison, Phys.Lett.B 665 (2008) 205

I. A Brief Review of Glueball Studies

- **QCD Sum Rules**

Tri-gluon glueballs in QCDSR

	0^{++}	0^{-+}	1^{-+}	1^{--}	2^{++}
Latorre *et. al.*	3.1 GeV	-	-	-	-
Liu *et. al.*	1.45 GeV	-	1.87 GeV	2.4 GeV	2.0 GeV
Hao *et. al.*	-	1.9-2.7 GeV	-	-	-

Latorre *et al*., PLB191 (1987) 437.

Liu, CPL15 (1998) 784.

G. Hao, CFQ, A.L. Zhang, PLB642 (2006) 53.

II. Oddballs via QCDSR

- Glueballs and Glueball Studies
- **Oddballs via QCDSR**
- Hunting for Oddballs
- Concluding remarks

II. Oddballs via QCDSR

- **Oddballs**

Oddballs: glueballs with exotic quantum numbers

$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}$ and so on

Trigluon glueballs

H. Chen, et al, 2103.17201; Alexandr Pimikov, et al, 1611.08691

V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009)

- $C = -1 \rightarrow$ **Trigluon glueballs.**
- **Exotic quantum numbers ➡ Do not mix with** $q\bar{q}$
- 0^{--} **oddball may be the lowest lying one. Besides, it has the simplest Lorentz structure.**

II. Oddballs via QCDSR

- **It can be produced in the decay of heavy vector quarkonium or quarkoniumlike states.**

[yellow] = exotic state (not easy to be detected, only $\pi_1(1400)$ with 1^{-+} in PDG)

[red] = unfavorable production channel

Like in football game [yellow] + [yellow] = [red]

$$1^{--} \to G_{0^{--}} + 1^{++}$$

$$1^{--} \to G_{0^{+-}} + 1^{-+}$$

$$1^{--} \to G_{1^{-+}} + 0^{--}$$

$$1^{--} \to G_{2^{+-}} + 1^{-+}/2^{-+}/3^{-+}$$

$$1^{--} \to G_{3^{-+}} + 2^{+-}$$

$$1^{++} = f_1(1285)/\chi_{c1}(3511)$$

II. Oddballs via QCDSR

➢ **QCDSR**

CFQ & Liang Tang, PRL113 (2014) 221601

CFQ & Liang Tang, NPB904 (2016) 282

➢ Field strength tensor

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(x) + g_s f^{abc} A^b_\mu(x) A^c_\nu(x)$$

➢ In coordinate gauge

$$A^a_\mu(x) \simeq \frac{1}{2} x^\nu G^a_{\nu\mu}(0); \qquad A^a_\mu(0) \simeq 0$$

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(0) + \frac{1}{4} g_s f^{abc} x^\rho x^\sigma G^b_{\rho\mu}(0) G^c_{\sigma\nu}(0)$$

$$G^a_{0\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x)$$

$$G^a_{\mu\nu}(0) = G^a_{0\mu\nu}(0) = G^a_{0\mu\nu}(x)$$

II. Oddballs via QCDSR

- Some contractions

$$G^a_{0\mu\nu}(x)G^i_{0\alpha\beta}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\Gamma_{\mu\nu\alpha\beta}(p)e^{-ip\cdot(x-y)}$$

$$\Gamma_{\mu\nu\alpha\beta}(p) = p_\mu p_\alpha g_{\nu\beta} + p_\nu p_\beta g_{\mu\alpha} - p_\mu p_\beta g_{\nu\alpha} - p_\nu p_\alpha g_{\mu\beta}$$

$$A^m_\mu(x)G^j_{0\beta\gamma}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{\delta^{mj}}{p_1^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$G^j_{0\beta\gamma}(x)A^m_\mu(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-\delta^{jm}}{p^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$\widetilde{G}^a_{0\mu\nu}(x)\widetilde{G}^i_{0\rho\sigma}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p)e^{-ip\cdot(x-y)}$$

$$\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p) = p_\mu p_\rho g_{\nu\sigma} + p_\nu p_\sigma g_{\mu\rho} - p_\mu p_\sigma g_{\nu\rho} - p_\nu p_\rho g_{\mu\sigma} + p^2(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})$$

II. Oddballs via QCDSR

- Gluon condensates

$$\delta^{ab}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)|0\rangle = \frac{1}{D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$\delta^{ab}\langle 0|\widetilde{G}^a_{\mu\nu}(0)\widetilde{G}^b_{\rho\sigma}(0)|0\rangle = \frac{2-D}{2D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$f^{abc}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)G^c_{\alpha\beta}(0)|0\rangle = \frac{1}{D(D-1)(D-2)}T^3_{\mu\nu\rho\sigma\alpha\beta}\langle GGG\rangle$$

$$\begin{aligned} T^3_{\mu\nu\rho\sigma\alpha\beta} \quad = \quad & g_{\mu\rho}g_{\nu\alpha}g_{\sigma\beta} - g_{\mu\rho}g_{\nu\beta}g_{\sigma\alpha} - g_{\mu\sigma}g_{\nu\alpha}g_{\rho\beta} + g_{\mu\sigma}g_{\nu\beta}g_{\rho\alpha} \\ & -g_{\mu\alpha}g_{\nu\rho}g_{\sigma\beta} + g_{\mu\alpha}g_{\nu\sigma}g_{\rho\beta} + g_{\mu\beta}g_{\nu\rho}g_{\sigma\alpha} - g_{\mu\beta}g_{\nu\sigma}g_{\rho\alpha} \end{aligned}$$

➢ Gluon condensates

$$
\begin{aligned}
& f^{abe} f^{cde} \langle 0 | G^a_{\mu\nu}(0) G^b_{\rho\sigma}(0) G^c_{\alpha\beta}(0) G^d_{\gamma\delta}(0) | 0 \rangle \\
= \quad & A \{ \, g_{\mu\rho} g_{\nu\alpha} g_{\sigma\gamma} g_{\beta\delta} - g_{\mu\rho} g_{\nu\alpha} g_{\sigma\delta} g_{\beta\gamma} - g_{\mu\rho} g_{\nu\beta} g_{\sigma\gamma} g_{\alpha\delta} + g_{\mu\rho} g_{\nu\beta} g_{\sigma\delta} g_{\alpha\gamma} \\
& - g_{\mu\rho} g_{\nu\gamma} g_{\sigma\alpha} g_{\beta\delta} + g_{\mu\rho} g_{\nu\gamma} g_{\sigma\beta} g_{\alpha\delta} + g_{\mu\rho} g_{\nu\delta} g_{\sigma\alpha} g_{\beta\gamma} - g_{\mu\rho} g_{\nu\delta} g_{\sigma\beta} g_{\alpha\gamma} \\
& - g_{\mu\sigma} g_{\nu\alpha} g_{\rho\gamma} g_{\beta\delta} + g_{\mu\sigma} g_{\nu\alpha} g_{\rho\delta} g_{\beta\gamma} + g_{\mu\sigma} g_{\nu\beta} g_{\rho\gamma} g_{\alpha\delta} - g_{\mu\sigma} g_{\nu\beta} g_{\rho\delta} g_{\alpha\gamma} \\
& + g_{\mu\sigma} g_{\nu\gamma} g_{\rho\alpha} g_{\beta\delta} - g_{\mu\sigma} g_{\nu\gamma} g_{\rho\beta} g_{\alpha\delta} - g_{\mu\sigma} g_{\nu\delta} g_{\rho\alpha} g_{\beta\gamma} + g_{\mu\sigma} g_{\nu\delta} g_{\rho\beta} g_{\alpha\gamma} \\
& - g_{\mu\alpha} g_{\nu\rho} g_{\sigma\gamma} g_{\beta\delta} + g_{\mu\alpha} g_{\nu\rho} g_{\sigma\delta} g_{\beta\gamma} + g_{\mu\alpha} g_{\nu\sigma} g_{\rho\gamma} g_{\beta\delta} - g_{\mu\alpha} g_{\nu\sigma} g_{\rho\delta} g_{\beta\gamma} \\
& + g_{\mu\beta} g_{\nu\rho} g_{\sigma\gamma} g_{\alpha\delta} - g_{\mu\beta} g_{\nu\rho} g_{\sigma\delta} g_{\alpha\gamma} - g_{\mu\beta} g_{\nu\sigma} g_{\rho\gamma} g_{\alpha\delta} + g_{\mu\beta} g_{\nu\sigma} g_{\rho\delta} g_{\alpha\gamma} \\
& + g_{\mu\gamma} g_{\nu\rho} g_{\sigma\alpha} g_{\beta\delta} - g_{\mu\gamma} g_{\nu\rho} g_{\sigma\beta} g_{\alpha\delta} - g_{\mu\gamma} g_{\nu\sigma} g_{\rho\alpha} g_{\beta\delta} + g_{\mu\gamma} g_{\nu\sigma} g_{\rho\beta} g_{\alpha\delta} \\
& - g_{\mu\delta} g_{\nu\rho} g_{\sigma\alpha} g_{\beta\gamma} + g_{\mu\delta} g_{\nu\rho} g_{\sigma\beta} g_{\alpha\gamma} + g_{\mu\delta} g_{\nu\sigma} g_{\rho\alpha} g_{\beta\gamma} - g_{\mu\delta} g_{\nu\sigma} g_{\rho\beta} g_{\alpha\gamma} \} \\
& + B [(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) (g_{\rho\gamma} g_{\sigma\delta} - g_{\rho\delta} g_{\sigma\gamma}) - (g_{\mu\gamma} g_{\nu\delta} - g_{\mu\delta} g_{\nu\gamma}) (g_{\rho\alpha} g_{\sigma\beta} - g_{\rho\beta} g_{\sigma\alpha})]
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{(D+1)\langle (f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma})^2 \rangle - \langle (f^{abc} G^a_{\mu\nu} G^b_{\rho\sigma})^2 \rangle}{(D+2)D(D-1)(D-2)(D-3)} \\
B &= \frac{-4\langle (f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma})^2 \rangle + (D-2)\langle (f^{abc} G^a_{\mu\nu} G^b_{\rho\sigma})^2 \rangle}{(D+2)D(D-1)(D-2)(D-3)}
\end{aligned}
$$

$$
f^{abe} f^{cde} \langle 0 | \widetilde{G}^a_{\mu\nu}(0) \widetilde{G}^b_{\rho\sigma}(0) \widetilde{G}^c_{\alpha\beta}(0) \widetilde{G}^d_{\gamma\delta}(0) | 0 \rangle = f^{abe} f^{cde} \langle 0 | G^a_{\mu\nu}(0) G^b_{\rho\sigma}(0) G^c_{\alpha\beta}(0) G^d_{\gamma\delta}(0) | 0 \rangle
$$

II. Oddballs via QCDSR

➢QCDSR

- **The two-point correlation function**

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\Big\{ j_{0^{--}}(x), j_{0^{--}}(0) \Big\}|0\rangle \ ,$$

- **The QCD side of the correlation function**

$$\Pi^{\rm QCD}(Q^2) = a_0 Q^{12} \ln \frac{Q^2}{\mu^2} + b_0 Q^8 \langle \alpha_s G^2 \rangle + \left(c_0 + c_1 \ln \frac{Q^2}{\mu^2} \right) Q^6 \langle g_s G^3 \rangle + d_0 Q^4 \langle \alpha_s G^2 \rangle^2 \ .$$

- **The phenomenological side of the correlation function**

$$\frac{1}{\pi} {\rm Im} \Pi^{\rm phe}(s) = f_G^2 M_{0^{--}}^{12} \delta(s - M_{0^{--}}^2) + \rho(s) \theta(s - s_0) \ .$$

II. Oddballs via QCDSR

- **The dispersion relation**

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s+Q^2} + \Big(\Pi(0) - Q^2\Pi'(0) + \frac{1}{2}Q^4\Pi''(0) - \frac{1}{6}Q^6\Pi'''(0)\Big) ,$$

- **The Borel transformation**

$$\hat{B}_\tau \equiv \lim_{\substack{Q^2\to\infty, n\to\infty \\ \frac{Q^2}{n}=\frac{1}{\tau}}} \frac{(-Q^2)^n}{(n-1)!}\left(\frac{d}{dQ^2}\right)^n ,$$

- **The quark-hadron duality approximation**

$$\frac{1}{\pi}\int_{s_0}^\infty e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \simeq \int_{s_0}^\infty \rho(s)e^{-s\tau}ds ,$$

II. Oddballs via QCDSR

- **The moments**

$$L_0(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$

$$L_1(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} se^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$

- **The mass function**

$$M^i_{0--}(\tau, s_0) = \sqrt{\frac{L_1(\tau, s_0)}{L_0(\tau, s_0)}}$$

- **Ratios to constrain the windows of**

$$R_i^{\mathrm{OPE}} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\langle g_s G^3\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds}$$

$$R_i^{\mathrm{PC}} = \frac{L_0(\tau, s_0)}{L_0(\tau, \infty)}\ .$$

II. Oddballs via QCDSR

➢ **Interpolating currents of 0^{--} oddballs**

- **Constraints: quantum number, gauge invariance, Lorentz invariance and SU$_c$(3) symmetry**

$$j^A_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$

$$j^B_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$

$$j^C_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$

$$j^D_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$

where $g^t_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\partial_\beta/\partial^2$ $\quad \tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\tau}G^a_{\kappa\tau}$

II. Oddballs via QCDSR

- **Interpolating currents of 0^{+-} oddballs**

$$
\begin{aligned}
J_A^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,, \\
J_B^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,, \\
J_C^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,, \\
J_D^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,;
\end{aligned}
$$

- **Interpolating currents of 1^{-+} oddballs**

$$
\begin{aligned}
J_{A,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[G^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,, \\
J_{B,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[G^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,, \\
J_{C,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[\tilde{G}^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,, \\
J_{D,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[\tilde{G}^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,,
\end{aligned}
$$

II. Oddballs via QCDSR

- **Interpolating currents of 2^{+-} oddballs**

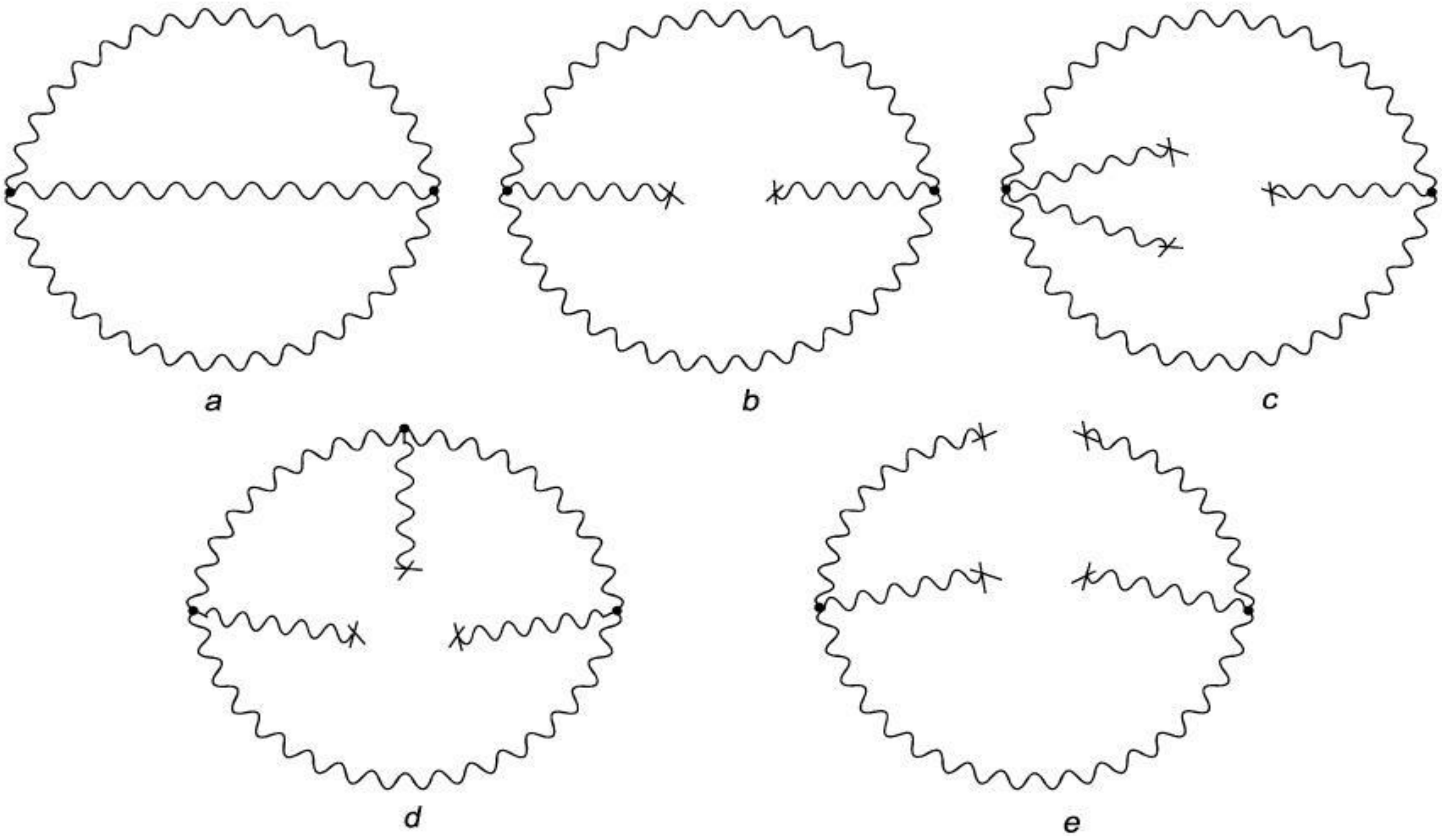
$$
\begin{aligned}
J^{2^{+-}}_{A,\mu\alpha}(x) &= g_s^3 d^{abc}[G^a_{\mu\nu}(x)G^b_{\nu\rho}(x)G^c_{\rho\alpha}(x)] \,, \\
J^{2^{+-}}_{B,\mu\alpha}(x) &= g_s^3 d^{abc}[G^a_{\mu\nu}(x)\tilde{G}^b_{\nu\rho}(x)\tilde{G}^c_{\rho\alpha}(x)] \,, \\
J^{2^{+-}}_{C,\mu\alpha}(x) &= g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)G^b_{\nu\rho}(x)\tilde{G}^c_{\rho\alpha}(x)] \,, \\
J^{2^{+-}}_{D,\mu\alpha}(x) &= g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)\tilde{G}^b_{\nu\rho}(x)G^c_{\rho\alpha}(x)] \,.
\end{aligned}
$$

- The two-point correlation function

$$
\Pi^{J^{PC},\,k}_{\alpha_1\cdots\alpha_j,\,\beta_1\cdots\beta_j}(q^2)=i\int d^4x e^{iq\cdot x}\langle 0|T\left\{j^{J^{PC},\,k}_{\alpha_1\cdots\alpha_j}(x),j^{J^{PC},\,k}_{\beta_1\cdots\beta_j}(0)\right\}|0\rangle \,,
$$

II. Oddballs via QCDSR

➢**Typical Feynman diagrams of trigluon glueballs**



II. Oddballs via QCDSR

- The QCD side of the correlation function

$$
\begin{aligned}
\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(q^2) &= a_0(q^2)^n \ln\frac{-q^2}{\mu^2} + \left(b_0 + b_1 \ln\frac{-q^2}{\mu^2}\right)(q^2)^{n-2}\langle\alpha_s G^2\rangle \\
&+ \left(c_0 + c_1 \ln\frac{-q^2}{\mu^2}\right)(q^2)^{n-3}\langle g_s G^3\rangle \\
&+ d_0(q^2)^{n-4}\langle\alpha_s G^2\rangle^2 ,
\end{aligned}
$$

where n represents the corresponding power of q^2 for each oddballs.

- The phenomenological side of the correlation function

$$
\begin{aligned}
\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{k,\,phe}}_{\mathrm{JPC}}(s) &= (f^k_{JPC})^2(M^k_{JPC})^{2n}\delta\left(s-(M^k_{JPC})^2\right) \\
&+ \rho^k_{JPC}(s)\theta(s-s_0) .
\end{aligned}
$$

- The dispersion relation

$$
\begin{aligned}
\Pi^k_{JPC}(q^2) &= \frac{1}{\pi}\int_0^\infty ds\frac{\mathrm{Im}\Pi^k_{JPC}(s)}{s-q^2} + \Big(\Pi^k_{JPC}(0) + q^2\Pi^{k\,\prime}_{JPC}(0) \\
&+ \frac{1}{2}q^4\Pi^{k\,\prime\prime}_{JPC}(0) + \frac{1}{6}q^6\Pi^{k\,\prime\prime\prime}_{JPC}(0)\Big) ,
\end{aligned}
$$

II. Oddballs via QCDSR

- The main function

$$\frac{1}{\pi}\int_0^{\infty}\frac{\mathrm{Im}\Pi^{\mathrm{k,\ QCD}}_{\mathrm{JPC}}(s)}{s-q^2}ds$$

$$= \frac{(f^k_{JPC})^2(M^k_{JPC})^{2n}}{(M^k_{JPC})^2-q^2}+\int_{s_0}^{\infty}\frac{\rho^k_{JPC}(s)\theta(s-s_0)}{s-q^2}ds\ .$$

- The Borel transformation

$$\hat{B}_\tau \equiv \lim_{\substack{-q^2\to\infty,n\to\infty\\ \frac{-q^2}{n}=\frac{1}{\tau}}}\frac{(q^2)^n}{(n-1)!}\left(-\frac{d}{dq^2}\right)^n\ ,$$

- The quark-hadron duality approximation

$$\frac{1}{\pi}\int_{s_0}^{\infty}e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\ QCD}}_{\mathrm{JPC}}(s)ds \simeq \int_{s_0}^{\infty}\rho^k_{JPC}(s)e^{-s\tau}ds\ ,$$

II. Oddballs via QCDSR

- The moments

$$L^k_{JPC,\,0}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds\ ,$$

$$L^k_{JPC,\,1}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} s e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds\ ,$$

- The mass function

$$M^k_{JPC}(\tau, s_0) = \sqrt{\frac{L^k_{JPC,\,1}(\tau, s_0)}{L^k_{JPC,\,0}(\tau, s_0)}}\ ,$$

- Ratio to constrain τ & s_0 by the pole contribution (PC)

$$R^{k,\,\mathrm{PC}}_J = \frac{L^k_{JPC,\,0}(\tau, s_0)}{L^k_{JPC,\,0}(\tau, \infty)}\ .$$

II. Oddballs via QCDSR

- Ratio to constrain τ and s_0 by convergence of the OPE

$$R_J^{k,\,\mathrm{G}^2} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k},\,\langle\alpha_s\mathrm{G}^2\rangle}_{\mathrm{JPC}}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k},\,\mathrm{QCD}}_{\mathrm{JPC}}(s)ds} ,$$

$$R_J^{k,\,\mathrm{G}^3} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k},\,\langle g_s\mathrm{G}^3\rangle}_{\mathrm{JPC}}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k},\,\mathrm{QCD}}_{\mathrm{JPC}}(s)ds} .$$

- Input parameters

$$\langle\alpha_s G^2\rangle = 0.06\,\mathrm{GeV}^4\ ,\ \langle g_s G^3\rangle = (0.27\,\mathrm{GeV}^2)\langle\alpha_s G^2\rangle\ ,$$

$$\Lambda_{\overline{\mathrm{MS}}} = 300\,\mathrm{MeV}\ ,\ \alpha_s = \frac{-4\pi}{11\ln(\tau\Lambda^2_{\overline{\mathrm{MS}}})}\ ,$$

II. Oddballs via QCDSR

- Wilson coefficients of 0^{--} in the QCD-side

$$a_0^i = \frac{487\alpha_s^3}{143\times 2^6\times 3^3\pi}\ ,\ b_0^i = -\frac{5\pi}{36}\alpha_s^2\ , c_0^A = -\frac{205}{12}\pi\alpha_s^3\ ,$$

$$c_1^A = -\frac{775}{144}\pi\alpha_s^3\ ,\ c_0^B = -\frac{2065}{48}\pi\alpha_s^3\ , c_1^B = -\frac{1075}{96}\pi\alpha_s^3\ ,$$

$$c_0^C = \frac{2275}{72}\pi\alpha_s^3\ ,\ c_1^C = \frac{2125}{144}\pi\alpha_s^3\ ,\ c_0^D = -\frac{1045}{144}\pi\alpha_s^3\ ,$$

$$c_1^D = -\frac{25}{32}\pi\alpha_s^3\ ,\ d_0^j = 0\ ,\ d_0^D = -\frac{5}{9}\pi^3\alpha_s\ ,$$

where, i=A, B, C, D; j=A, B, C; with A, B, C and D corresponding to the above four currents.

There are symmetries within Wilson coefficients a_0^i, b_0^i and d_0^j. The position and number of $\tilde{G}$ do not influence the perturbative and $\langle\alpha_s G^2\rangle$ contributions, whereas they influence $\langle g_s G^3\rangle$ term. Since $\langle\alpha_s G^2\rangle^2$ involves no loop contribution, d_0^j are governed by the number of $\tilde{G}$.

➢Wilson coefficients of 0^{+-} in the QCD-side

$$a_0^A=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^A=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^A=0\,,$$
$$c_0^A=-\frac{325}{72}\pi\alpha_s^3\,,\quad c_1^A=-\frac{2125}{144}\pi\alpha_s^3\,,\quad d_0^A=0\,;$$
$$a_0^B=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^B=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^B=0\,,$$
$$c_0^B=\frac{7445}{144}\pi\alpha_s^3\,,\quad c_1^B=\frac{1075}{96}\pi\alpha_s^3\,,\quad d_0^B=0\,;$$
$$a_0^C=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^C=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^C=0\,,$$
$$c_0^C=\frac{1955}{72}\pi\alpha_s^3\,,\quad c_1^C=\frac{775}{144}\pi\alpha_s^3\,,\quad d_0^C=0\,;$$
$$a_0^D=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^D=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^D=0\,,$$
$$c_0^D=\frac{235}{72}\pi\alpha_s^3\,,\quad c_1^D=\frac{25}{32}\pi\alpha_s^3\,,\quad d_0^D=0\,,$$

where we notice that except for c_0^k and c_1^k, a_0^k, b_0^k , b_1^k, and d_0^k are equal for case A to D. This situation is similar to the 0^{--} oddballs.

➢Wilson coefficients of 1^{-+} in the QCD-side

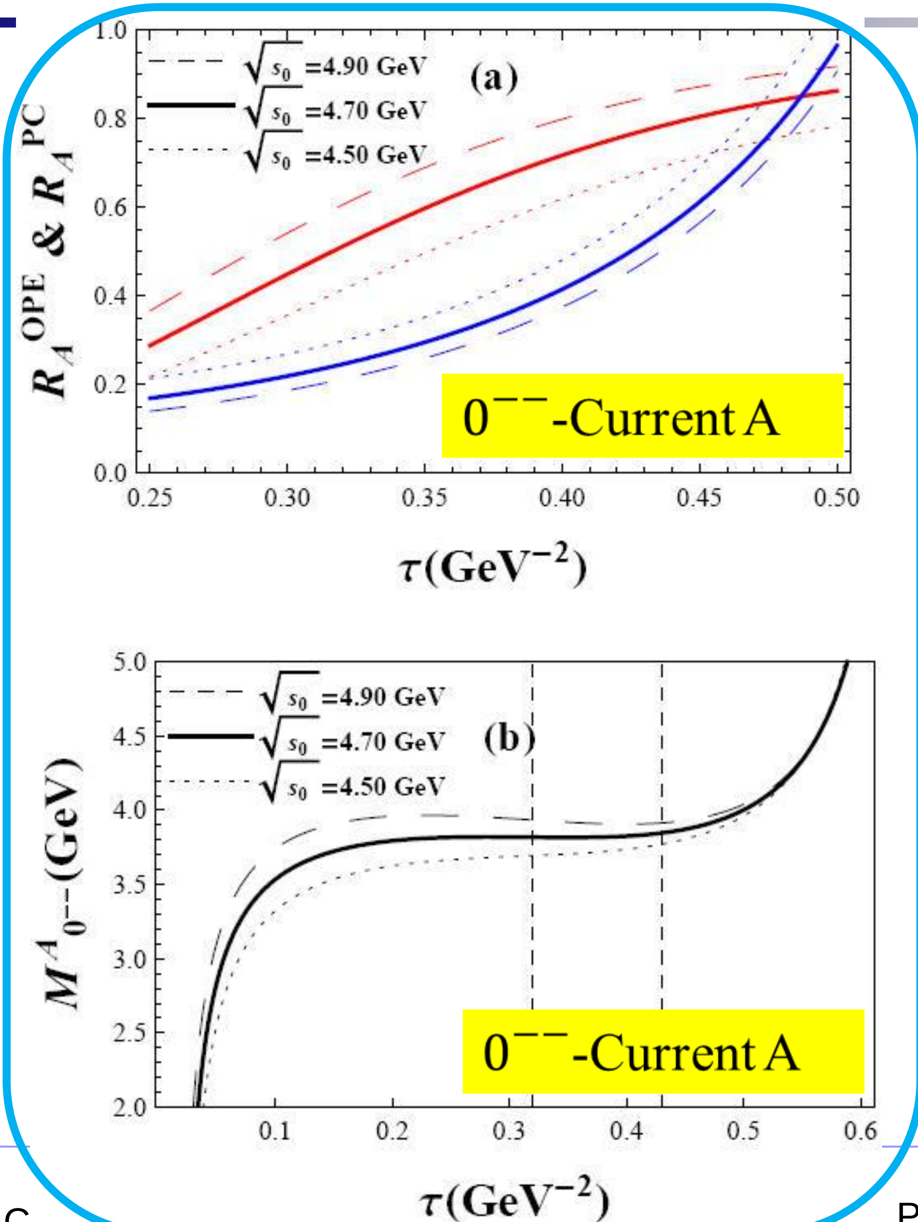
$$a_0^A=\frac{1}{1008}\frac{\alpha_s^3}{\pi}\ , \quad b_0^A=-\frac{1}{72}\pi\alpha_s^2\ , \quad b_1^A=\frac{1}{12}\pi\alpha_s^2\ ,$$
$$c_0^A=\frac{71}{96}\pi\alpha_s^3\ , \quad c_1^A=\frac{23}{48}\pi\alpha_s^3\ , \quad d_0^A=\frac{1}{3}\pi^3\alpha_s\ ;$$
$$a_0^B=\frac{1}{1008\pi}\frac{\alpha_s^3}{\pi}\ , \quad b_0^B=\frac{23}{72}\pi\alpha_s^2\ , \quad b_1^B=\frac{1}{12}\pi\alpha_s^2\ ,$$
$$c_0^B=\frac{89}{64}\pi\alpha_s^3\ , \quad c_1^B=\frac{27}{128}\pi\alpha_s^3\ , \quad d_0^B=\frac{1}{3}\pi^3\alpha_s\ ;$$
$$a_0^C=\frac{1}{112}\frac{\alpha_s^3}{\pi}\ , \quad b_0^C=-\frac{1}{8}\pi\alpha_s^2\ , \quad b_1^C=\frac{3}{4}\pi\alpha_s^2\ ,$$
$$c_0^C=\frac{79}{48}\pi\alpha_s^3\ , \quad c_1^C=\frac{845}{384}\pi\alpha_s^3\ , \quad d_0^C=3\pi^3\alpha_s\ ;$$
$$a_0^D=\frac{1}{1008}\frac{\alpha_s^3}{\pi}\ , \quad b_0^D=\frac{23}{72}\pi\alpha_s^2\ , \quad b_1^D=\frac{1}{12}\pi\alpha_s^2\ ,$$
$$c_0^D=-\frac{47}{64}\pi\alpha_s^3\ , \quad c_1^D=-\frac{1}{64}\pi\alpha_s^3\ , \quad d_0^D=\frac{1}{3}\pi^3\alpha_s\ ,$$

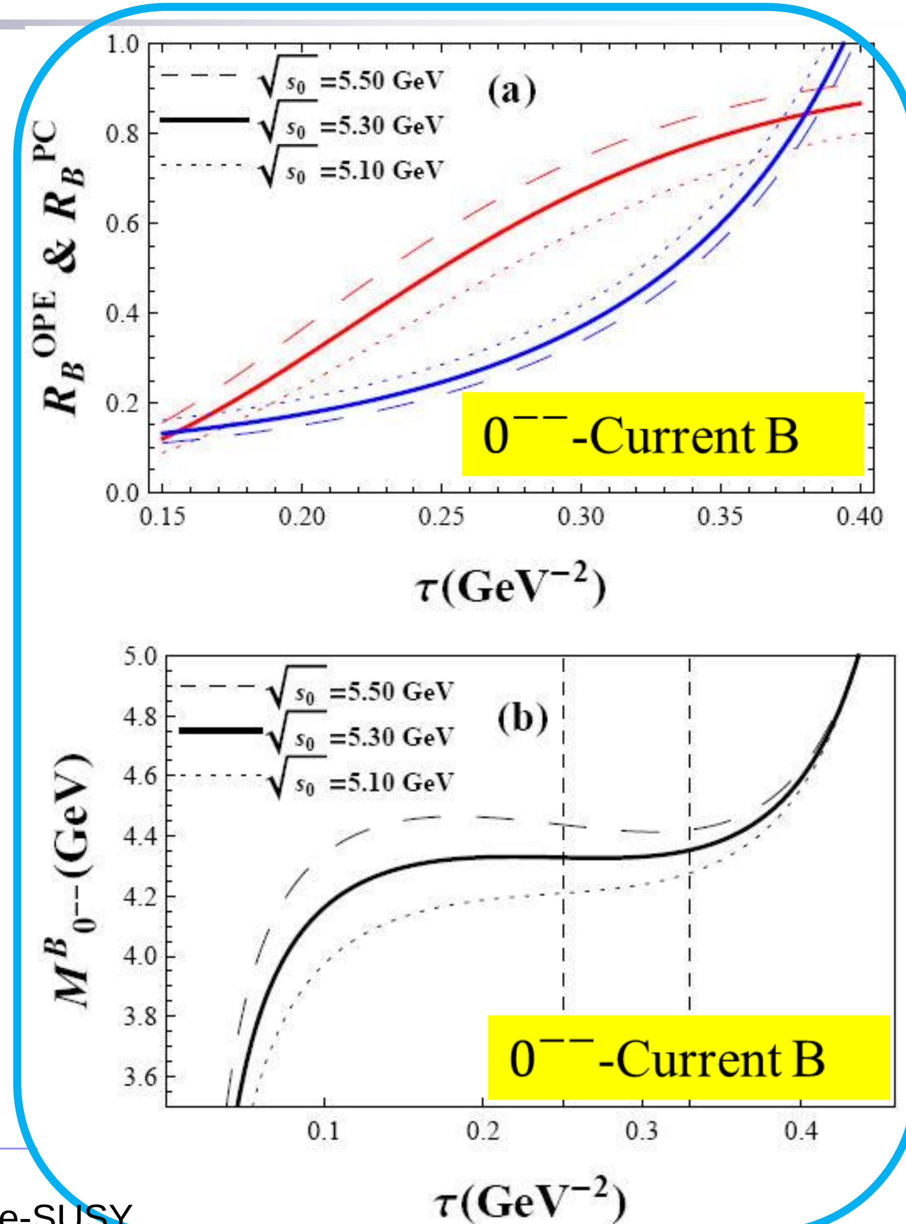
where the ratios of a_0^k to b_1^k are equal for case A to D. This implies that the mass curves of case A to D will be very similar, since if we neglect the $\langle g_s G^3\rangle$ term which is much smaller than the $\langle \alpha_s G^2\rangle$ term in mass Eq., the mass of the oddball only depends on the ratio of a_0^k to b_1^k.

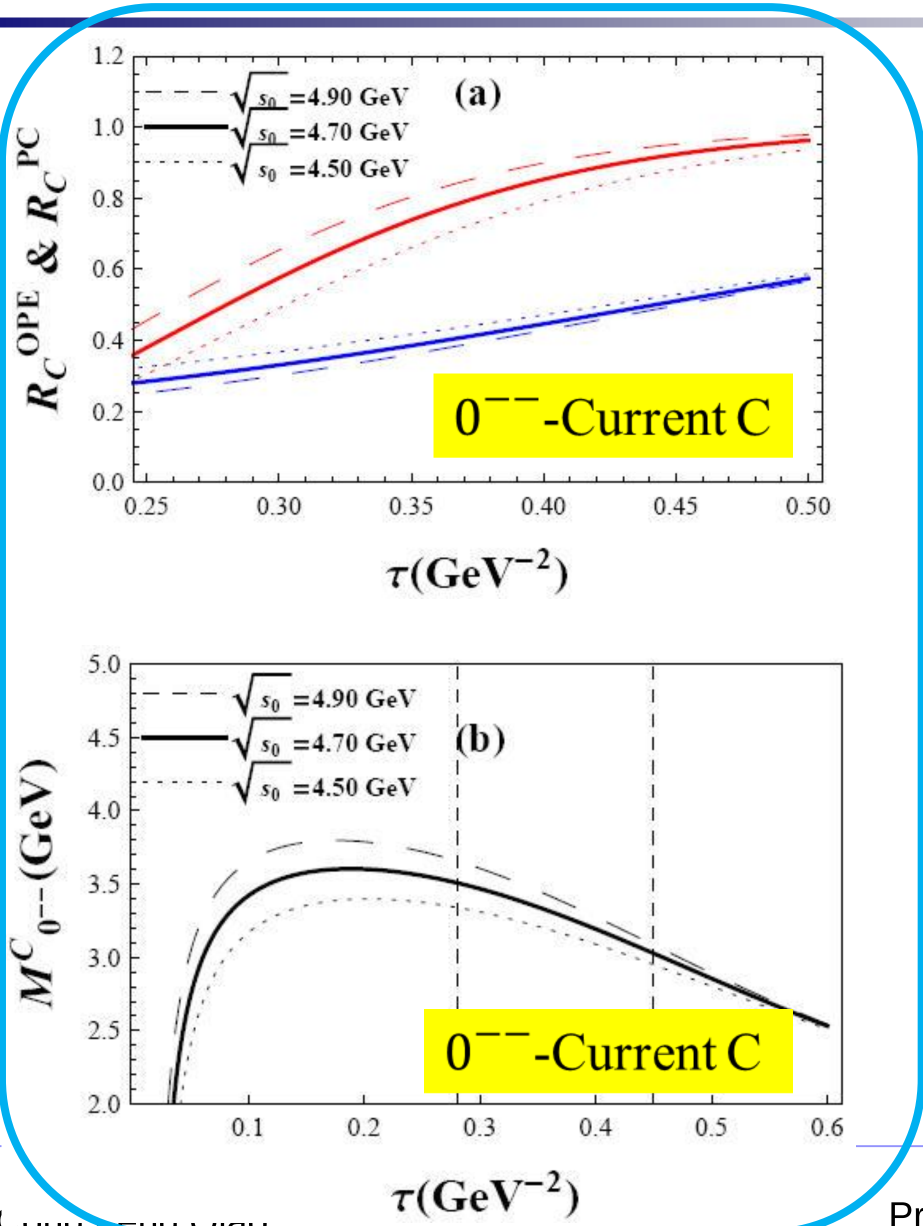
➢Wilson coefficients of 2^{+-} in the QCD-side

$$a_0^A = -\frac{2}{81}\frac{\alpha_s^3}{\pi}, \quad b_0^A = \frac{20}{3}\pi\alpha_s^2, \quad b_1^A = -\frac{20}{9}\pi\alpha_s^2,$$
$$c_0^A = \frac{205}{54}\pi\alpha_s^3, \quad c_1^A = -\frac{40}{9}\pi\alpha_s^3, \quad d_0^A = \frac{20}{9}\pi^3\alpha_s;$$
$$a_0^B = -\frac{1}{324}\frac{\alpha_s^3}{\pi}, \quad b_0^B = \frac{5}{81}\pi\alpha_s^2, \quad b_1^B = \frac{10}{27}\pi\alpha_s^2,$$
$$c_0^B = \frac{415}{162}\pi\alpha_s^3, \quad c_1^B = \frac{20}{27}\pi\alpha_s^3, \quad d_0^B = \frac{10}{9}\pi^3\alpha_s;$$
$$a_0^C = -\frac{1}{324}\frac{\alpha_s^3}{\pi}, \quad b_0^C = -\frac{115}{81}\pi\alpha_s^2, \quad b_1^C = \frac{10}{27}\pi\alpha_s^2,$$
$$c_0^C = -\frac{65}{162}\pi\alpha_s^3, \quad c_1^C = \frac{20}{27}\pi\alpha_s^3, \quad d_0^C = \frac{10}{9}\pi^3\alpha_s;$$
$$a_0^D = -\frac{1}{324}\frac{\alpha_s^3}{\pi}, \quad b_0^D = \frac{5}{81}\pi\alpha_s^2, \quad b_1^D = \frac{10}{27}\pi\alpha_s^2,$$
$$c_0^D = \frac{415}{162}\pi\alpha_s^3, \quad c_1^D = \frac{20}{27}\pi\alpha_s^3, \quad d_0^D = \frac{10}{9}\pi^3\alpha_s,$$

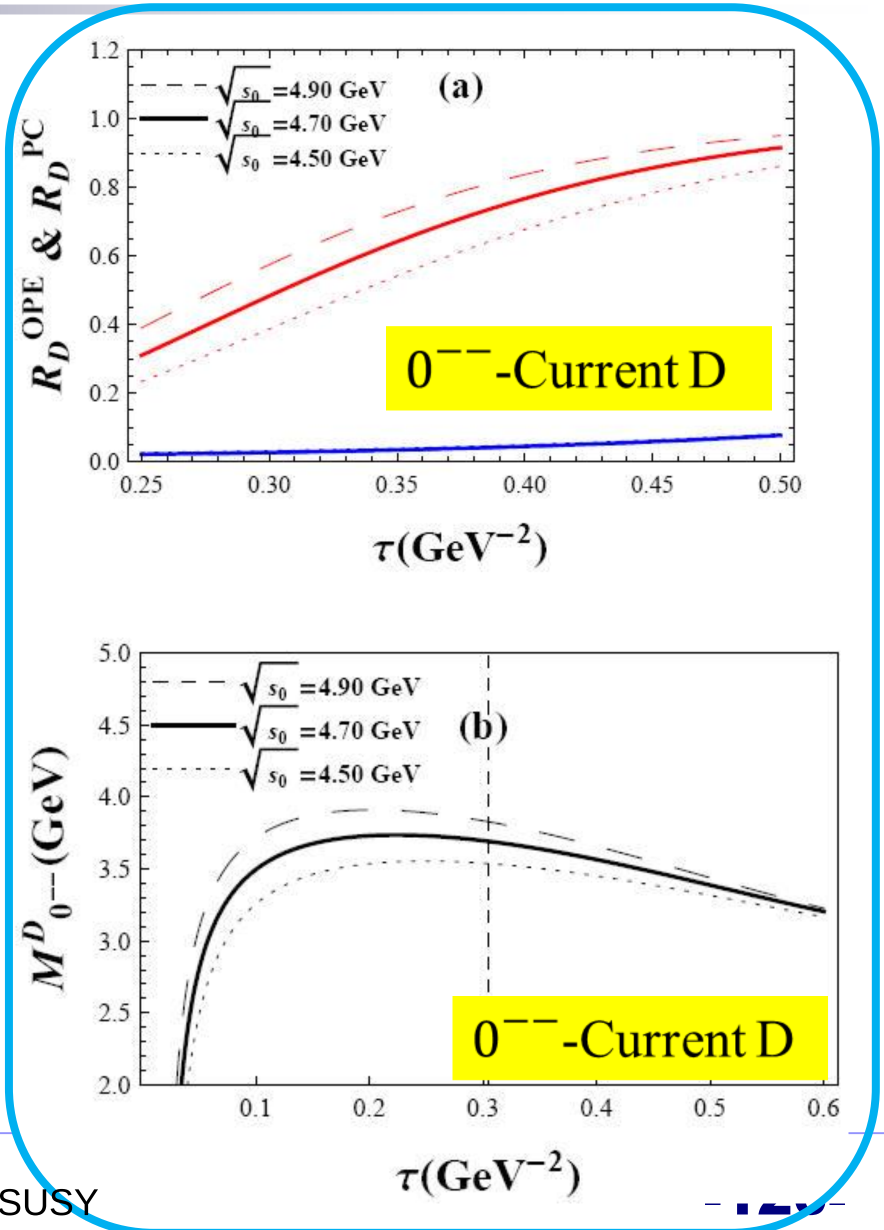
where a_0^k, b_1^k, and c_1^k are equal for case B to D. This implies that the mass curves of case B to D will be exactly equal, because they are determined by the Wilson coefficients a_0^k, b_1^k, and c_1^k.


(a)
$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
R_A^{OPE} & R_A^{PC}
$\tau(\text{GeV}^{-2})$
0^{--}-Current A
(b)
$M^A_{0^{--}}$(GeV)
$\tau(\text{GeV}^{-2})$
0^{--}-Current A

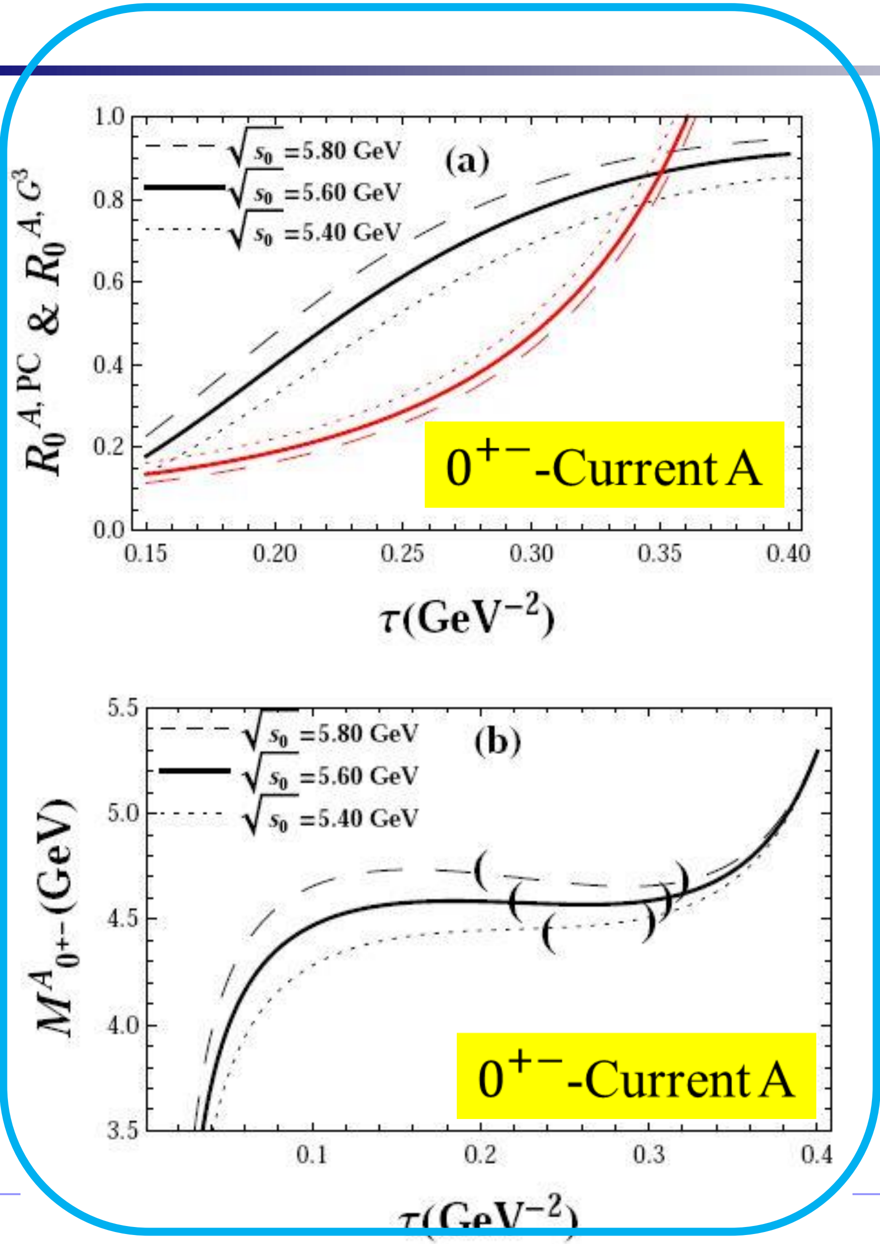

(a)
$\sqrt{s_0}$ =5.50 GeV
$\sqrt{s_0}$ =5.30 GeV
$\sqrt{s_0}$ =5.10 GeV
R_B^{OPE} & R_B^{PC}
$\tau(\text{GeV}^{-2})$
0^{--}-Current B
(b)
$M^B_{0^{--}}$(GeV)
$\tau(\text{GeV}^{-2})$
0^{--}-Current B

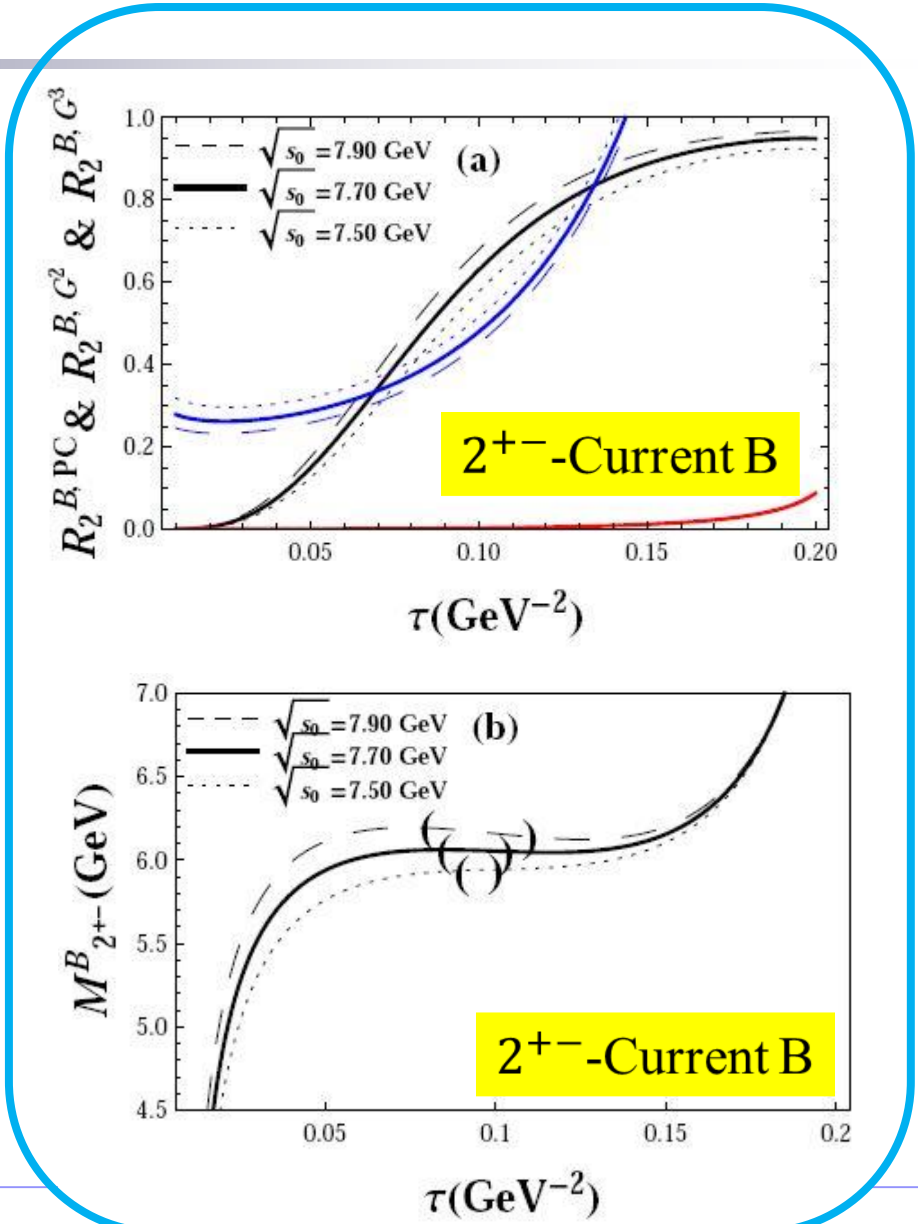


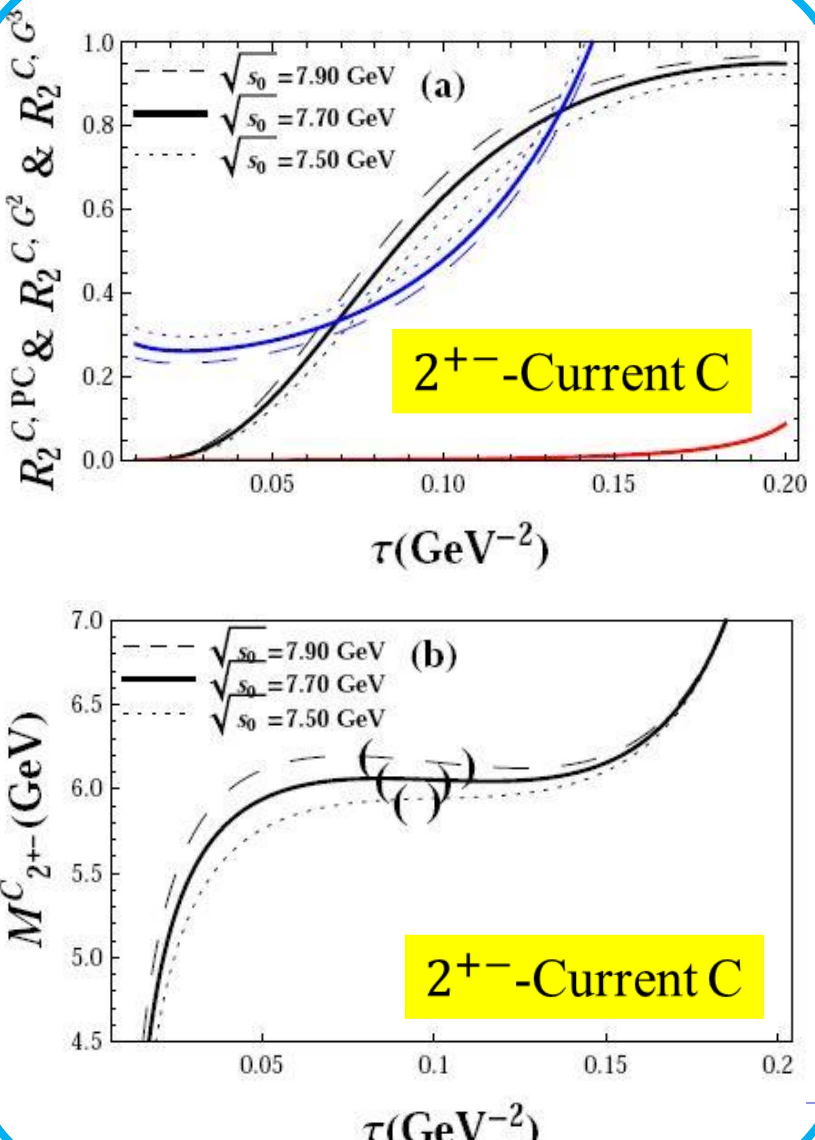
(a)
$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
R_C^{OPE} & R_C^{PC}
0^{--}-Current C
$\tau(\mathrm{GeV}^{-2})$
(b)
$M^C_{0^{--}}$ (GeV)
0^{--}-Current C
$\tau(\mathrm{GeV}^{-2})$

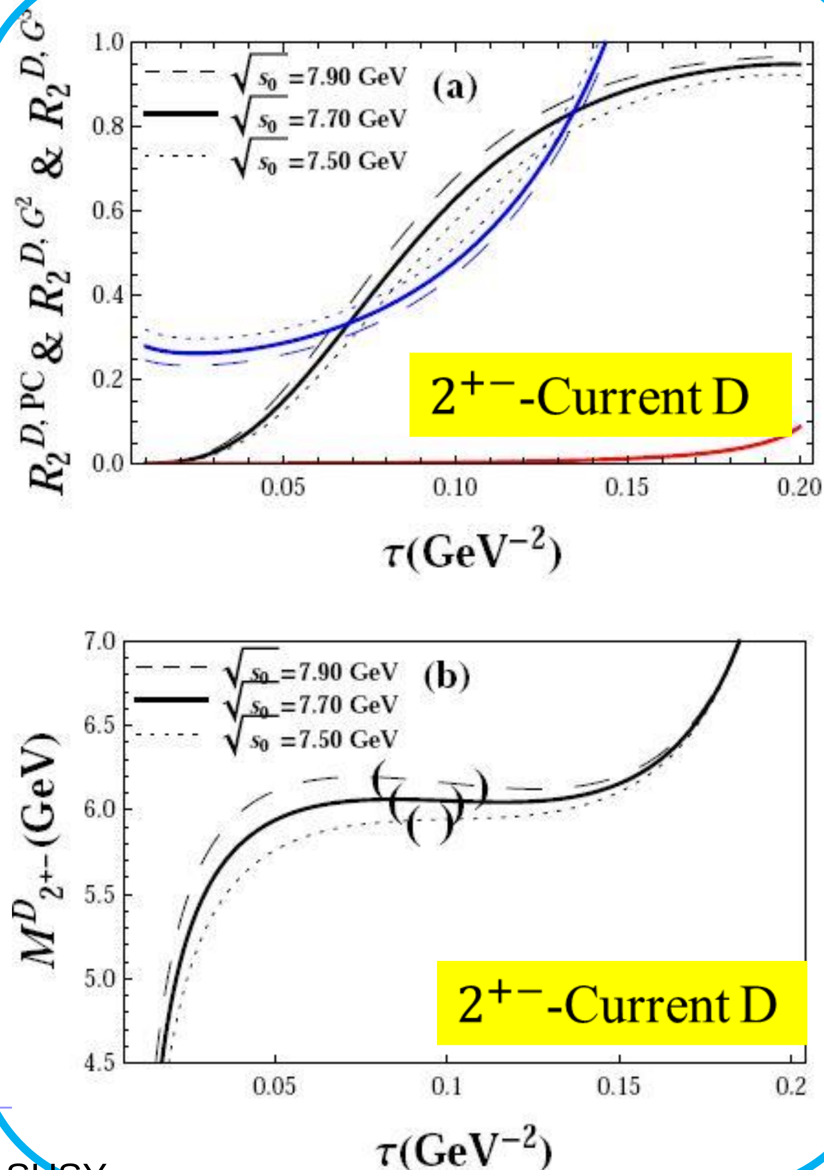


(a)
$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
R_D^{OPE} & R_D^{PC}
0^{--}-Current D
$\tau(\mathrm{GeV}^{-2})$
(b)
$M^D_{0^{--}}$ (GeV)
0^{--}-Current D
$\tau(\mathrm{GeV}^{-2})$


(a)
√s0 =5.80 GeV
√s0 =5.60 GeV
√s0 =5.40 GeV
0+−-Current A
τ(GeV−2)
(b)
√s0 =5.80 GeV
√s0 =5.60 GeV
√s0 =5.40 GeV
0+−-Current A


(a)
√s0 =7.90 GeV
√s0 =7.70 GeV
√s0 =7.50 GeV
2+−-Current B
τ(GeV−2)
(b)
√s0 =7.90 GeV
√s0 =7.70 GeV
√s0 =7.50 GeV
2+−-Current B
τ(GeV−2)


$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(a)
2^{+-}-Current C
$R_2^{C,\,PC}$ & $R_2^{C,\,G^2}$ & $R_2^{C,\,G^3}$
$\tau(\mathrm{GeV}^{-2})$
(b)
2^{+-}-Current C
$M^C_{2^{+-}}$(GeV)
$\tau(\mathrm{GeV}^{-2})$


$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(a)
2^{+-}-Current D
$R_2^{D,\,PC}$ & $R_2^{D,\,G^2}$ & $R_2^{D,\,G^3}$
$\tau(\mathrm{GeV}^{-2})$
(b)
2^{+-}-Current D
$M^D_{2^{+-}}$(GeV)
$\tau(\mathrm{GeV}^{-2})$

- Comparison with other methods, in unit of GeV.

J^{PC}	Flux tube model	Lattice QCD	Holography QCD	Our results (QCDSR)
0^{--}	2.79 [1]	5.166 [2]	2.8 [6], 3.817 [7]	3.81, 4.33
0^{+-}	2.79 [1]	5.45 [2], 4.74 [3], 4.78 [4]	X	4.57
1^{-+}	X	1.68 [5]	X	X
2^{+-}	X	4.14 [3], 4.23 [4]	2.786 [7]	6.06

[1] N.Isgur and J.E.Paton, PRD 31, 2910 (1985).
[2] E.Gregory, et al., JHEP1210, 170 (2012).
[3] C.J.Morningstar and M.J.Peardon,PRD60, 034509 (1999).
[4] Y.Chen et al.,PRD73, 014516 (2006).
[5] K.Ishikawa, et al., PLB120, 387 (1983).
[6] L. Bellantuono, et al., JHEP 1510 (2015) 137.
[7] Y.-D Chen and Mei Huang, arXiv:1511.07018.

III. Hunting for Oddballs

- Glueballs and Glueball Studies
- 0^{--} Oddballs via QCDSR
- **Hunting for Trigluon glueballs**
- Concluding remarks

III. Hunting for Oddballs

- Proposed production channels of 0^{--} oddball G(3810)

$$X(3872) \to \gamma + G_{0^{--}}(3810), \qquad \Upsilon(1S) \to f_1(1285) + G_{0^{--}}(3810),$$

$$\Upsilon(1S) \to \chi_{c_1} + G_{0^{--}}(3810), \qquad \chi_{b_1} \to J/\psi + G_{0^{--}}(3810),$$

$$\chi_{b_1} \to \omega + G_{0^{--}}(3810).$$

- Proposed decay channels of 0^{--}oddball

$$G_{0^{--}}(3810) \to \gamma + f_1(1285),$$

$$G_{0^{--}}(3810) \to \omega + f_1(1285).$$

$$G_{0^{--}}(3810) \to \gamma + \chi_{c_1},$$

III. Hunting for Oddballs

➢ Proposed production channels of 0^{+-} and 2^{+-} oddballs

J^{PC}	S-wave	P-wave
0^{+-}	$h_b \to \left\{f_1(1285),\ \chi_{c1},\ X(3872)\right\} + G_{0^{+-}}(4570)$	$\Upsilon(1S) \to \left\{f_1(1285),\ \chi_{c1},\ X(3872)\right\} + G_{0^{+-}}(4570)$ $\chi_{bJ} \to \left\{\gamma,\ \omega,\ \phi,\ J/\psi,\ \psi(2S)\right\} + G_{0^{+-}}(4570)$ $h_b \to \left\{\eta,\ \eta',\ \eta_c\right\} + G_{0^{+-}}(4570)$
2^{+-}	$\Upsilon(1S) \to \eta_2(1645) + G_{2^{+-}}(6060)$ $\chi_{b1,2} \to \left\{h_1(1170),\ h_c\right\} + G_{2^{+-}}(6060)$ $h_b \to \left\{f_1(1285),\ f_2(1270),\ \chi_{c1,2},\right\} + G_{2^{+-}}(6060)$	$\Upsilon(1S) \to f_1(1285) + G_{2^{+-}}(6060)$

➢ Proposed decay channels of 0^{+-} and 2^{+-} oddballs

J^{PC}	S-wave	P-wave
0^{+-}	$G_{0^{+-}}(4570) \to h_1(1170) + f_1(1285)$	$G_{0^{+-}}(4570) \to \left\{\gamma,\ \omega,\ \phi,\ J/\psi\right\} + f_0(980)$ $G_{0^{+-}}(4570) \to h_1(1170) + \left\{\eta, \eta',\ \eta_c\right\}$ $G_{0^{+-}} \to h_c + \left\{\eta, \eta'\right\}$
2^{+-}	$G_{2^{+-}}(6060) \to \left\{h_1(1170),\ h_c\right\} + f_1(1285)$	$G_{2^{+-}}(6060) \to \left\{\gamma,\ \omega,\ \phi, J/\psi,\ \psi(2S)\right\} + f_1(1285)$

III. Hunting for Oddballs

- **BESIII Collaboration, by …**
- **Belle Collaboration, by C.P. Shen, …**
- **Fermilab, Mike Albrow**
- **LHCb Collaboration, Paolo Gandini.**
- **phys.org, "*Long-searched-for glueball could soon be detected*", by Lisa Zyga.**

Contents:

- Before the advent of QCD
- The establishment of QCD
- Typical natures of QCD
- Applications of QCD
- **Concluding remarks**

Concluding remarks

- 在宇宙中，可见物质绝大多数是原（强）子物质，弄清这部分物质的构成和性质对于理解宇宙非常重要

- 强相互作用中的一些基本问题还不清楚，有待解决，如强CP问题，禁闭问题等

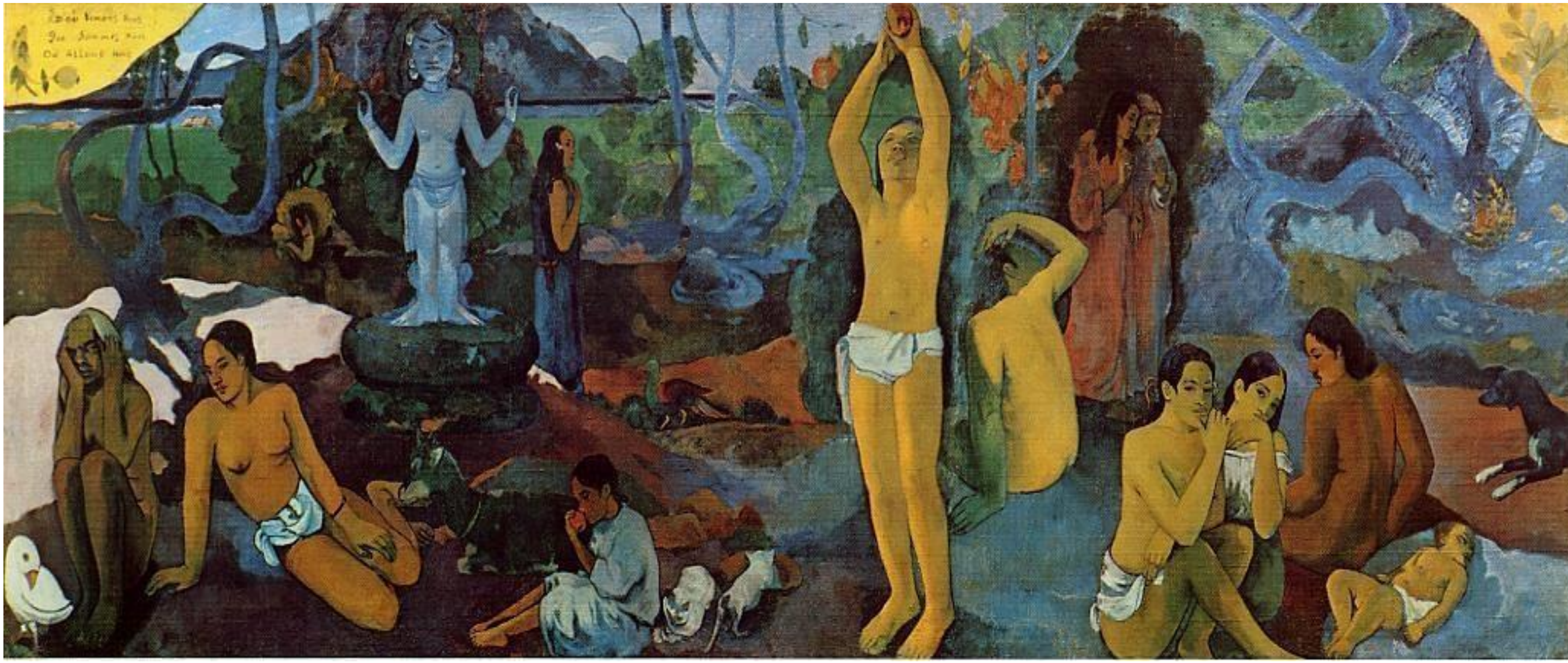
- 越来越多的实验证据表明，强相互左右在一定程度上制约着新物理的发现，如g-2问题

Concluding remarks

- 纵向来说，夸克（轻子）是否到普朗克能标都是基本的物质构成，它们是否还有结构？如果是有结构的，那对宇宙的演化有什么影响，还远没有答案

- 横向来说，除通常的介子和重子外，其它可能的强子态，如多夸克态、混杂态、胶球等是否存在，性质如何？发展和研究适合不同能标的有效理论还有很大的发展空间

- 极端条件下QCD性质研究，如夸克胶子等离子体的性质等，均有待研究。



Where Do We Come From? What Are We? Where Are We Going? ---Paul Gauguin

Enjoy your stay at the pre-SUSY summer school!

THANKS