Introduction to madgraph

A user viewpoint on MC tools for TeV frontier

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Keywords: TeV, Tools, Monte Carlo, Parton Shower

PreSUSY School 2021, Beijing, 9-13, August, 2021

Books

- 马文淦，《计算物理学》
- 杜东生，杨茂志，**《粒子物理导论》**
- ***R.K. Ellis, W.J. Stirling, and B.R. Webber*，QCD and Collider Physics**
- ***Bo Andersson,* The Lund Model**

前置课程：**Quantum Field Theory,**
Group Theory，
Particle Physics

Resources

- Madgraph school 2013 Beijing, https://indico.cern.ch/event/253947/
- ACAT 2013 Beijing, http://acat2013.ihep.ac.cn
- MC4BSM 2016 Beijing, https://indico.ihep.ac.cn/event/5301/
- Madgraph school 2015 Shanghai, https://www.physics.sjtu.edu.cn/madgraphschool/
- Madgraph school 2018 Hefei, https://indico.ihep.ac.cn/event/7822/page/0
- MCnet School 2021 Beijing, https://indico.ihep.ac.cn/event/11202/overview
- ***Talks of Qiang Li,* 2021**年对撞机暑期学校，https://indico.ihep.ac.cn/event/11211/

For the memory of Dr. Cen Zhang

张岑博士（1984-2021）

研究方向：高能对撞机唯象学，超出标准模型的新物理，精准计算

2002-2006： 北京大学物理学院（学士）
2006-2011： 美国伊利诺伊大学香槟分校（博士）
2012-2017： 比利时法语天主教鲁汶大学（博士后）
美国布鲁克海文国家实验室（博士后）
2017年回国： 高能所理论室（副研究员）
国科大老师

具体研究内容包括大型强子对撞机上顶夸克和希格斯粒子的相关过程，标准模型的有效场论，以及针对新物理信号的幅射修正等。近几年通过在MadGraph5_aMC@NLO框架中实现自动化计算，将基于有效场论的顶夸克物理精准唯象研究系统地推进到了次领头阶精度。

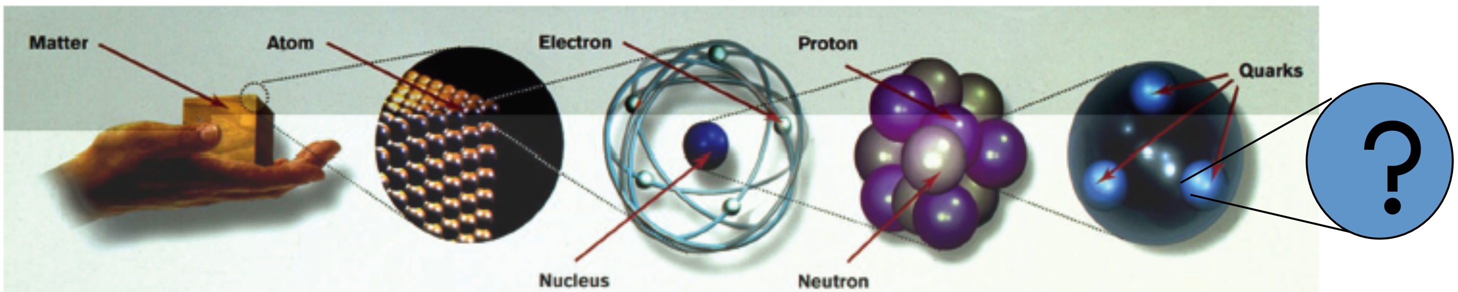
Outline

- TeV frontier
- MC tools
- Madgraph
  - ME
  - PS integration (Monte Carlo)
  - Parton Shower

Introduction to Modern Particle Physics

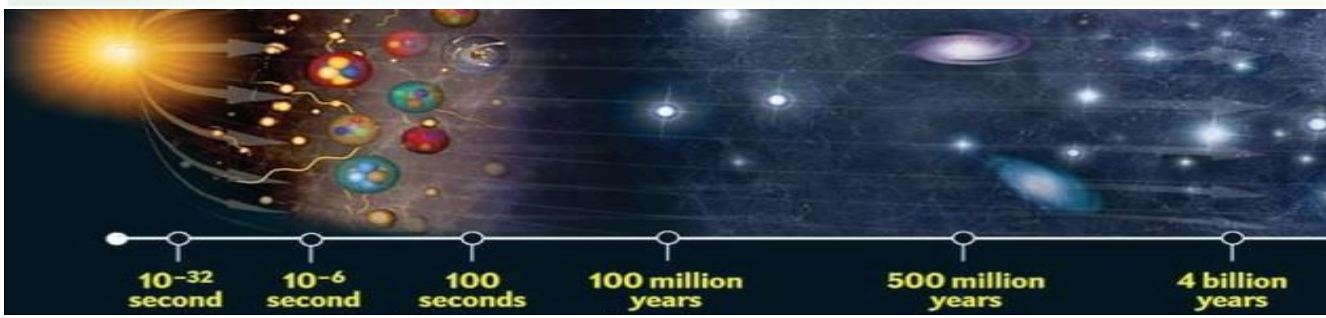
现代粒子物理是以实验为基础，研究物质世界基本组成及相互作用、时空及宇宙演化的一门自然科学。

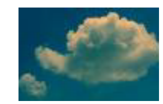
D.S.Yang
X. Zhuang



Q. Wang
B. Zhu
Yakefu

Y.G. Gong
Z.Z. Xianyu
Z.H. Zhao





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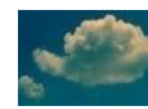

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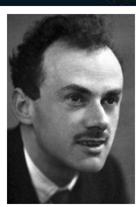

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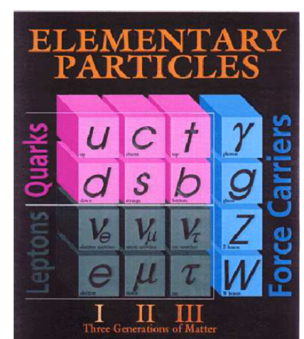
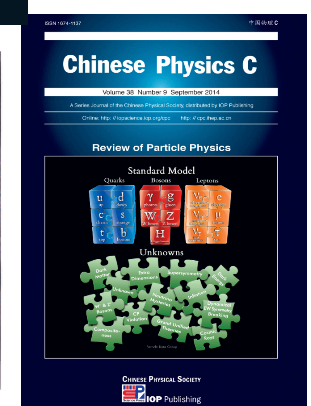
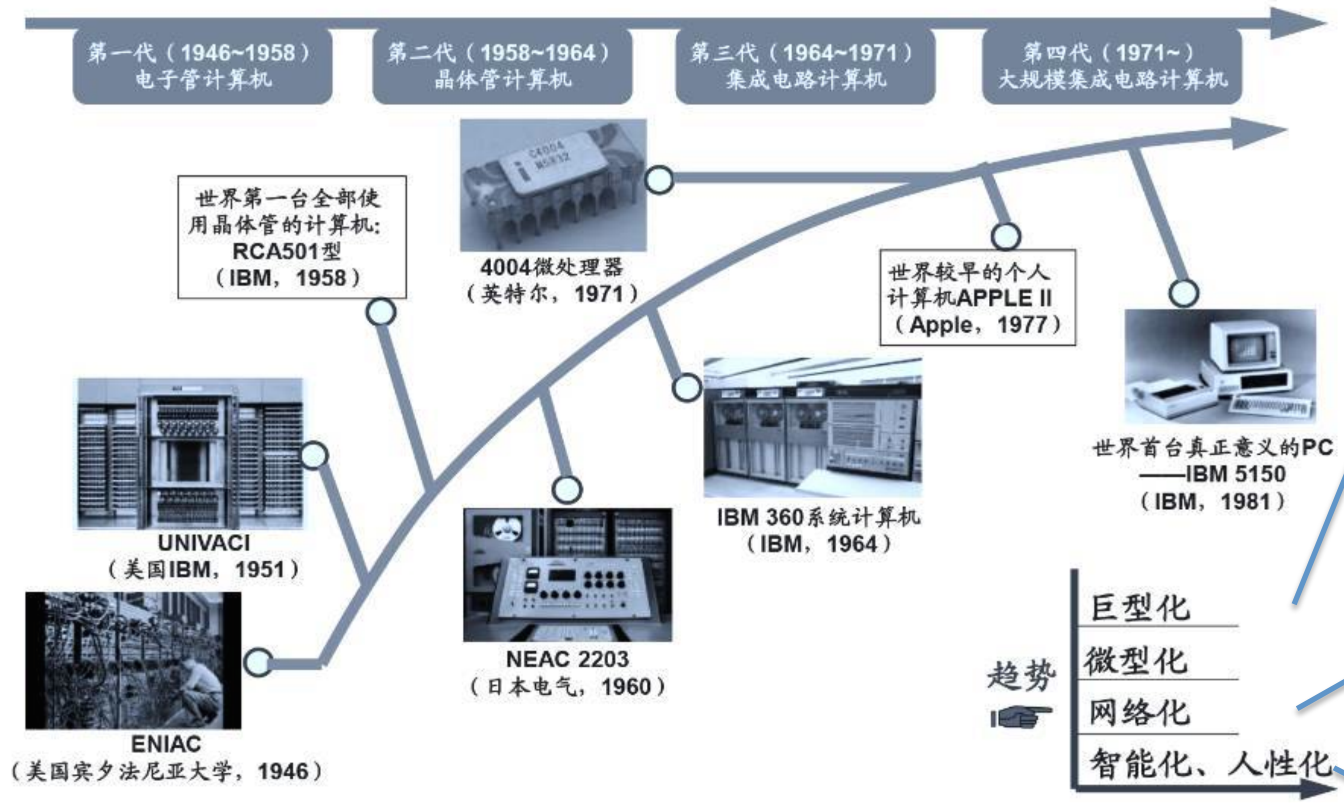
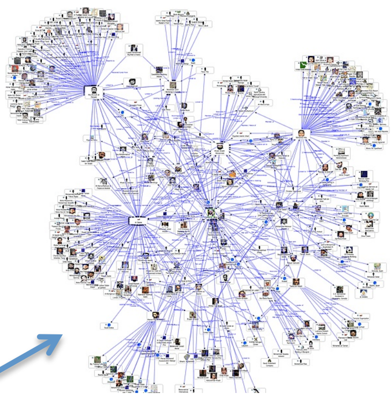




Computer Technology and Science



SUPERCOMPUTER



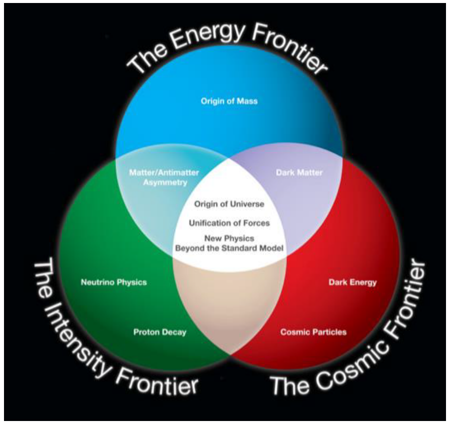
05-0.6 million patents

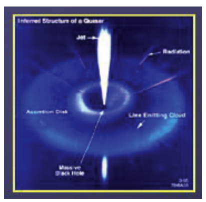
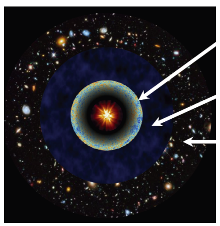
Physics (HEP) & Computer Science

- 1) Computations in Theoretical Physics: Techniques and Methods (MC Event Generators, multi-leg and multi-loop high precision computations, computer algebra techniques and applications, Lattice QCD, Structure formation of early Universe, climate, earth quake prediction, …)
- 2) Data Analysis - Algorithms and Tools (Machine Learning, Neural network pattern recognition, MVA, data mining, quantum computing, …)
- 3) Computing Technology for Physics Research (Computer language, new architecture, data taking, triggers, GPU computing, online computing, parallel computing, visualization,…)

Computational HEP, with HEP-TH and HEP-EXP, becomes the 3rd path to boost the evolution of high energy physics!

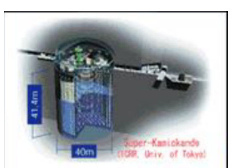
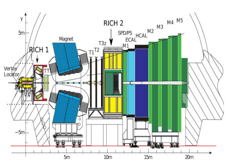
Particle physics and Astrophysics



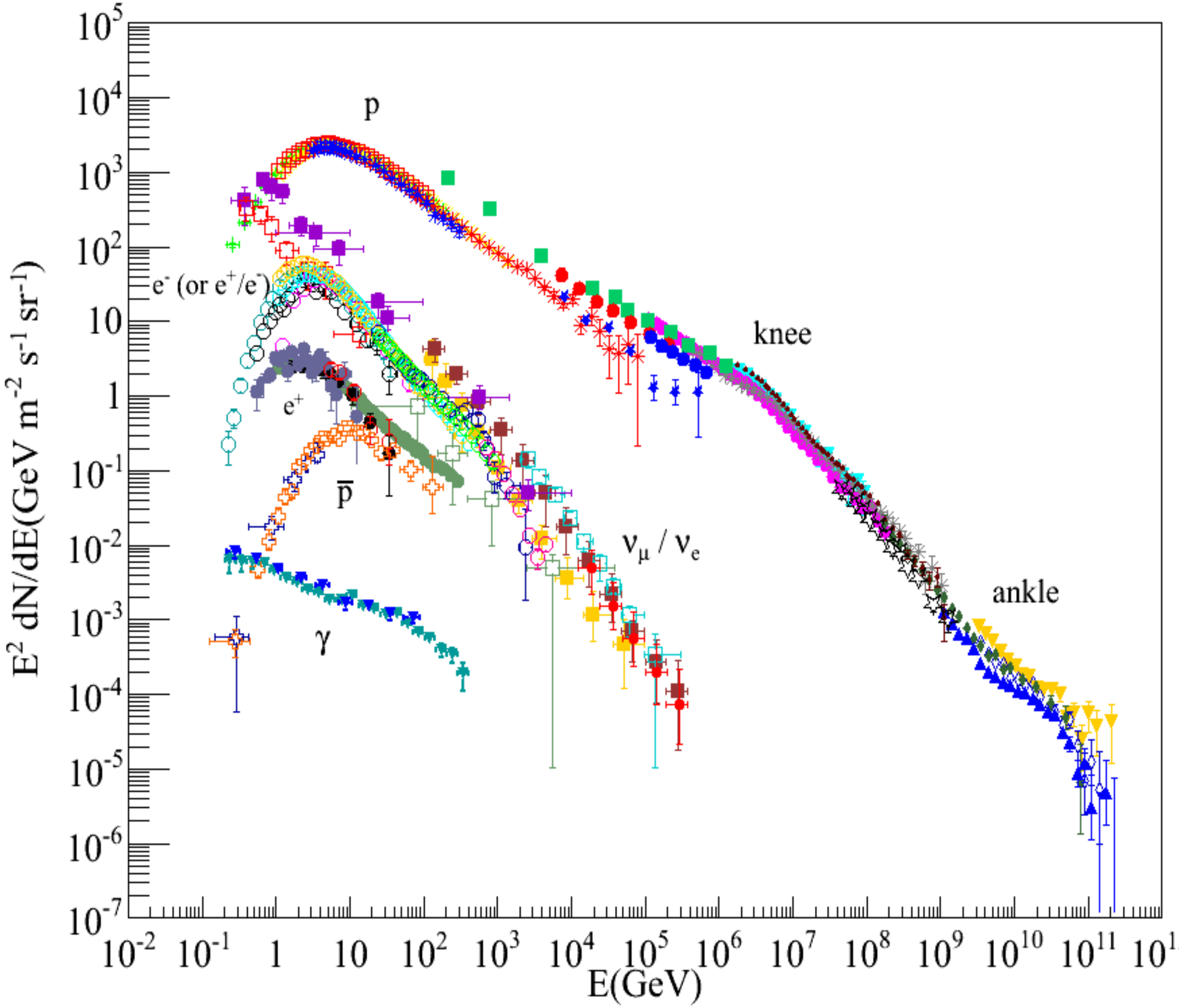
B.Q. Ma

Z.Z.Xianyu

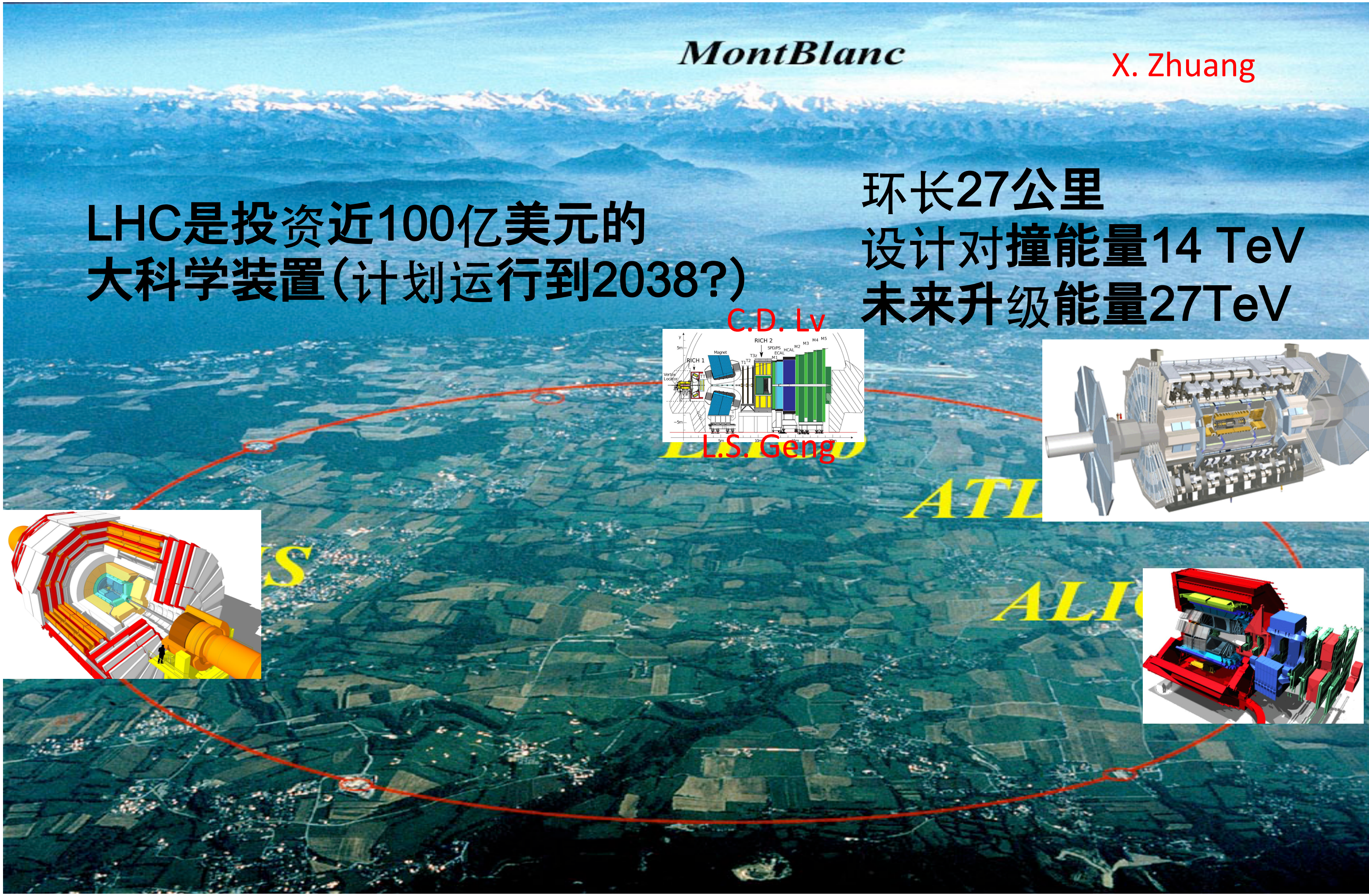



Y. Reyimuaji

C.D. Lv
L.S. Geng



Ultra-High energy Cosmic Rays provide important clues and guides.



LHC是人类探测TeV物质结构的重要手段和工具
TeV物理自然也成为现代粒子物理研究的前沿

The success of HEP

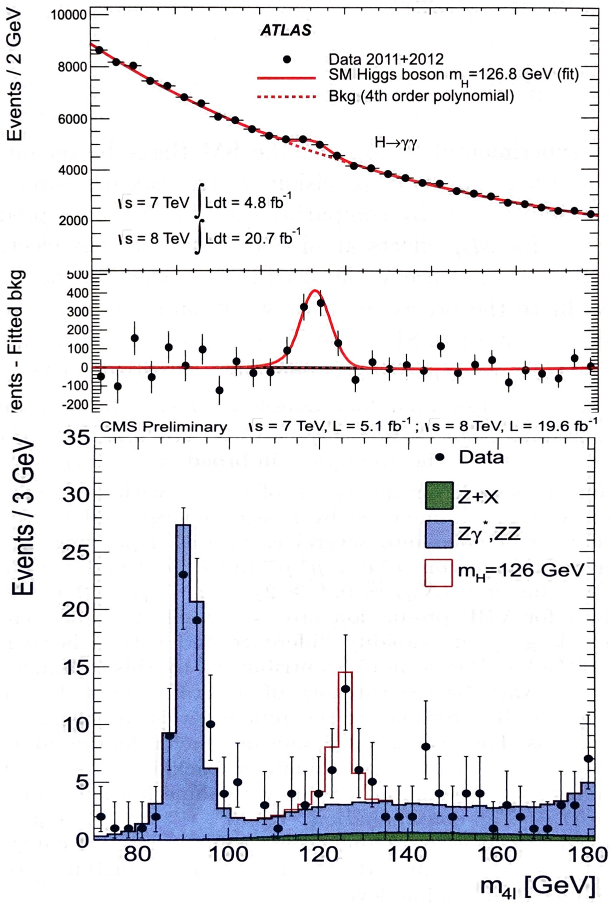


Photo: A. Mahmoud

François Englert

Prize share: 1/2

Photo: A. Mahmoud

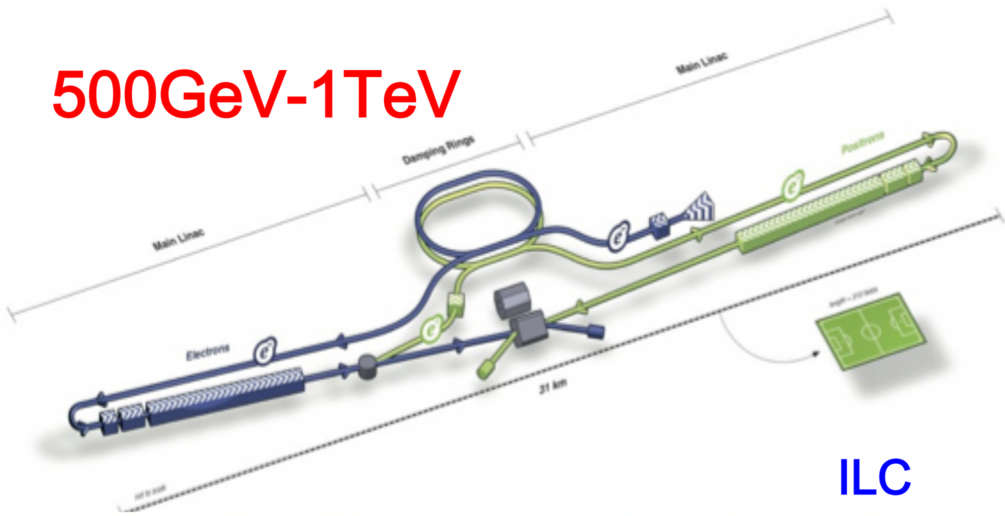
Peter W. Higgs

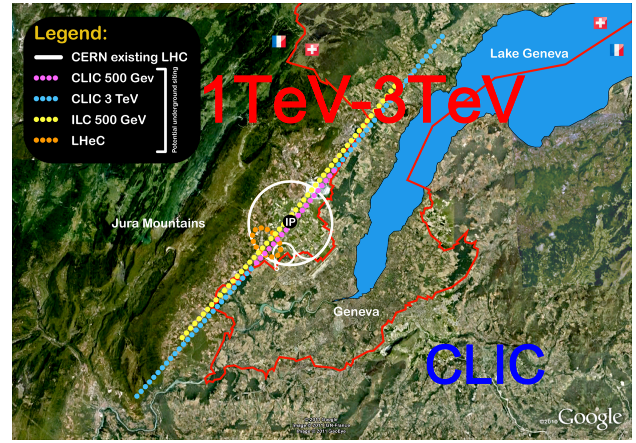
Prize share: 1/2

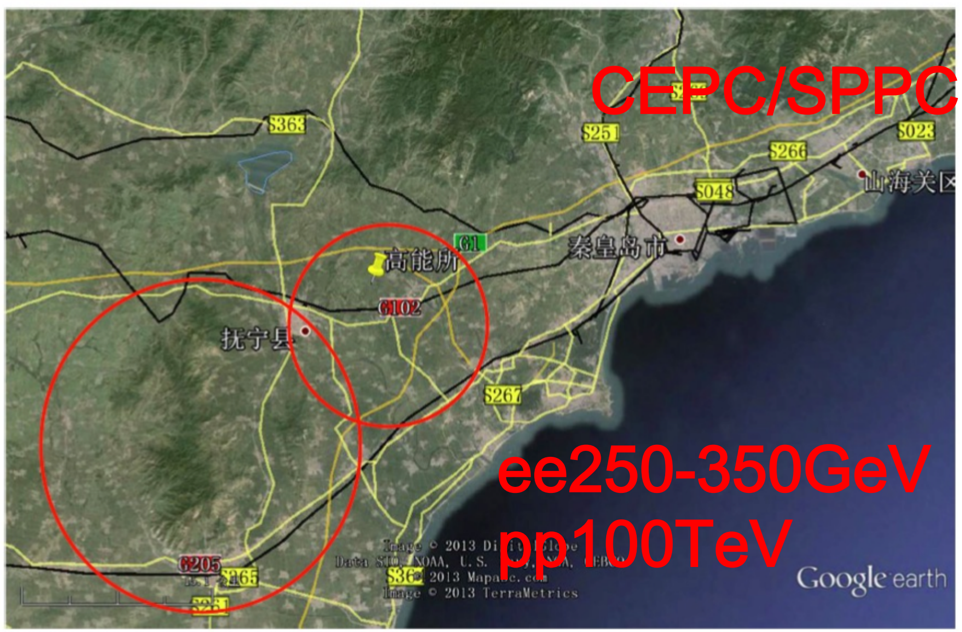
经过50多年的寻找，2012年在LHC发现了Higgs玻色子。
2013年物理诺贝尔奖授予Englert和Higgs，以表彰他们提出的电弱自发破缺理论有助于我们理解基本粒子质量起源。
Higgs玻色子的发现开启了人类对TeV物质世界认识新篇章。

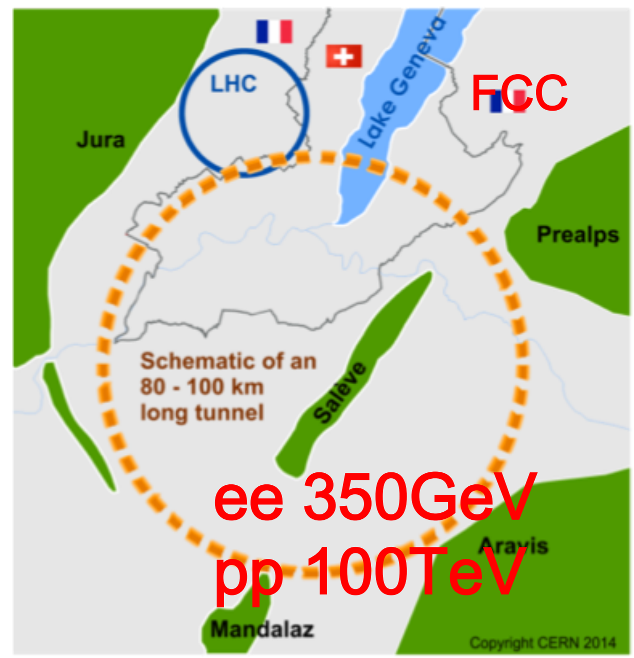
Future HEP collider projects

X.A. Zhuang









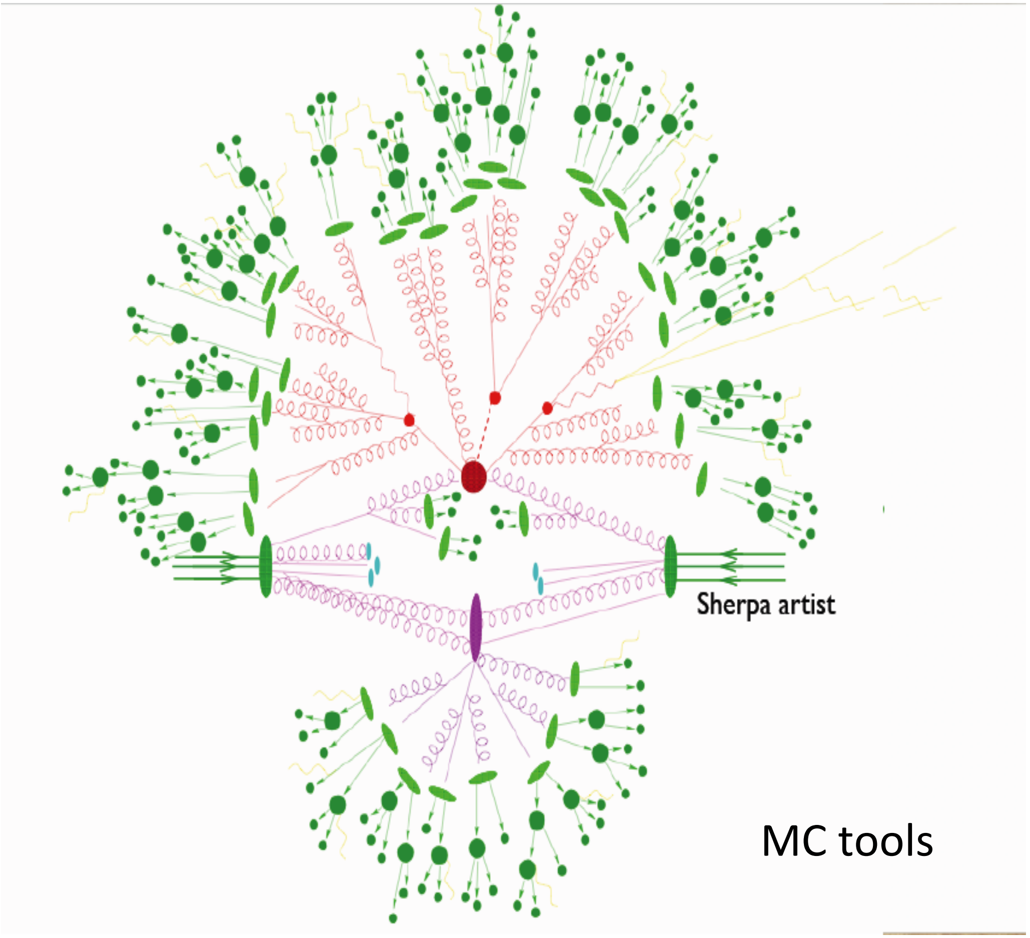
TeV物理：是LHC以后运行及未来对撞机研究计划、现代粒子物理学科的核心问题。

Simplicity v.s. Complexity, and Beauty

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$



Simplicity,
Profound,
and **Beauty**

Complexity,
Comprehensibility,
and **Beauty**

Are tools making necessary for us?

- Human being was defined as a kind of animals who can make and use tools to a remarkable high level.
- Tools shape our science, arts, culture, society, and civilization.
- Tools encode the wisdom of our human beings and are indispensable for our daily life.
- Computing/MC tools are more and more important for HEP.

Are tools making necessary for us?

智人 类人猿

人属

人科

- Humans (variously Homo sapiens) are primates of the family Hominidae, and the only extant species of the genus Homo. Humans are distinguished from other primates by their bipedal locomotion, and especially by their relatively larger brain with its particularly well developed noecortex, prefrontal cortex and temporal lobes, which enable high levels of abstract reasoning, language, problem solving, and culture through social learning. Humans use tools to a much higher degree than any other animal, and are the only extant species known to build fires and cook their food, as well as the only known species to clothe themselves and create and use numerous other technologies and arts.

前额皮质

---From Wikipedia

Are MC tools making necessary for us?

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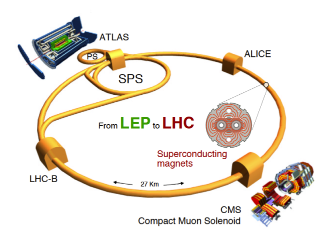


Sakurai Prize 2012

Altarelli

Sjöstrand

Webber

For key ideas leading to the detailed confirmation of the SM of particle physics, enabling high energy experiments to extract precise information about QCD, EW interactions and possible new physics

Pythia

Herwig

HEPPP of EPS Prize 2021

Pythia

Sjöstrand

Herwig

Webber

The 2021 High Energy and Particle Physics Prize of the EPS for an outstanding contribution to High Energy Physics is awarded to Torbjörn Sjöstrand and Bryan Webber for the conception, development and realisation of **parton shower Monte Carlo simulations**, yielding an accurate description of particle collisions in terms of **QCD and EW interactions**, and thereby enabling the experimental validation of the SM, particle discoveries and searches for **new physics**.

Are MC tools making necessary for us?

- 2011, May, 16-18, HeFei, Prof. Y.P. Kuang and Prof. Q. Wang (THU) initialized the idea to form a computing working group (CWG) to develop MC tools for HEP community in China.
- LHC 14TeV runs and Higgs Factory projects (CEPC/SPPC) provide a unique opportunity for the CWG to commence its mission.
- Schools have been being organized to train our young minds.

Objectives of LHC for TeV Physics

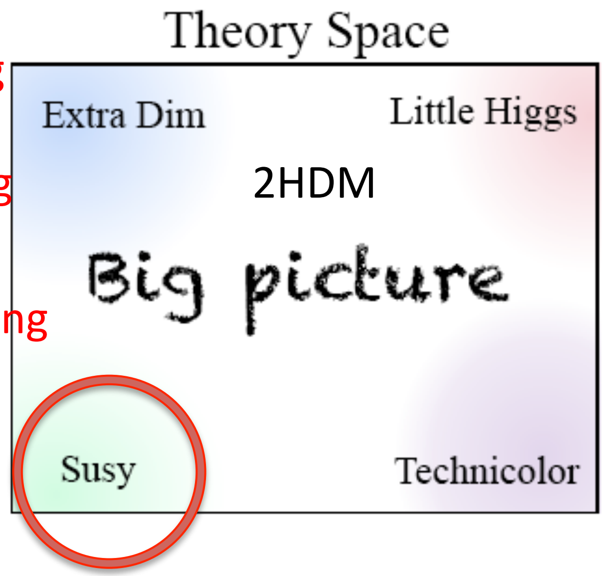
To discover NP, i.e. SUSY, DM, and Exotics, …

Q.Wang

L. Wang

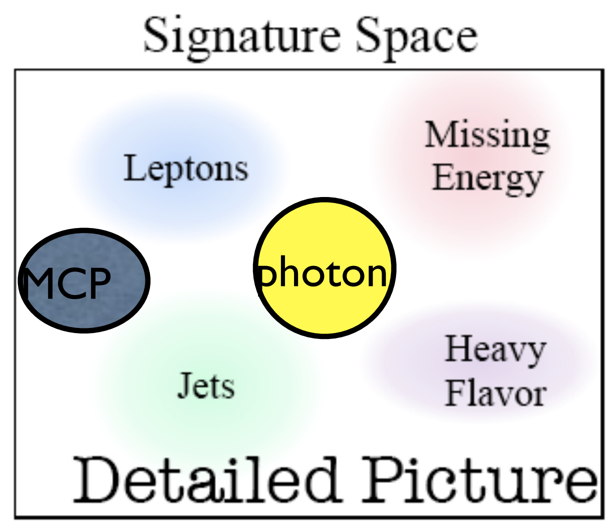
H.H Zhang

J.Cao,
Z. Sun
X.Gao



Constructing Signature Space
from Theory Space
not easy/efficient

Need axes for
Theory Space to map into
Signature Space



X. Zhuang

MC tools bridges TH and EXP (HEP)

New generation (LHC) of MC tools

Theory

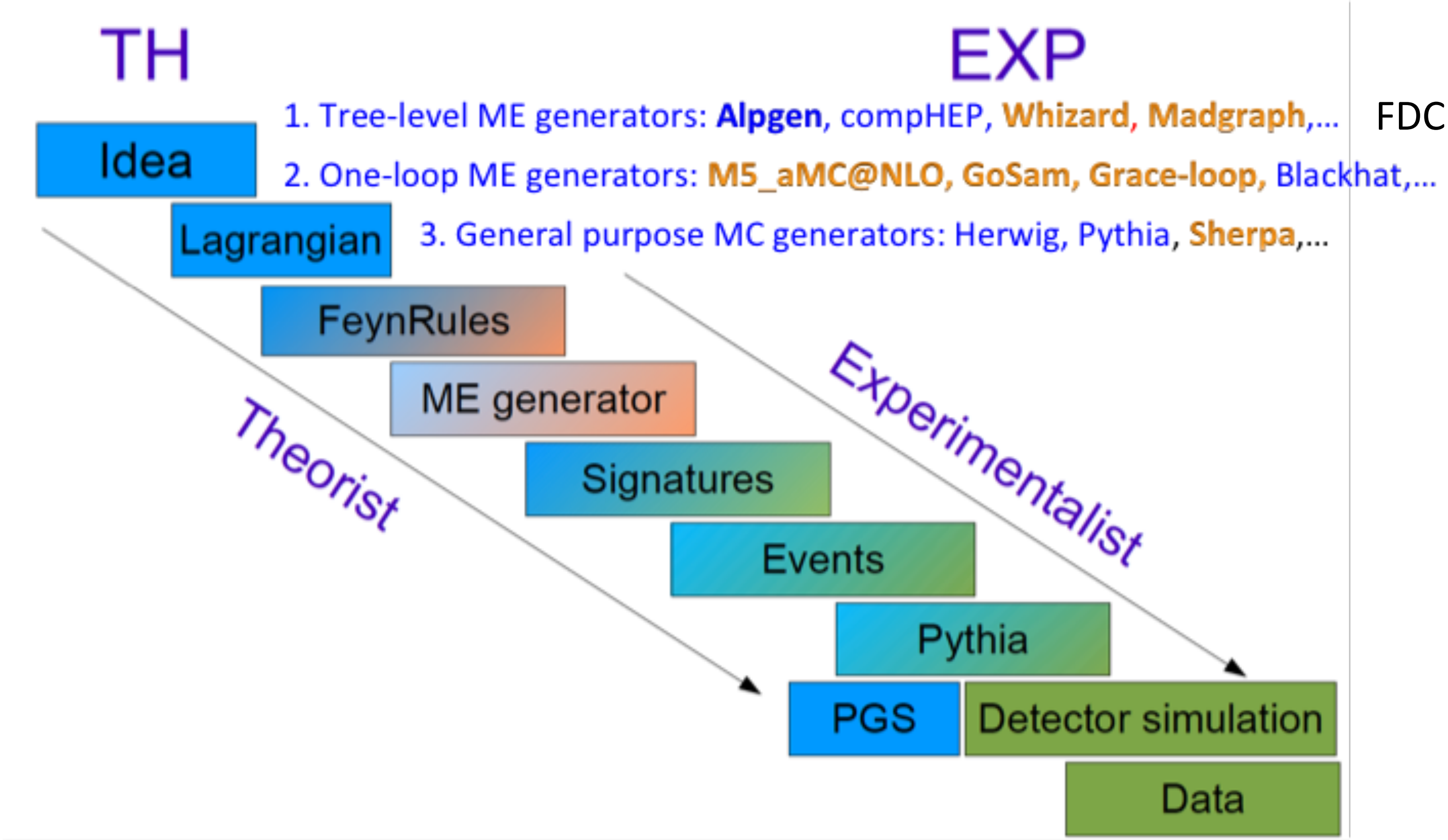
Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation
...

Automation packages: repeatable, process independent, efficient, integration (multi-leg and multi-loop, exp data, ...)

Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network
...

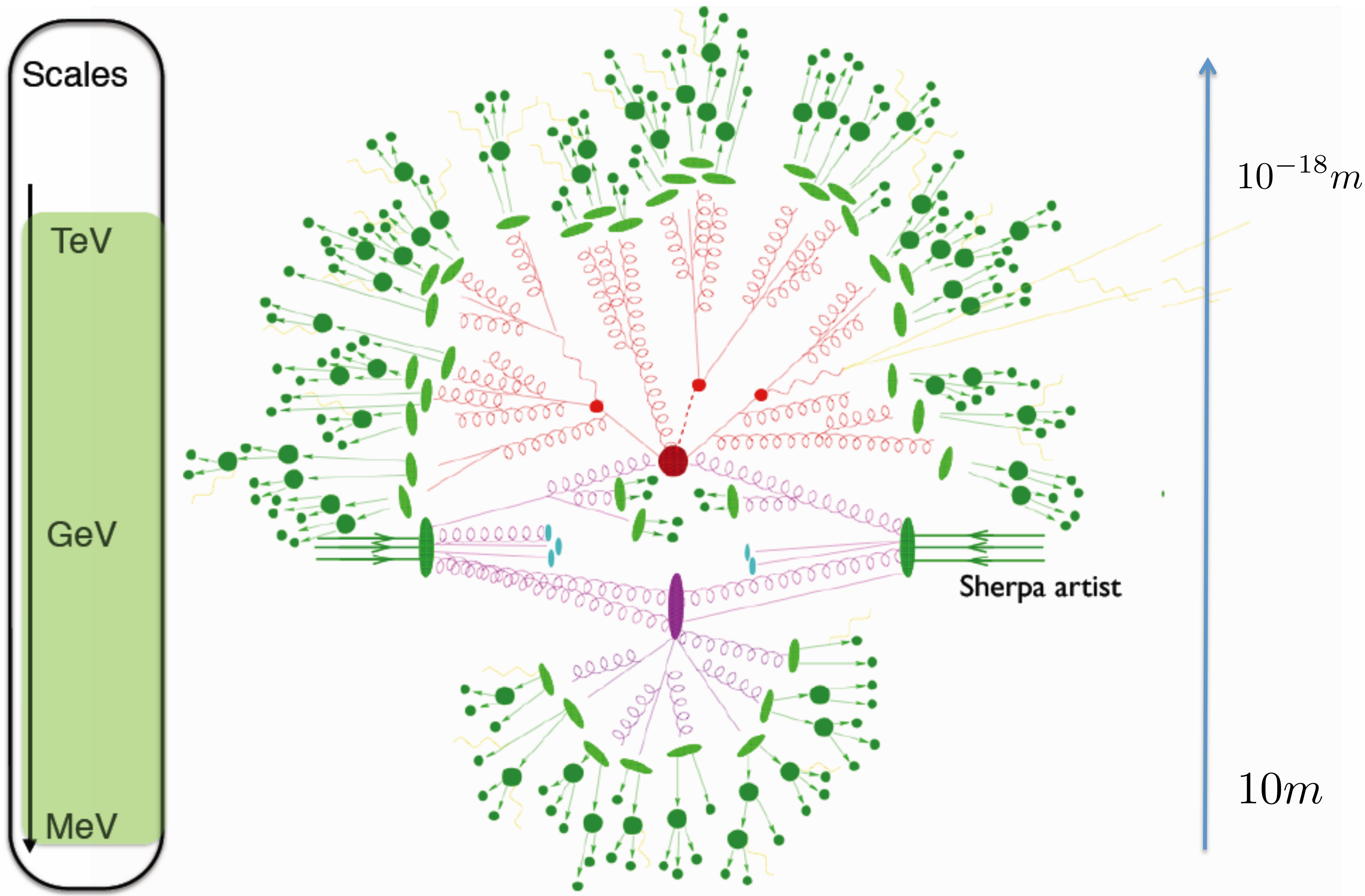
Experiment

The bridge of HEP-MC tools



MC tools bridge theory and experiment and promote the communication and collaboration.

How can the complexity at the LHC be comprehensible?



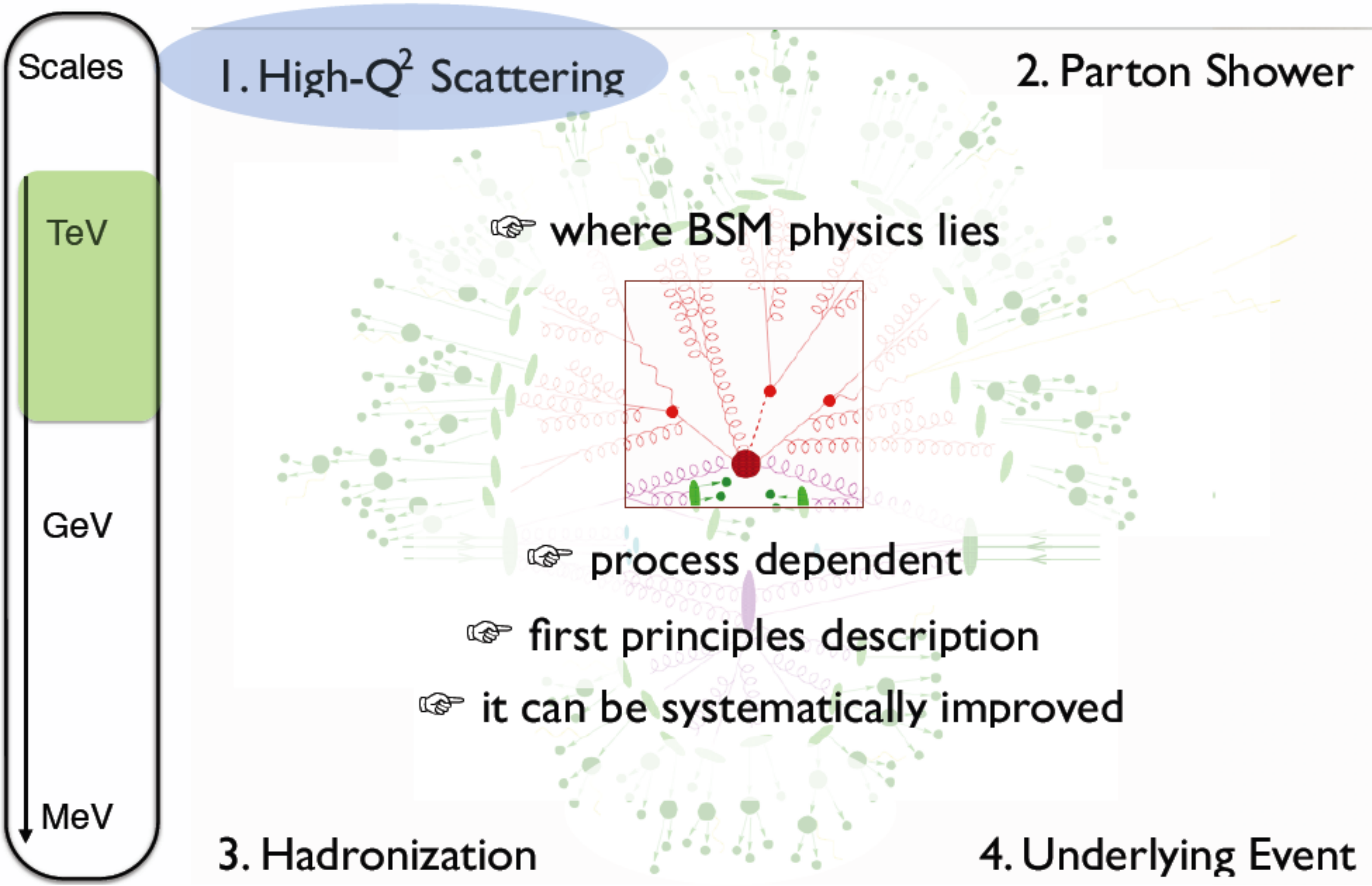
How to comprehend the complexity at the LHC?

Divide and Conquer → Split the problem into many (nested) pieces

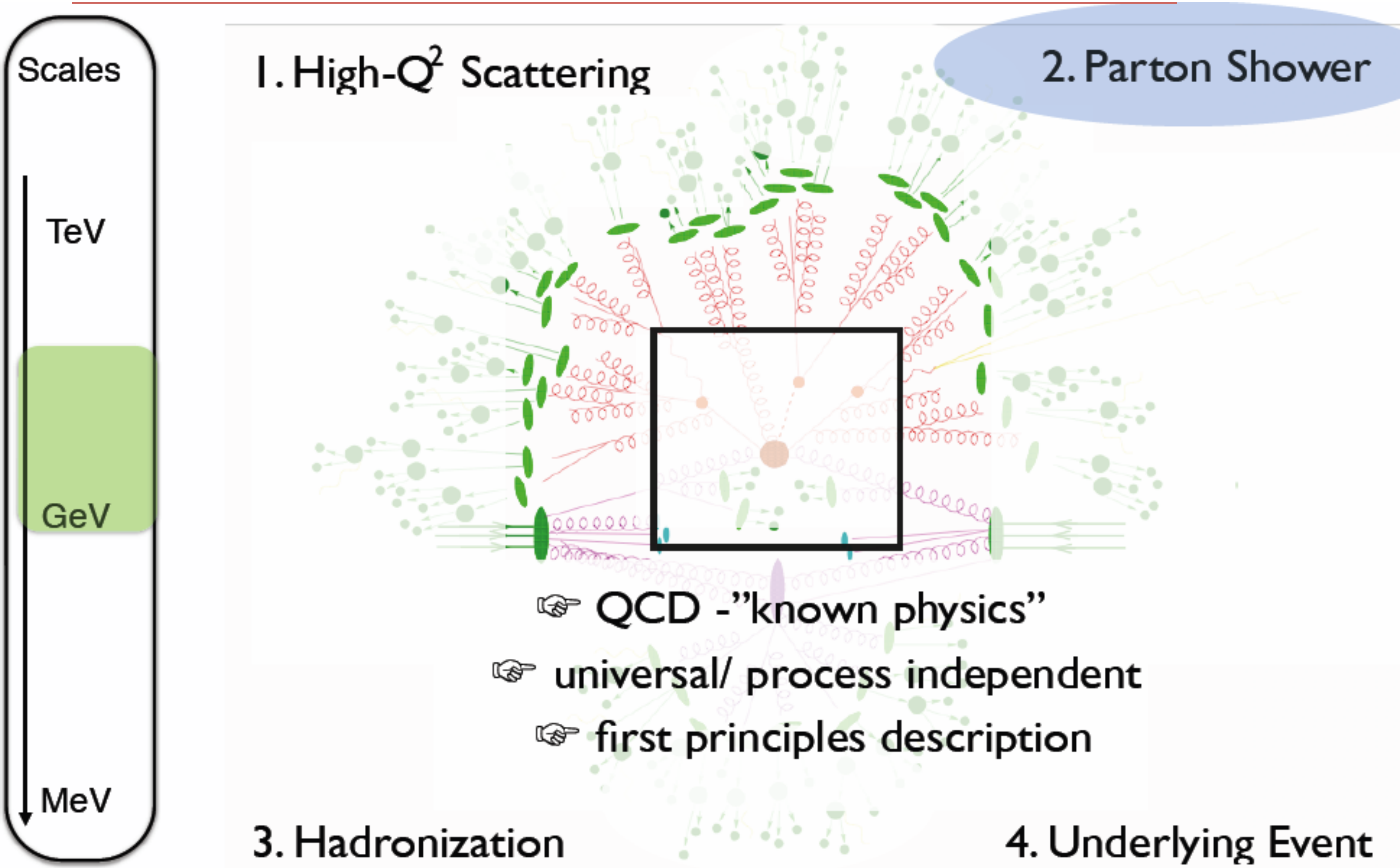
Physics Separation of time scales ➤ Factorisations Maths

$$\mathcal{P}_{\text{event}} = \boxed{\mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}}} \otimes \mathcal{P}_{\text{ISR}} \otimes \boxed{\mathcal{P}_{\text{FSR}}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \ldots$$

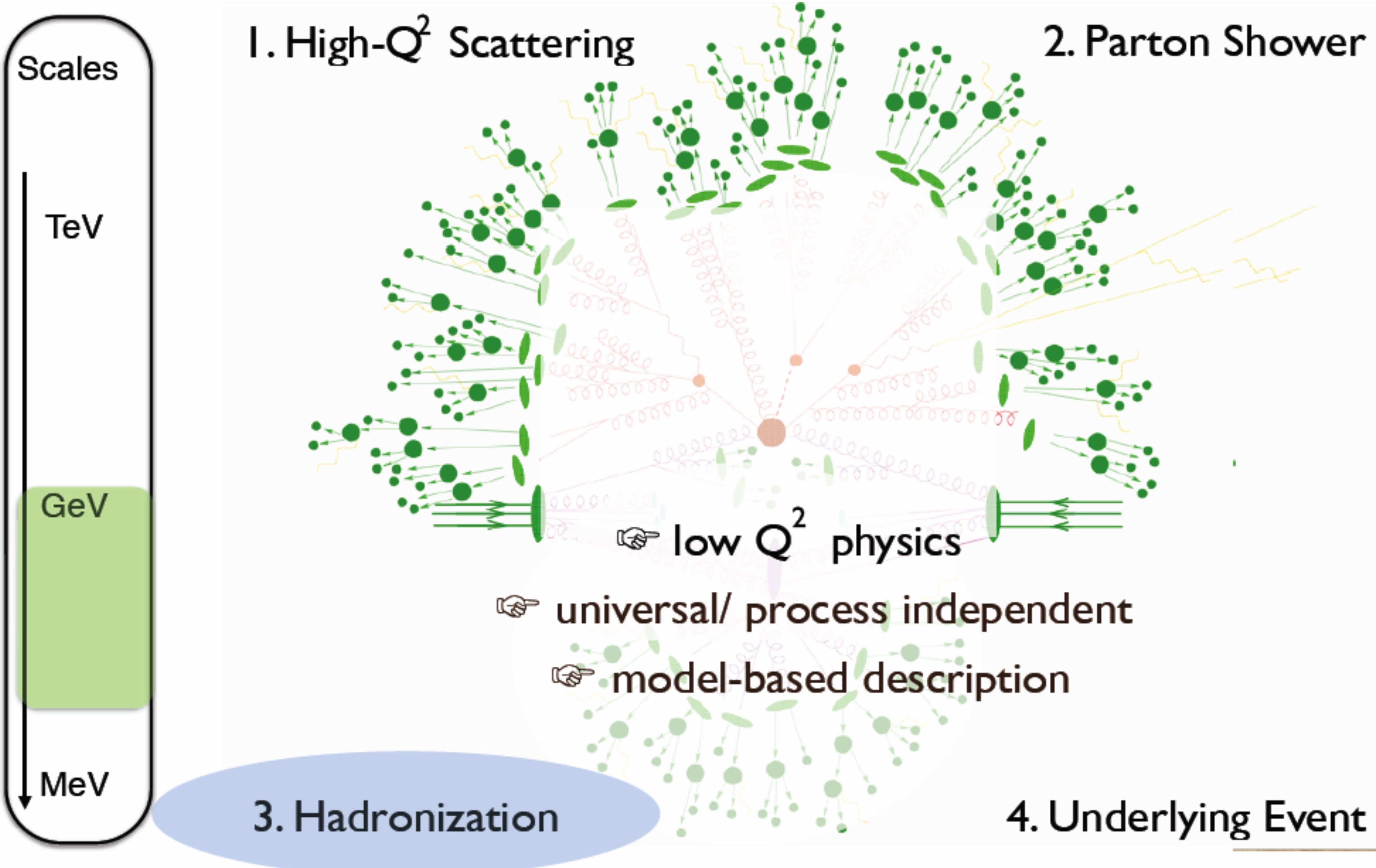
1. Hard processes



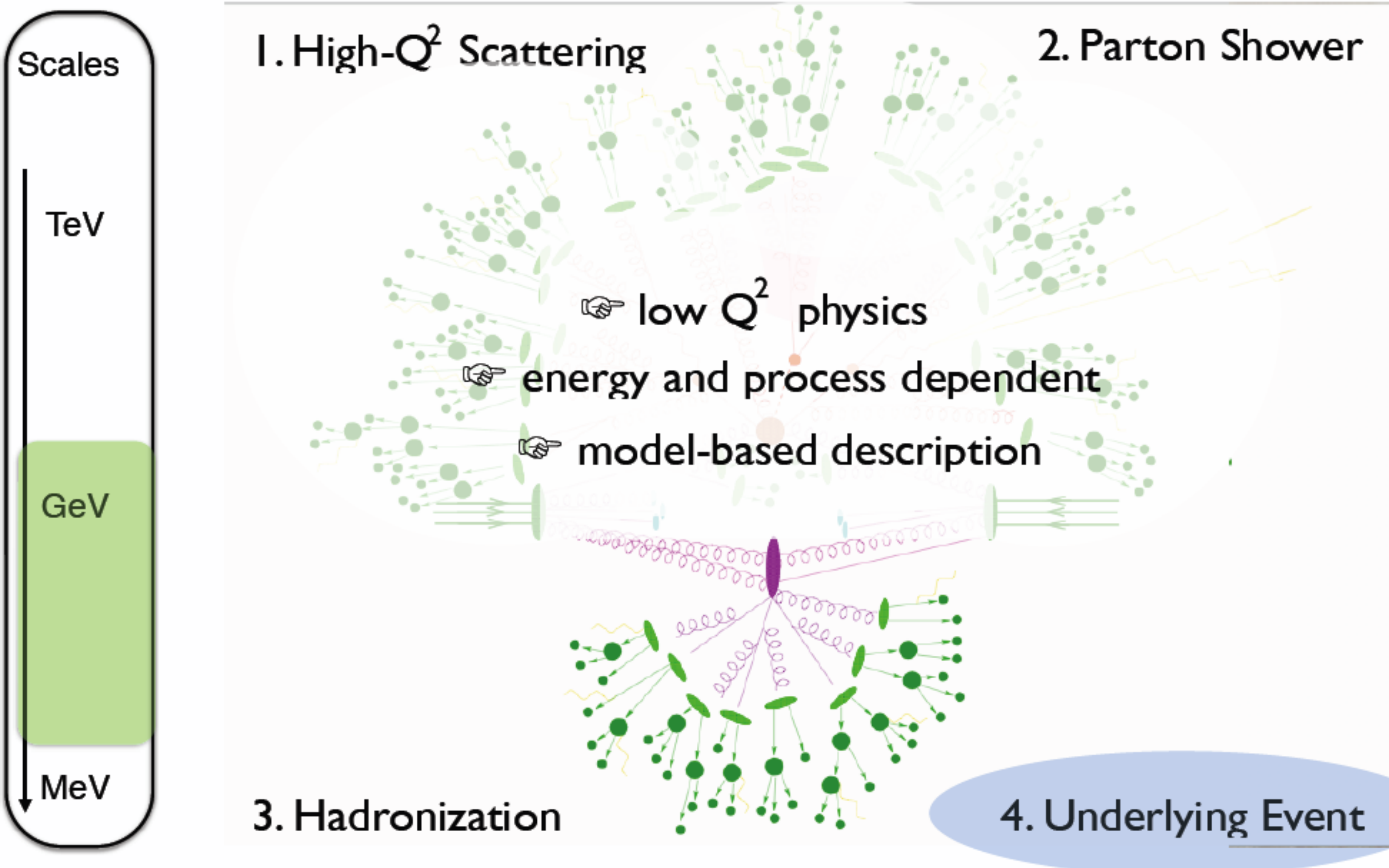
2. QCD shower process



3. Fragmentation/Hadronization/Decay



4. Underlying processes, multi-parton interactions, pileups



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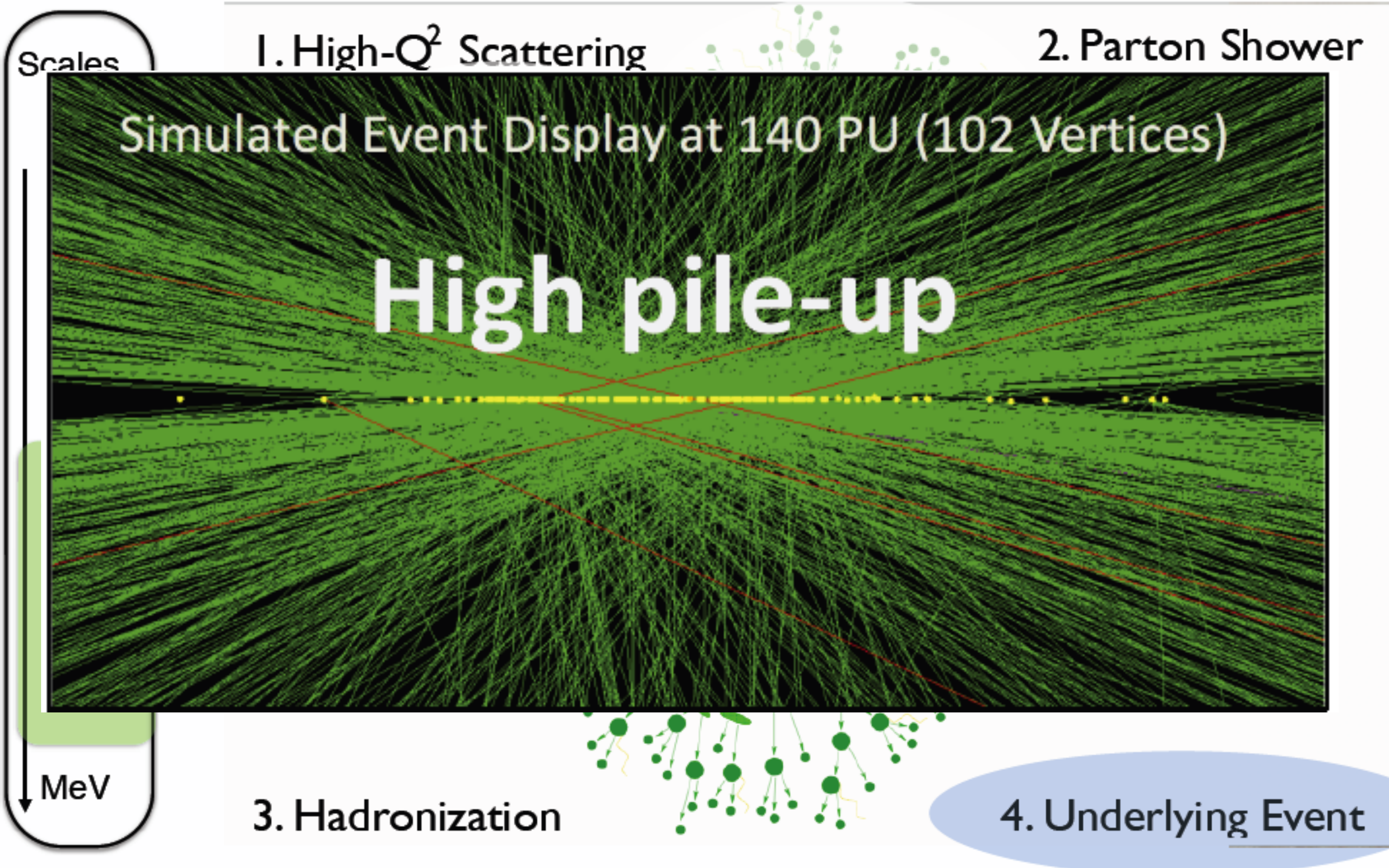
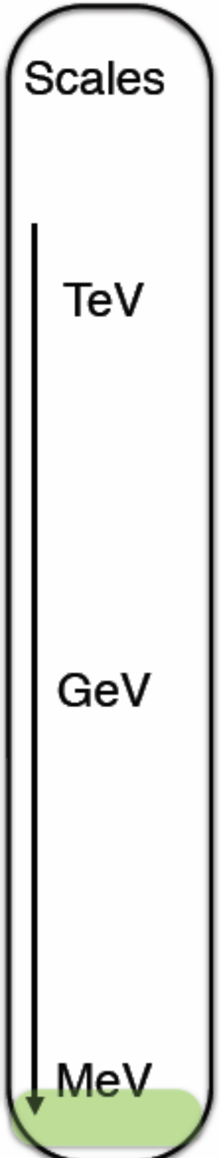
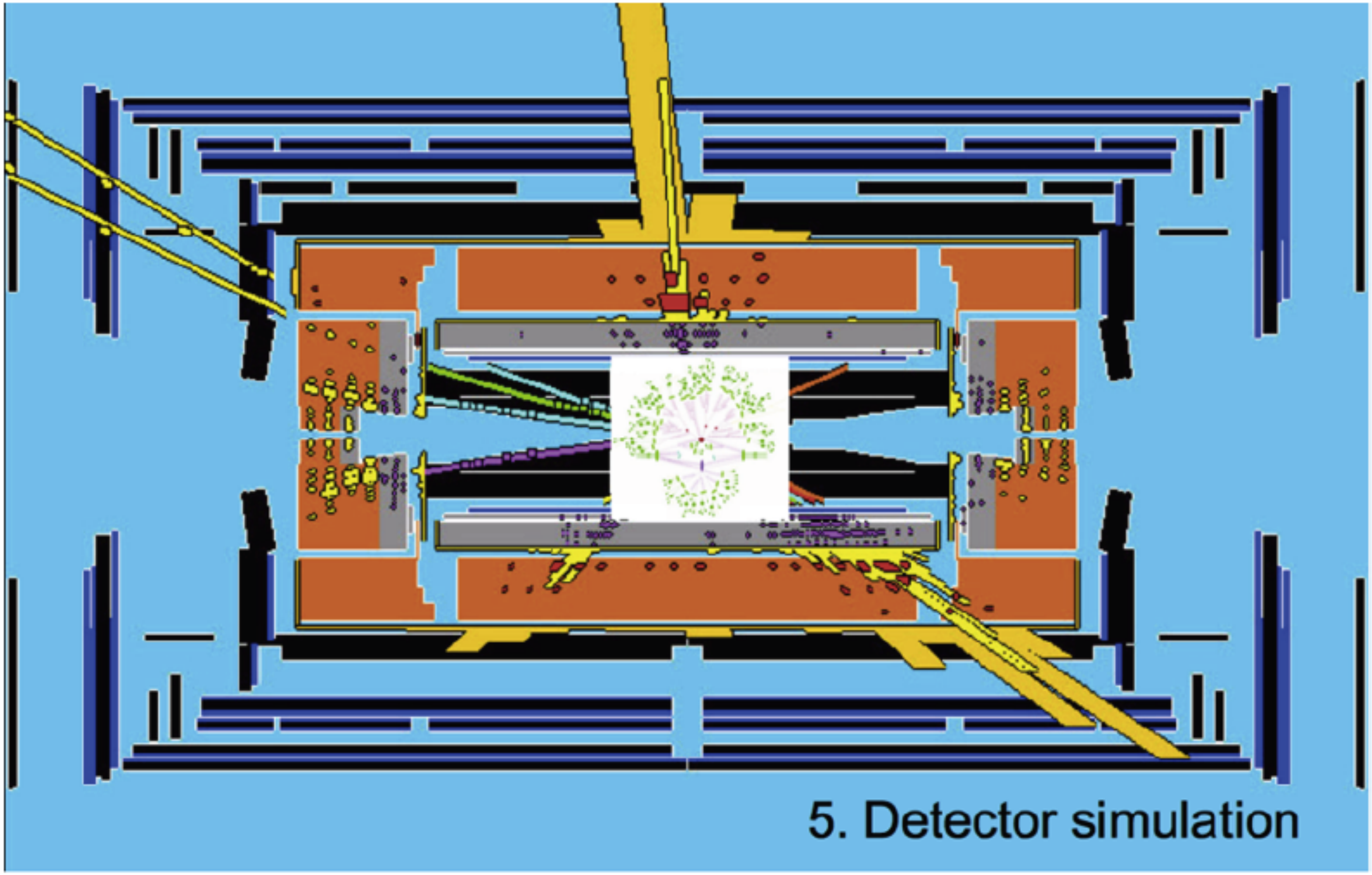
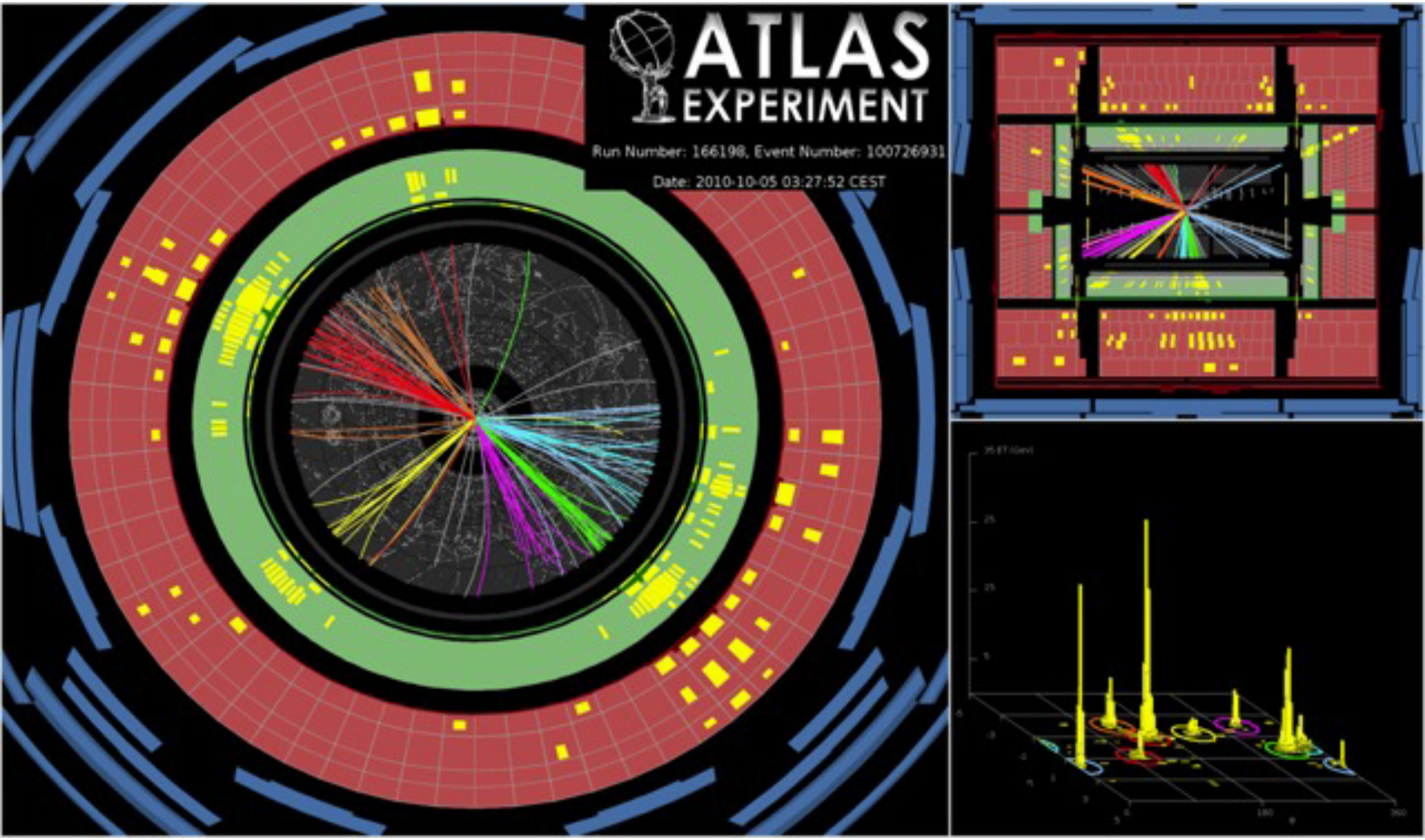


5. Detector simulation and approximations



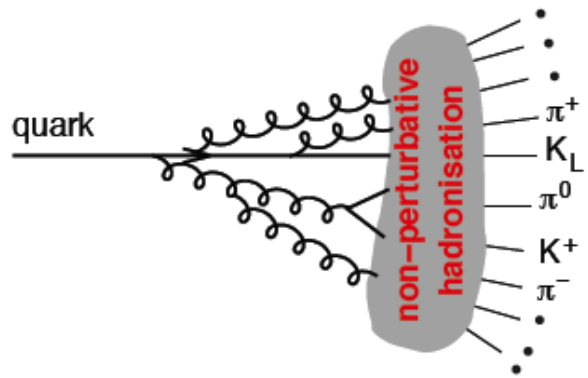


A typical event at ATLAS

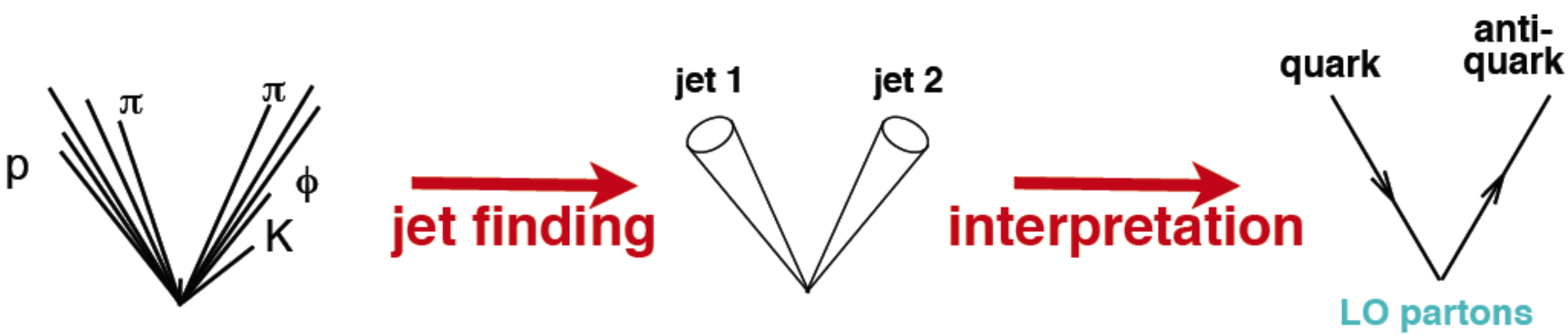


Typically, a hard process can produce around 1000 particles.

How jets come to have the structure they do



How we "reconstruct" jets

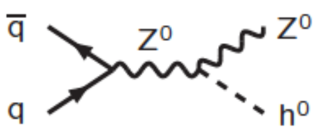


Parton shower plays a central role at the LHC!

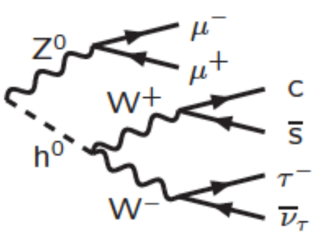
How can we know the physics at short distance?

Matrix elements (ME):

1) Hard subprocess: $|\mathcal{M}|^2$, Breit-Wigners, parton densities.

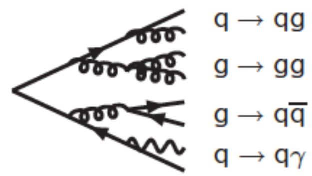


2) Resonance decays: includes correlations.

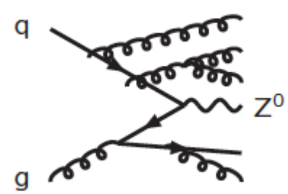


Parton Showers (PS):

3) Final-state parton showers.

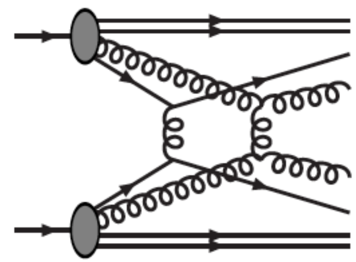


4) Initial-state parton showers.

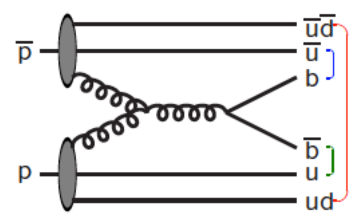


pQCD

5) Multiple parton–parton interactions.

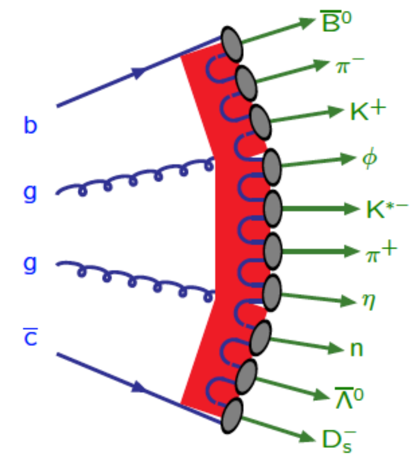


6) Beam remnants, with colour connections.

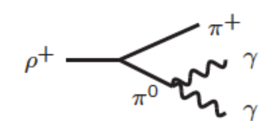


5) + 6) = Underlying Event

7) Hadronization



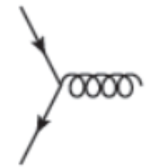
8) Ordinary decays: hadronic, τ, charm, . . .



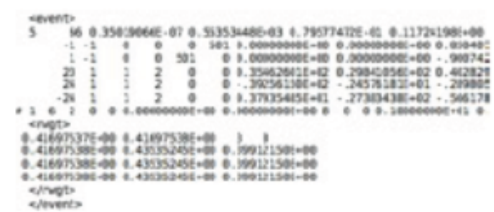
The physics from 3)-8) is universal for SM and NP, and had been proven by earlier HEP data (LEP, Tevatron).

A well-educated and more precise description

SU(3)XSU(2)XU(1) • **Symmetries** $G^{\mu\nu}G_{\mu\nu} + \imath\bar{q}_{(i)}D_{\mu}\gamma^{\mu}q_{(i)} + \cdots$

Standard Model • **Model**  $= \imath\gamma^{\mu}t^{a}_{ij}$, ...

p p > j j QCD=2 • **Matrix Element** $\mathcal{M}^2_{gg\to d\bar{d}}$, ...

matrix.f • **Partonic Events** 

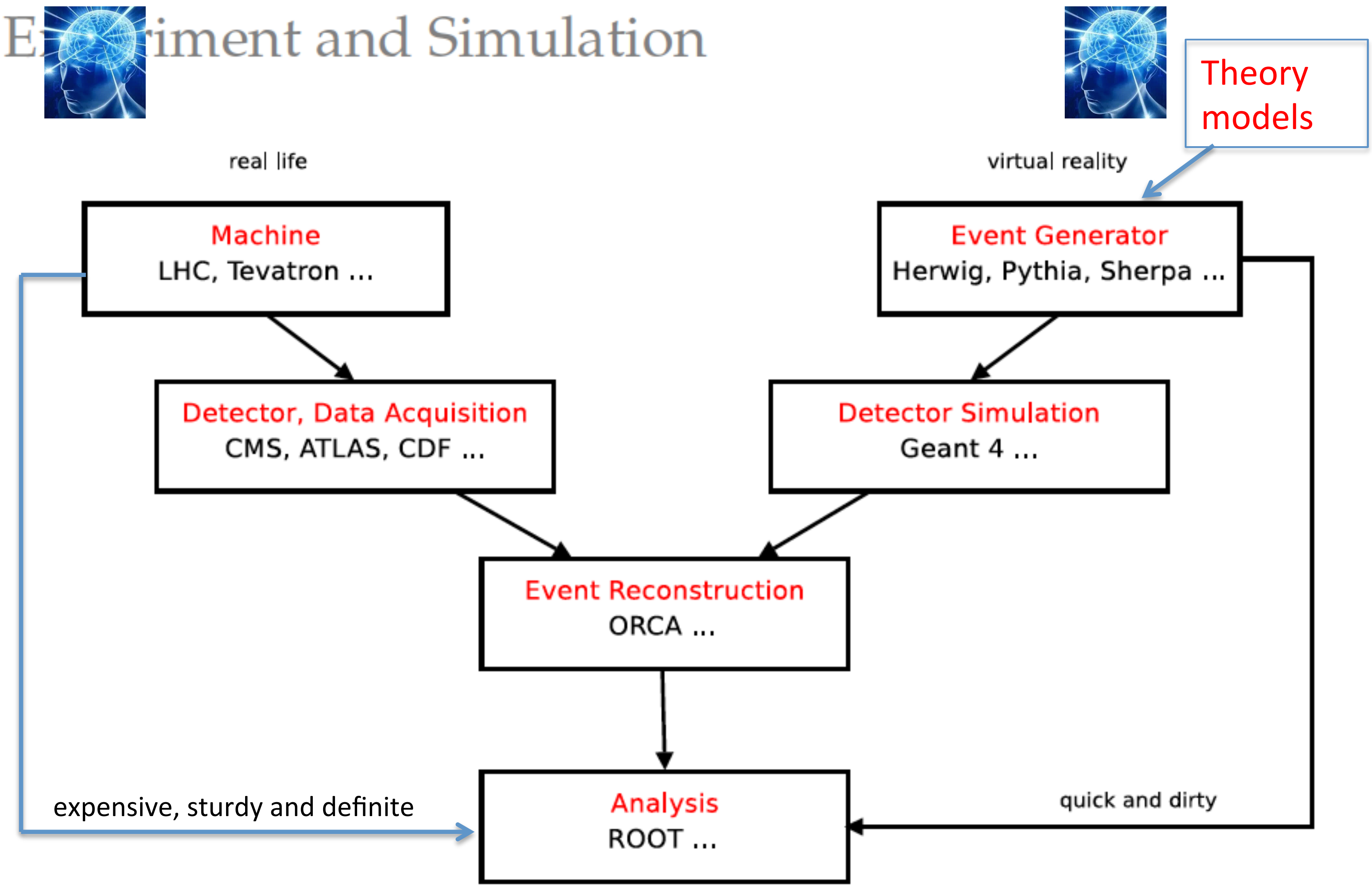
events.lhe • **Hadron Level** $\{\pi^0, K^+, e^+, p, \cdots\}$

events.hep • **Detector Level** 

events.lhco

How a NP model is tested by HEP-EXP?

X. Zhuang



Madgraph in a nutshell

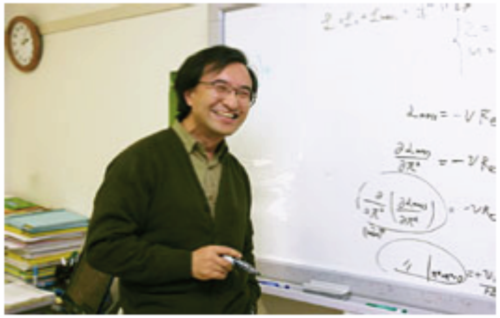
1. **Some history of Madgraph**
2. **ME calculation**
3. **Phase Space integration (MC)**
4. **Parton Shower (MC)**

Not cover: UFO interface for NP,QCD NLO, EW NLO,
Jet-PS matching, Jet physics,
detector simulation, MadAnalysis, root,
MadDM, …

A short history of Madgraph

- 1991 HELAS
- 1994 MadGraph
- 2002 MadEvent
- 2006 MG/MEv4
- 2011 MadGraph5
- 2014 MadGraph5_aMC@NLO

J.X. Wang KEK Grace PDF



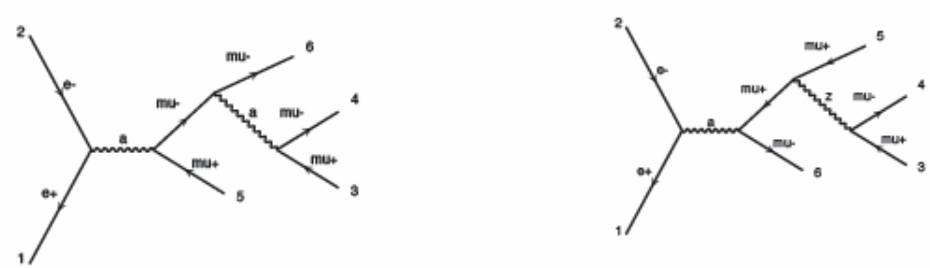
- Computing Matrix Element for a fixed Helicity and sum over the felicities.

- Suite of Routine, which allow to write the matrix element for any (SM) process

A short history of Madgraph

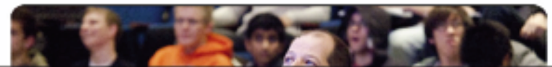
1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Automate the creation of the diagram generation and the writing of the HELAS routine



A short history of Madgraph

1991 HELAS



199

200

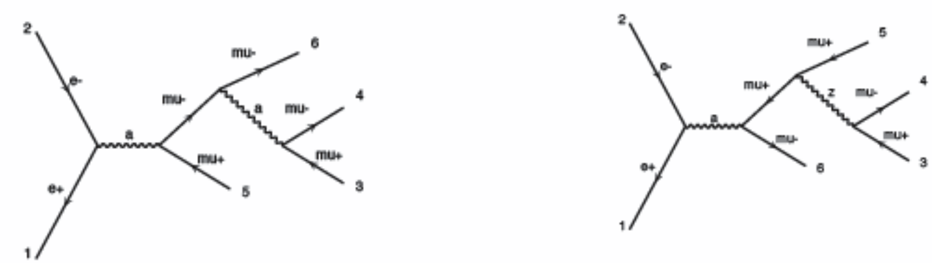
200

MAD stands for Madison

gram

2011 MadGraph5

2014 MadGraph5_aMC@NLO



A short history of Madgraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Multi-Channel Method!
- Automatic phase-space Integration
- Generation of Events

- Support for the MSSM (SMADGRAPH)

A short history of Madgraph

1991 HELAS

1994 MadGraph

2002 MadEvent

2006 MG/MEv4

2011 MadGraph5

2014 MadGraph5_aMC@NLO

- Support for BSM
- Decay Chain
- Pass to a platform (MadOnia/MadWeight/...)
- Link to Pythia/PGS
- Matching/Merging

- Official/Main SM generator for CMS

A short history of Madgraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_aMC@NLO

- Full restart of the MadGraph part in Python
- Fully Automatic BSM
- Various Output Format
- Huge Improvement

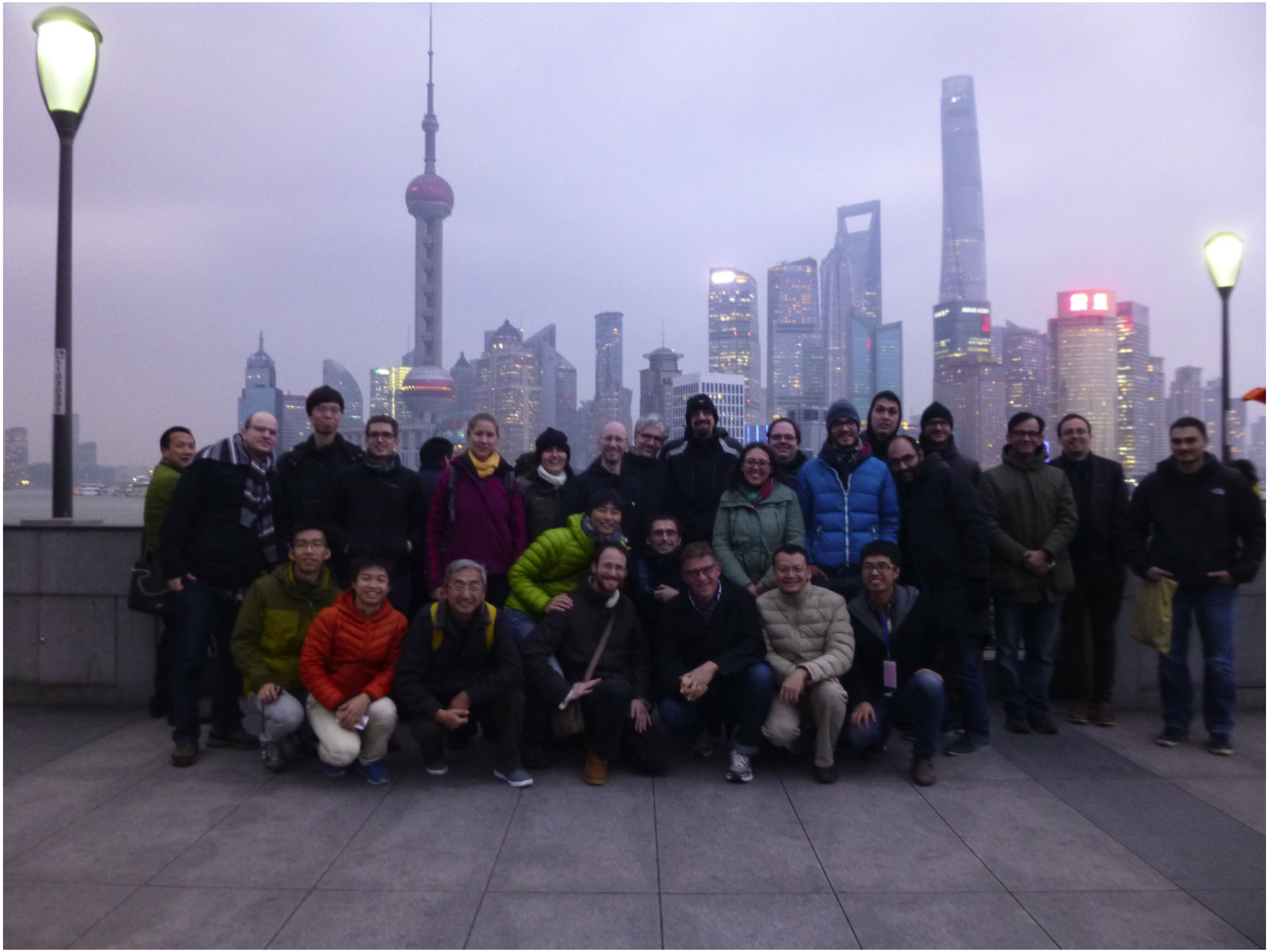
A short history of Madgraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

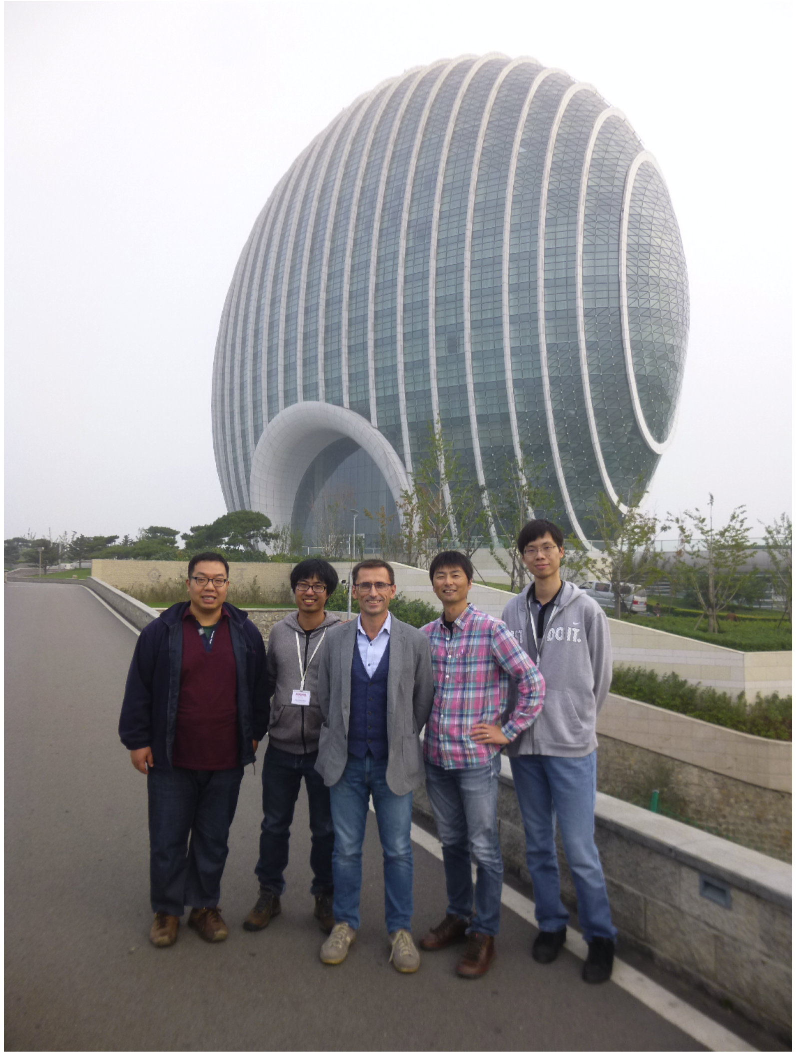
- Fully Automatic computation at
  - NLO* (cross-section)
  - NLO* matched to PS

*NLO= NLO in QCD

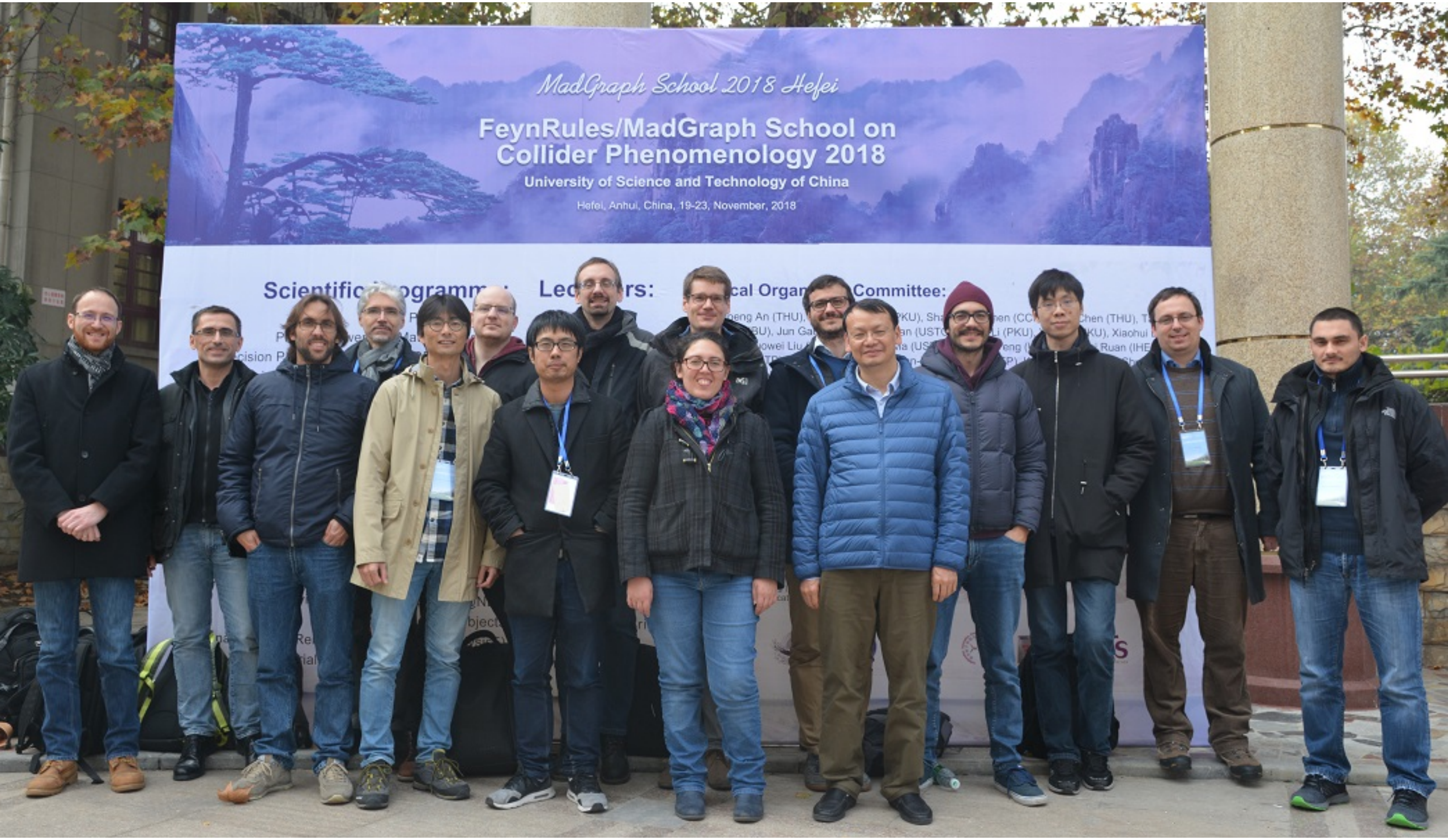
A short history of Madgraph (2015, Nov, Shanghai)



Several Madgraph family members (2016)



A short history of Madgraph



Madgraph school 2018 HeFei

The SMEFT paradigm

* In the last decade, the SMEFT has become a **key part of the LHC physics programme**
* Paradigm shift: we want to measure the parameters of the

Standard Model $\Rightarrow$ **Standard Model Effective Field Theory**

* How?

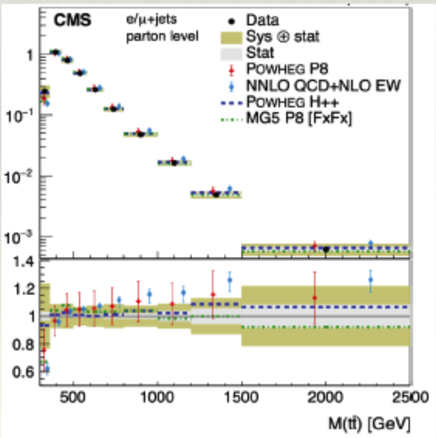
1. Measure observables (O) that depend on them: e.g. particle production at the LHC
2. Compare to theoretical prediction

$$\Delta O_n = O_n^{\mathrm{EXP}} - O_n^{\mathrm{SM}} = \sum_i \frac{a_{n,i}^{(6)} c_i^{(6)}}{\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Difference between experiment & SM theory = **New contributions from SMEFT operators**

Data/Theory

Theory/Data



Requires:

a) Good measurements

$\Rightarrow O_n^{\mathrm{EXP}}$

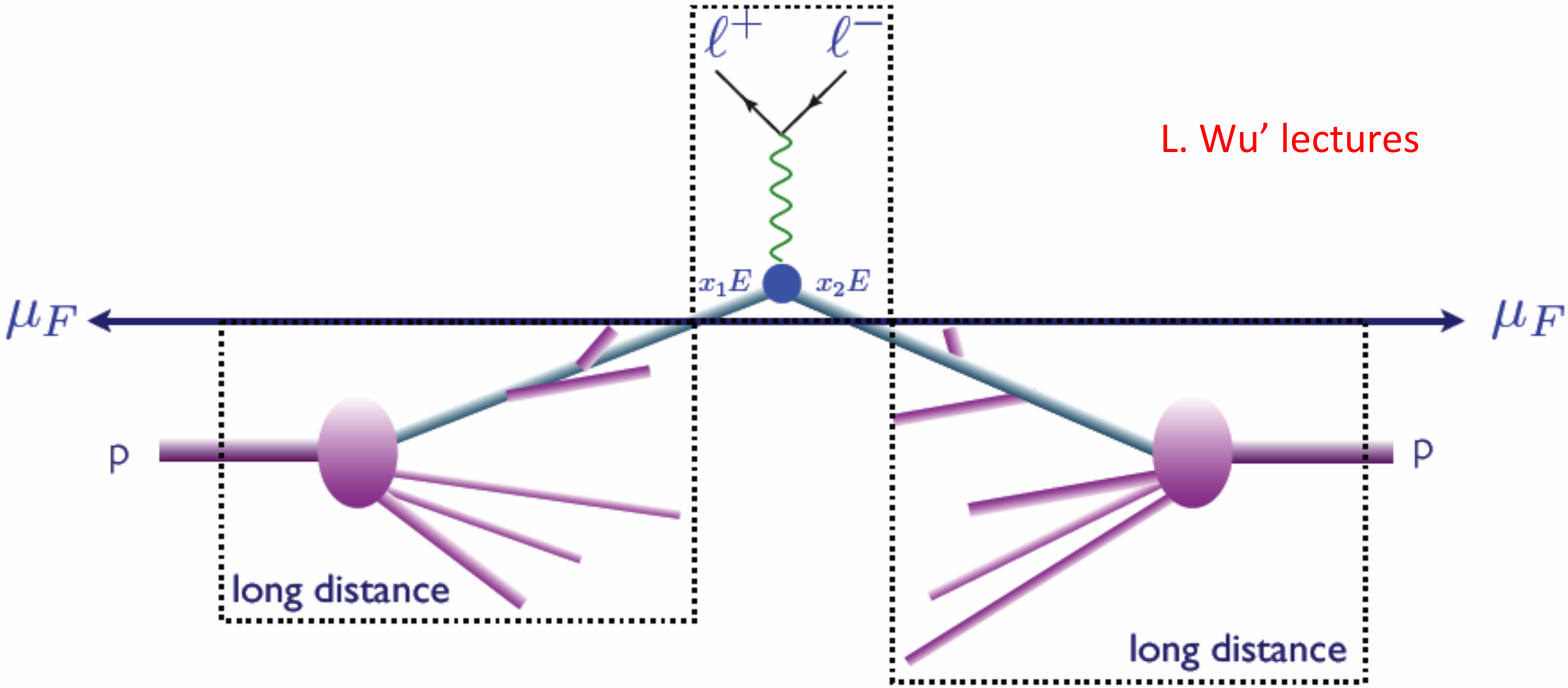
b) Reliable predictions

$\Rightarrow O_n^{\mathrm{SM}},\ a_{n,i}^{(6)}$

Cen's expertise

Understand the complexity at the LHC from QFT

L. Wu' lectures



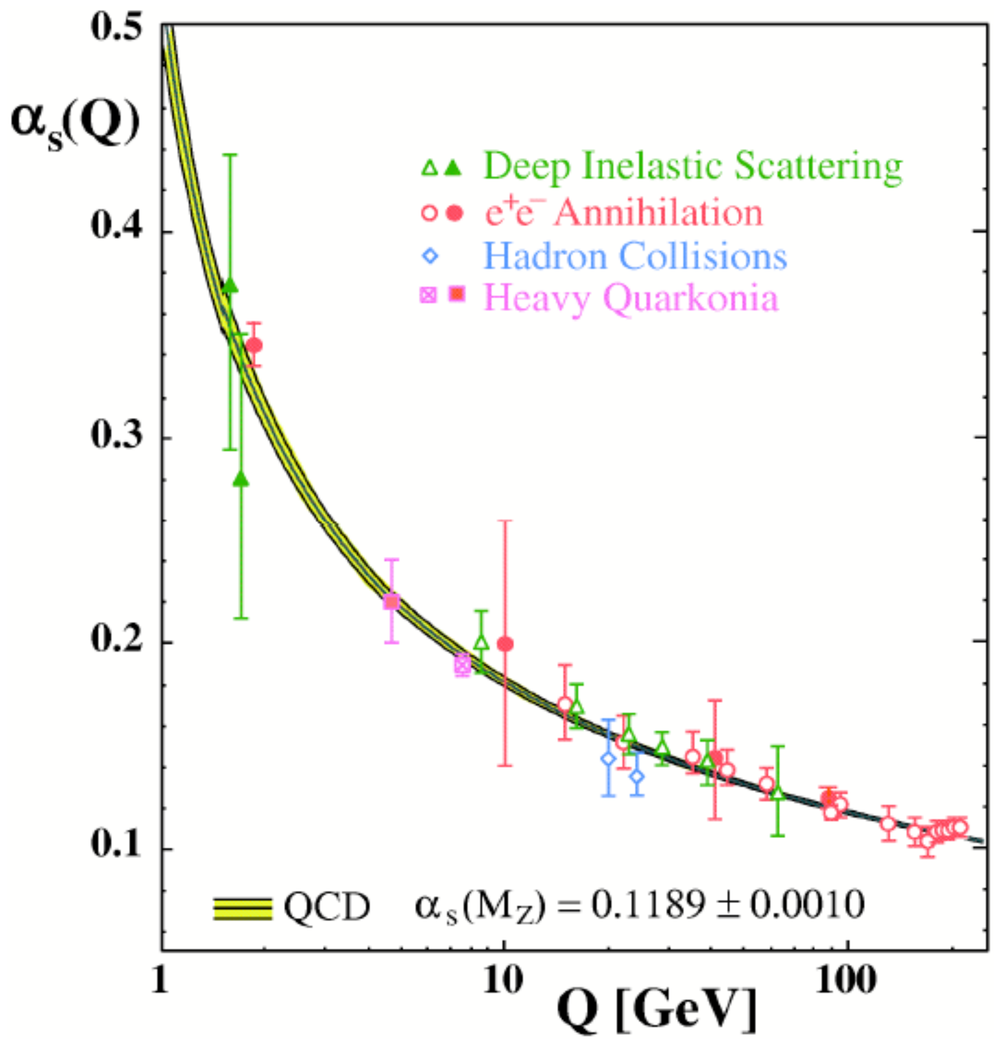
$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

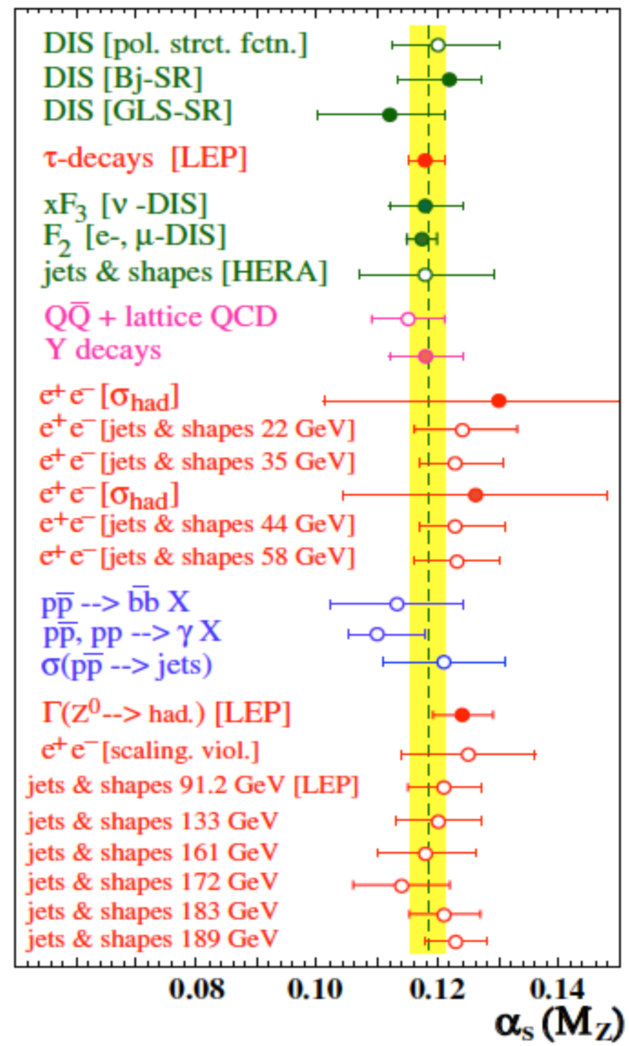
Phase-space integral | Parton density functions | Parton-level cross section

QCD: Asymptotic freedom

C.F. Qiao's lecture
X.G. Wu's lecture

$$q^2\frac{\partial a_s}{\partial q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$





Perturbation calculation of QCD at high energy scale

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

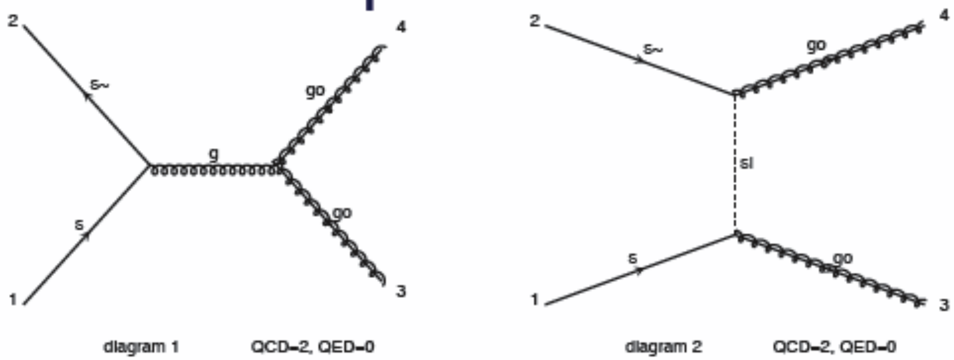
LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties

Total and Differential Cross Section

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

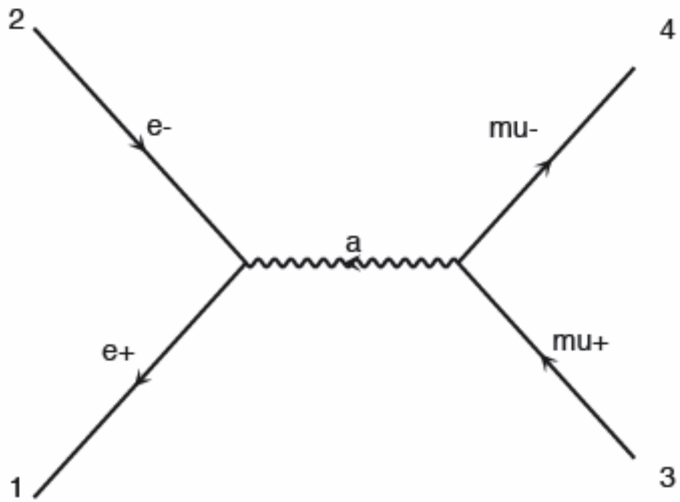
$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Very Hard (in general)

Standard Textbook Method (Klein and Nishina)



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \quad \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \quad \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!

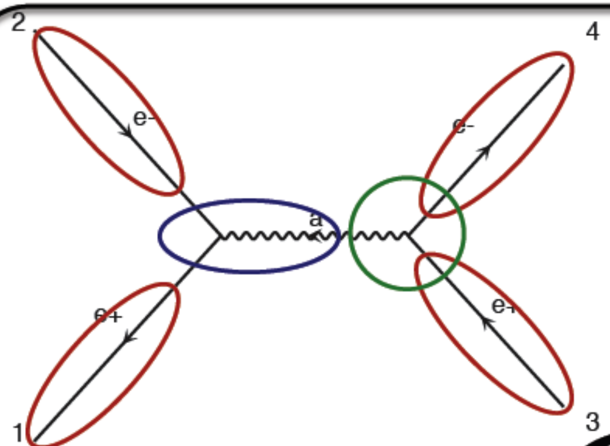
Only for 2→2 and 2→3

Because the number of terms rises as N^2

S Matrix Element is just a complex number!

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Sum over helicity configurations and average the results



$$\mathcal{M} = (\bar{v}\, e\, \gamma^{\mu}\, v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^{\nu}\, u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m \\ \mathcal{M} = fct(v_3, \bar{u}_4, W \end{cases}$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_{\lambda}(\vec{p}) \\ \omega_{\lambda}(p)\chi_{\lambda}(\vec{p}) \end{pmatrix}$$

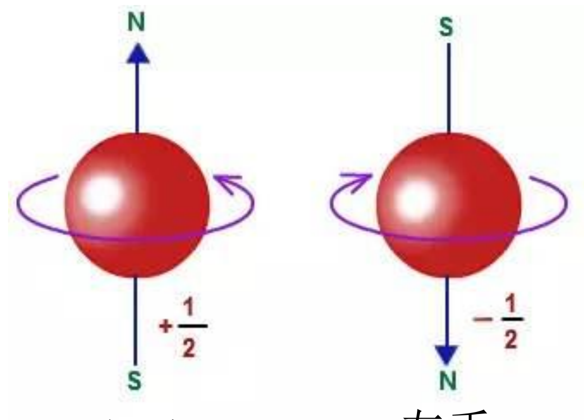
$$\omega_{\pm}(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_{+}(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_{-}(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Spinors for a fermion

$$h := \frac{\mathbf{p}\cdot\mathbf{S}}{|\mathbf{p}|}$$



右手　　左手

p_z

$$u_\pm(k) = \frac{1\pm\gamma_5}{2}u(k), \quad v_\mp(k) = \frac{1\pm\gamma_5}{2}v(k),$$

$$\overline{u_\pm(k)} = \overline{u(k)}\frac{1\mp\gamma_5}{2}, \quad \overline{v_\mp(k)} = \overline{v(k)}\frac{1\mp\gamma_5}{2}$$

$$|p^\mu, m_e, Q=-1e, h=\pm\frac{1}{2}\rangle \qquad |p^\mu, m_e, Q=+1e, h=\pm\frac{1}{2}\rangle$$

电子态　　正电子态

Helicity bases can be used for massless particles to describe scattering amplitudes. On-shell recursion relation can be powerful.

Computational Efficiency dependence on M and N

	M diag	N particle	2 > 6
Analytical	M^2	$(N!)^2$	1.6e9
Helicity	M	$(N!)\,2^N$	1.0e7
Recycling	M	$(N-1)!\,2^{(N-1)}$	6.5e5
Recursion Relation	$log(M)$	$2^N\,2^{(N-1)}$	3.2e4

Whizard

Monte Carlo method comes to the play.

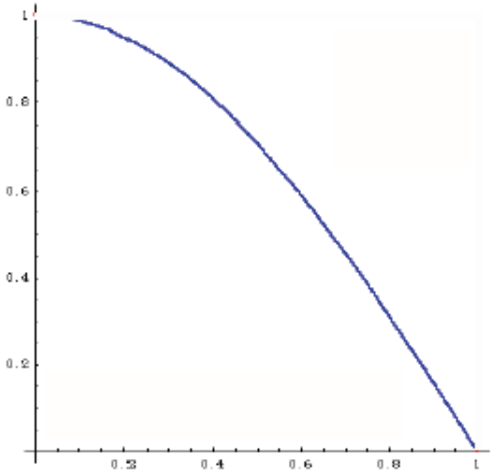
Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$$Dim[\Phi(n)] \sim 3n$$

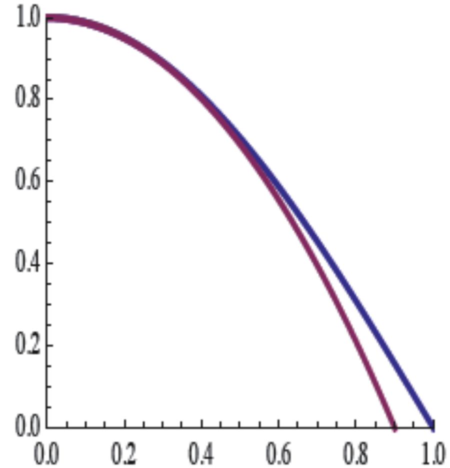
General and flexible method is needed

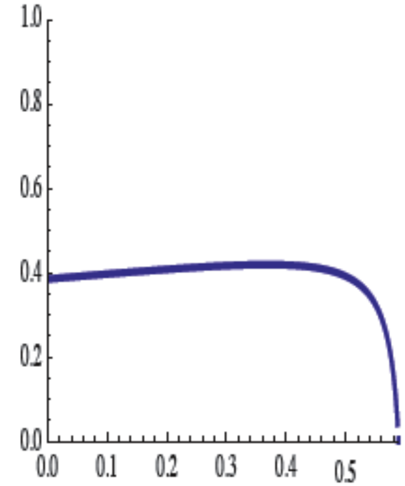
Monte Carlo 1



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$





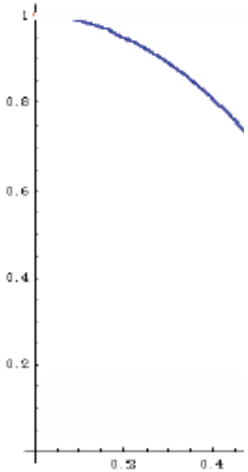
$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2 c}$$

$$\simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

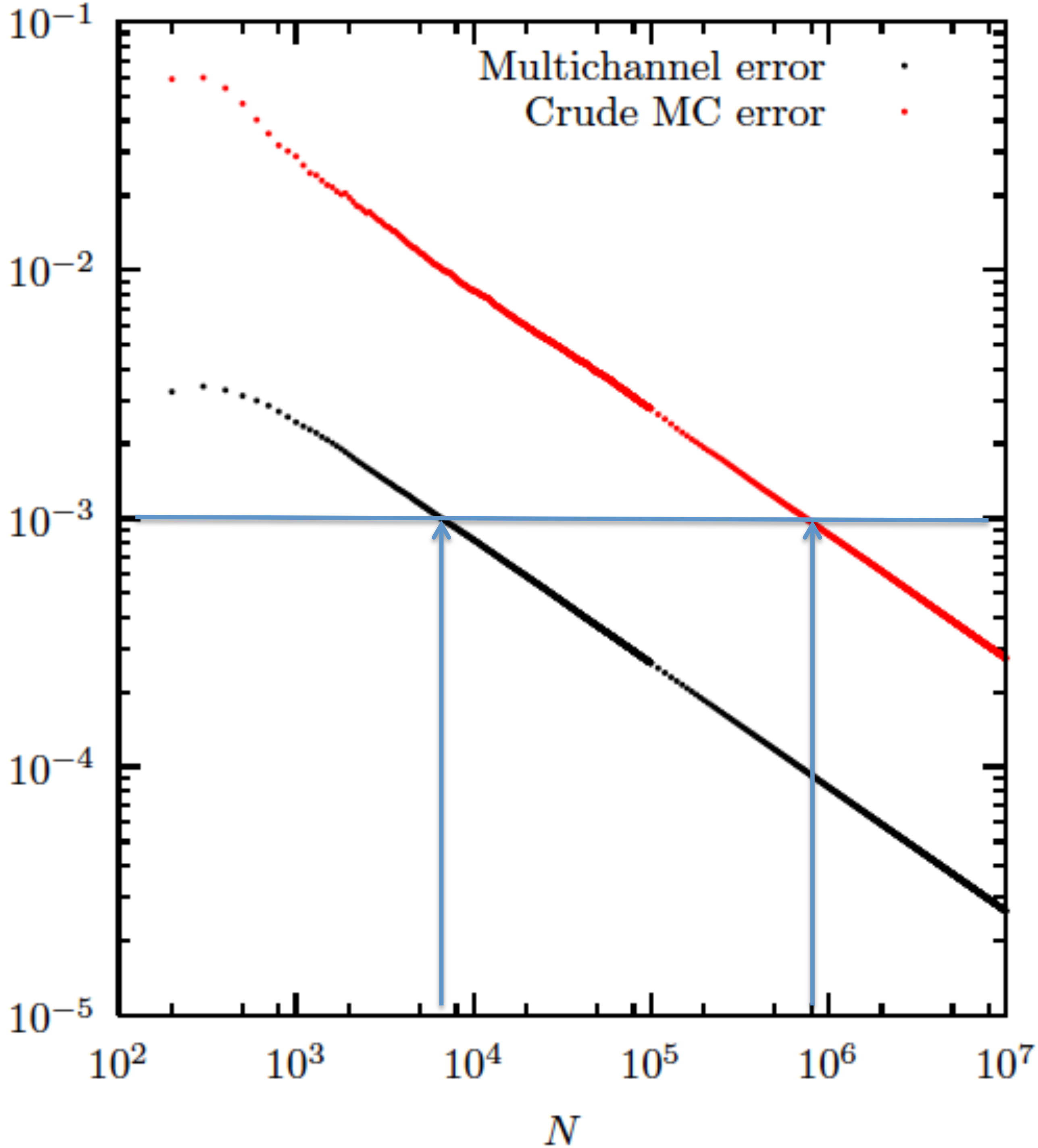
The Phase-Space parametrization is important to have an efficient computation!

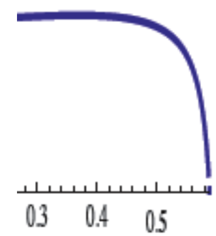
Importa… … 1



$$I = \int_0^1 c\ldots$$

$I_N = 0.637 \ldots$





$$\frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

$$\simeq 1$$

The … an

effic…

Using each Feynman Diagram as importance — Monte Carlo 1

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\boxed{\sum_j |M_j|^2} } \boxed{|M_{tot}|^2}$$

≈ 1

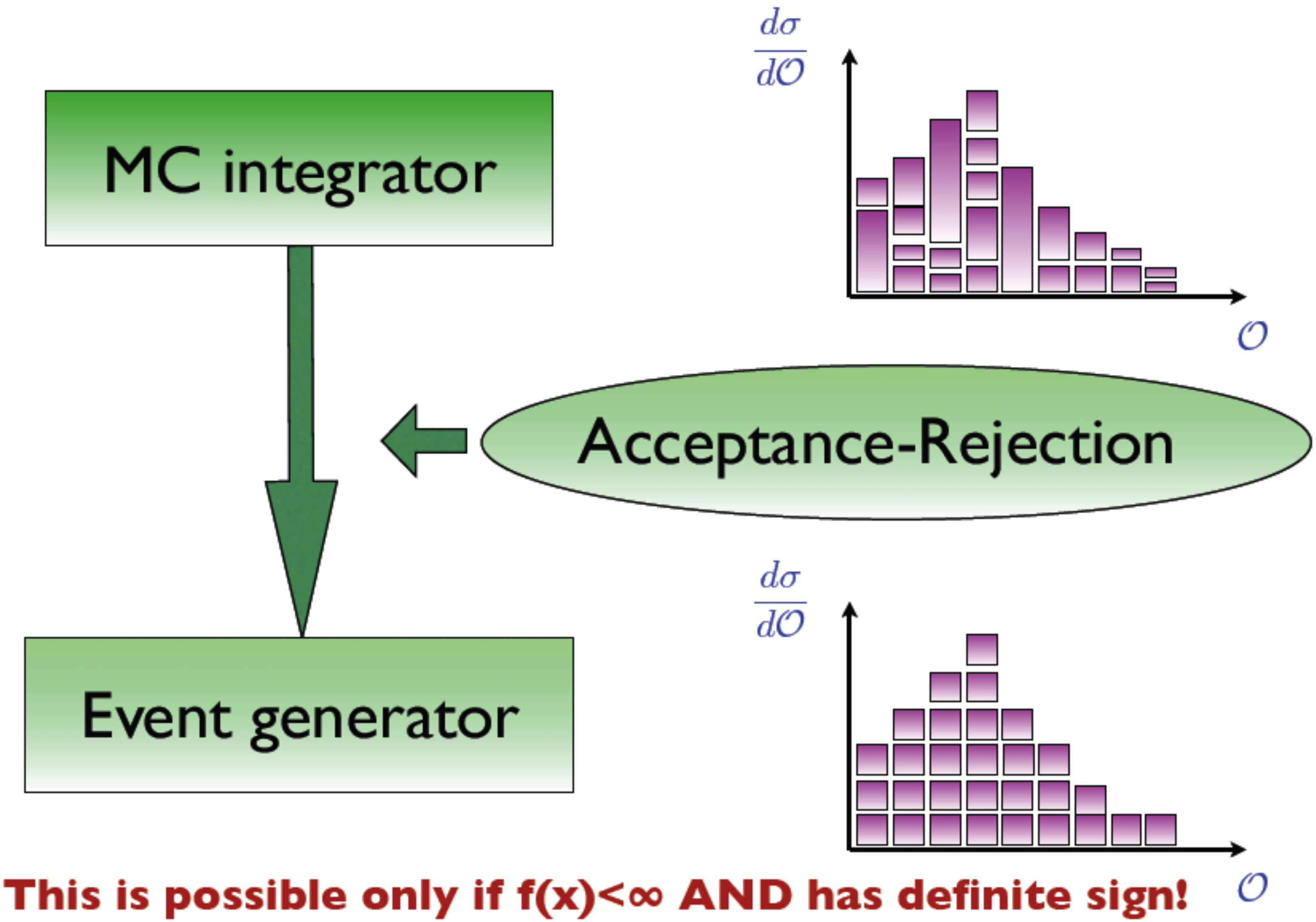
Key Idea

- Any single diagram is "easy" to integrate (pole structures/suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

N Integral

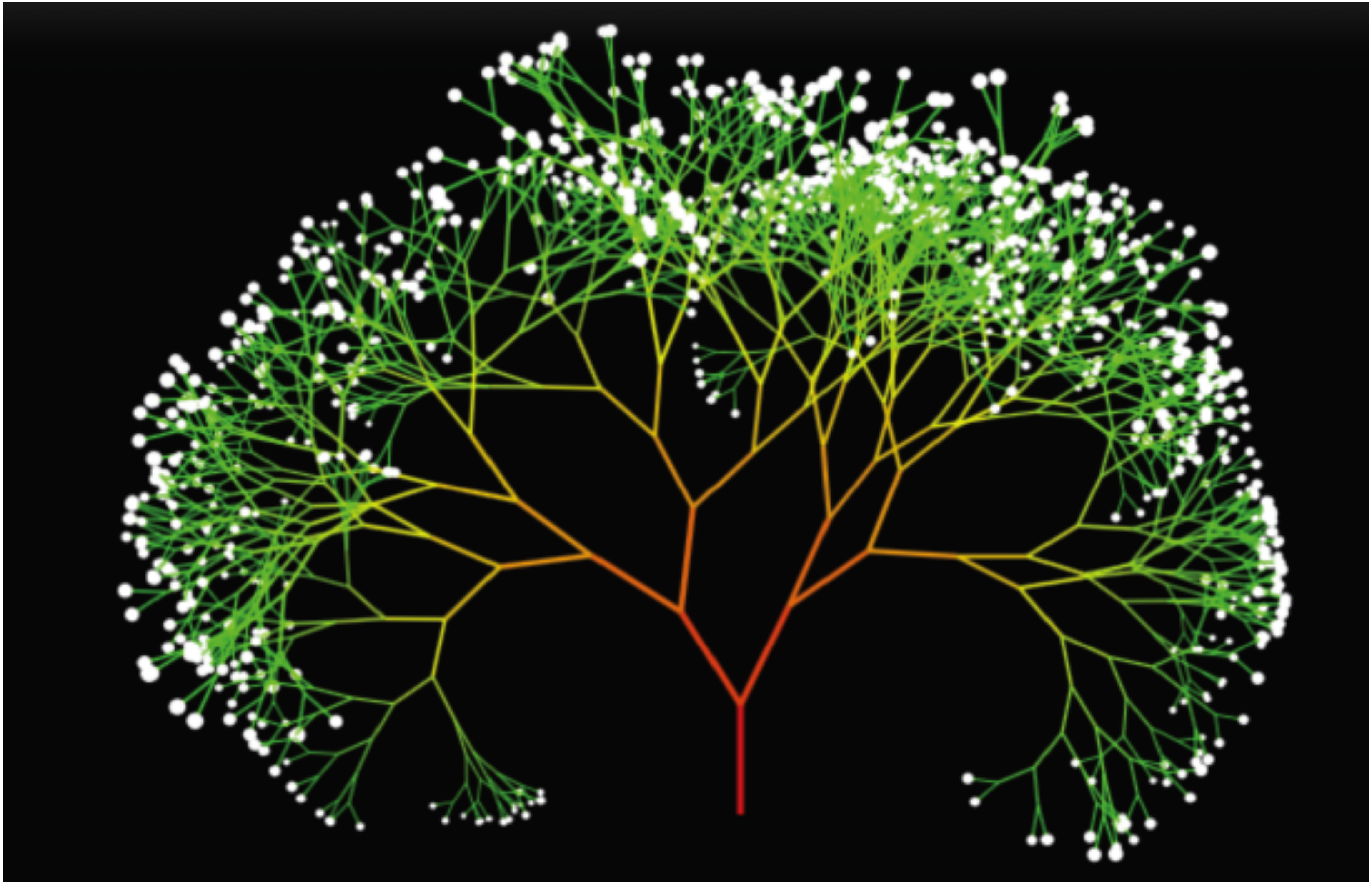
- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

Uniform weighted event generation



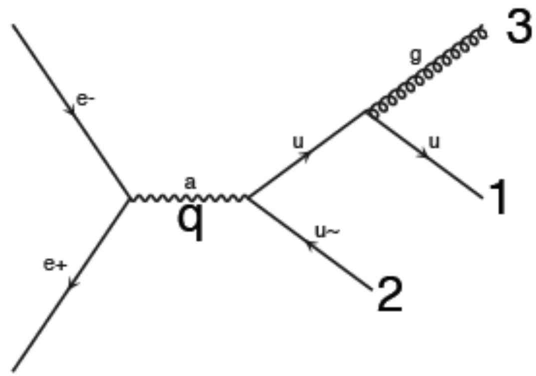
☞ **This is possible only if f(x)<∞ AND has definite sign!**

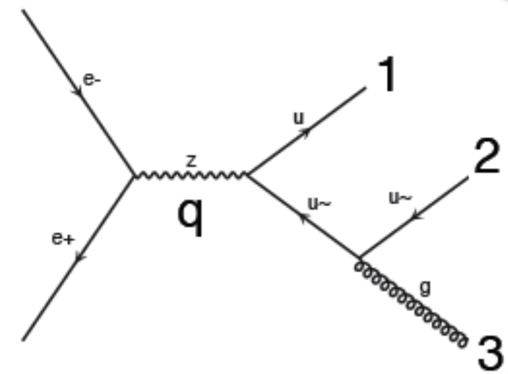
How to understand Parton Shower?



A constructive example: 2 -> 3

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Divergent at $x_1 = 1$ and $x_2 = 1$
    - Soft Divergencies
    - Collinear Divergencies

$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

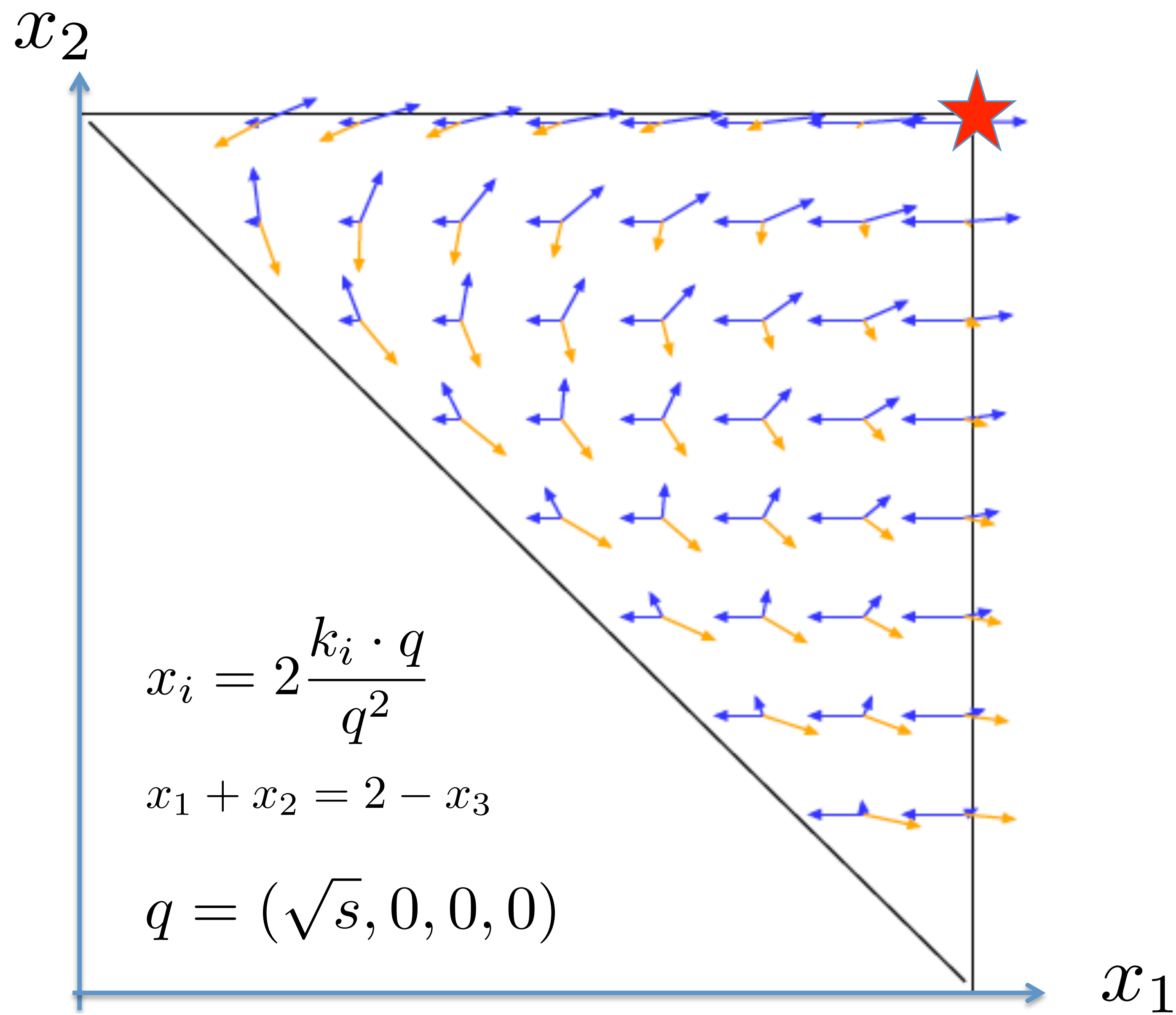
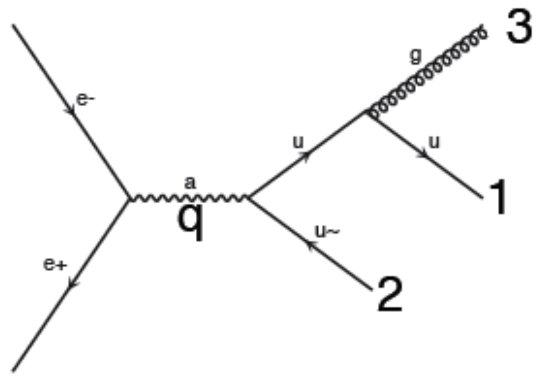
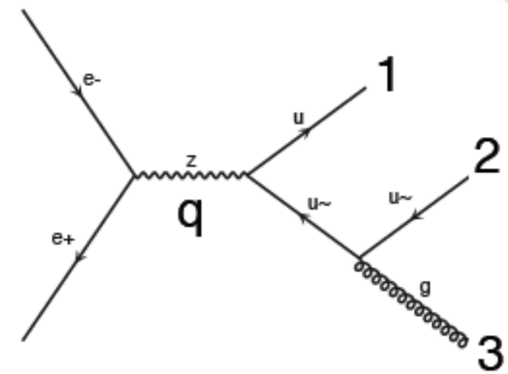


Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1, x_2), $\bar{q}$ direction fixed.

A constructive example: 2 -> 3

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$

$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$

$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$

$$x_1 + x_2 + x_3 = 2$$

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$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

Collinear limit of 2to3 process with 2to2 process

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi}\left(\frac{2}{\sin^2\theta_{13}}\frac{1-(1-x_3)^2}{x_3} - x_3\right)$$

- Collinear limit
  - Split our integral in two

$$\frac{2\,dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1-cos\theta_{13}} + \frac{dcos\theta_{13}}{1+cos\theta_{13}}$$

$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$

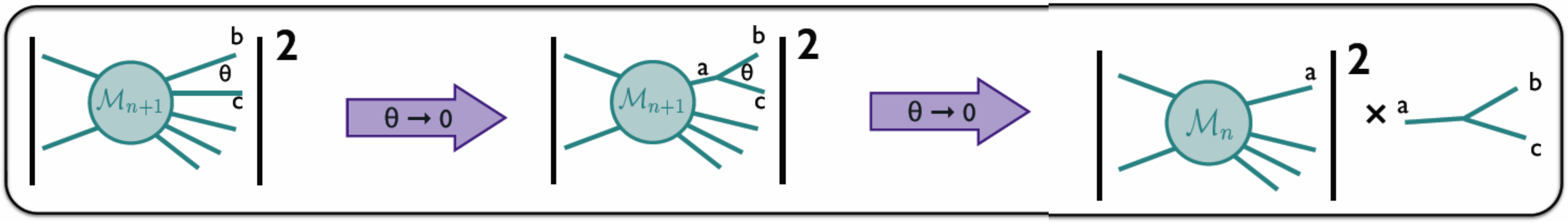
$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

☞ z fraction of energy

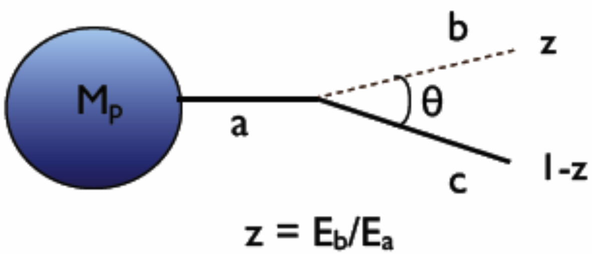
☞ **Generic Formula**

Recursive relation for 2ton and 2ton+1



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2E_b E_c(1 - \cos\theta)} = \frac{1}{t}$$



soft and collinear divergencies

Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

when θ is small.

A close look to the recursive relation: t

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality $\mathbf{m^2}$ of particle a or its $\mathbf{p_T}^2$ or $\mathbf{E^2\theta^2}$ …

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$

$$p_T^2 \simeq z m^2$$

  - It represents the hardness of the branching and tends to 0 in the collinear limit.
  - Different choice of 'evolution parameter' in different Parton-shower code

A close look to the recursive relation: z and phi

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

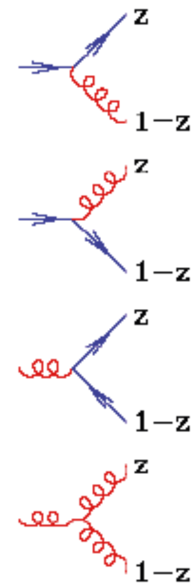
- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a.** It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)
- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

A close look to the recursive relation: splitting function

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} \boxed{P_{a\to bc}(z)}$$

The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\begin{aligned}
\hat{P}_{qq}(z) &= C_F \left[\frac{1+z^2}{(1-z)}\right], \\
\hat{P}_{gq}(z) &= C_F \left[\frac{1+(1-z)^2}{z}\right], \\
\hat{P}_{qg}(z) &= T_R \left[z^2+(1-z)^2\right], \\
\hat{P}_{gg}(z) &= C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z)\right].
\end{aligned}$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Comments:

* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* P_{qg} has no soft divergences.

A close look to the recursive relation: coupling strength

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_{a\longrightarrow bc}(z) \longrightarrow P_{a\longrightarrow bc}(z) + \alpha_S P'_{a\longrightarrow bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a\longrightarrow bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a\longrightarrow bc}(z)$ (just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1+\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1-\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}
\alpha_S(Q^2)\left(P_{a\to bc}(z) + \alpha_S(Q^2) P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1-\alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z) \\
&\sim \alpha_S(z(1-z)Q^2) P_{a\to bc}(z) \qquad t = z(1-z)Q^2
\end{aligned}$$

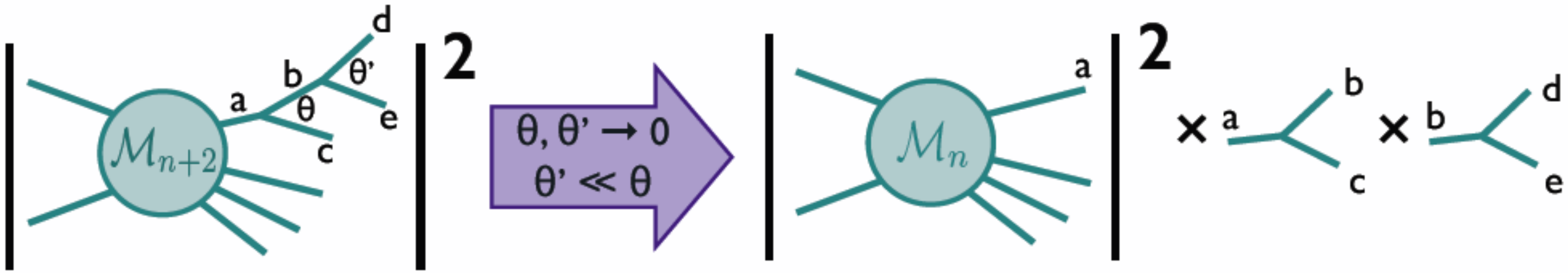
The core recursive relation on collinear radiation

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
- **alpha_s** need to be evaluated at the scale t
- **P** is the splitting Kernel (control the soft behaviour)

Application the recursive relation from n to n+k emissions



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement:
  $\theta \gg \theta' \gg \theta''$...
  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_s^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_s}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Sudakov form factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non}-\text{branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)}$$

Sudakov form factor

$$\boxed{\Delta(Q^2, t)} \simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz \frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

➡Property: $\Delta(A,B) = \Delta(A,C)\,\Delta(C,B)$

Monte Carlo 3

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define dP_k as the probability for k ordered splittings from leg a at given scales

$$\begin{aligned} dP_1(t_1) &= \Delta(Q^2, t_1)\, dp(t_1)\Delta(t_1, Q_0^2), \\ dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\, dp(t_1)\, \Delta(t_1, t_2)\, dp(t_2)\, \Delta(t_2, Q_0^2)\Theta(t_1 - t_2), \\ \ldots &= \ldots \\ dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l) \end{aligned}$$

- Q_0^2 is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.

Unitarity is guaranteed

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

- Hence, the total probability is conserved

Soft and collinear limit (IR and collinear safety)

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission:
  Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

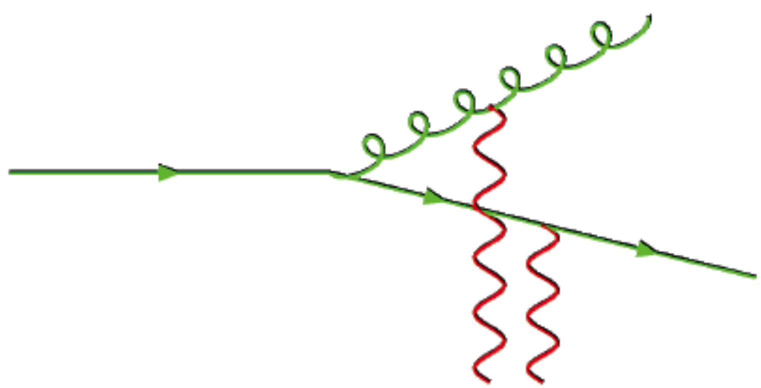
- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.

What is about the pure soft limit?

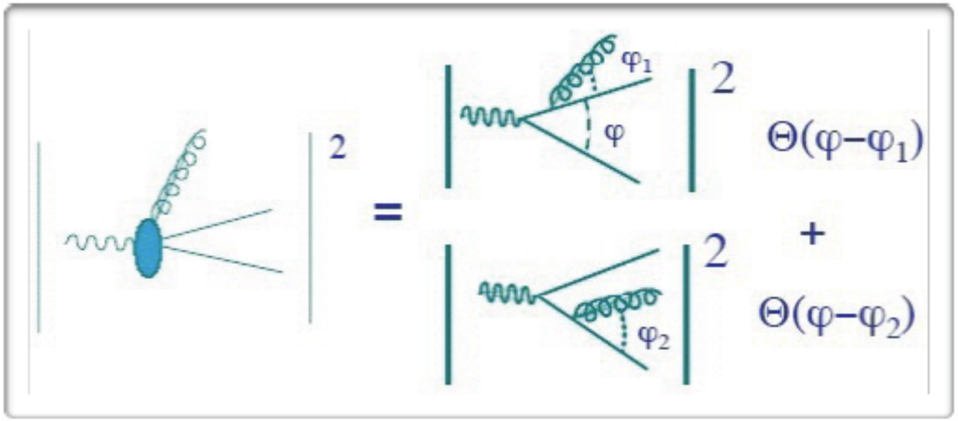
$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2}\frac{dt'}{t'}dz\frac{d\phi}{2\pi}\frac{\alpha_s}{2\pi}P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its p_T^2 or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 0,1$) they behave differently
- Can we chose it such that we get the correct soft limit?
- Soft gluon comes from the full event!

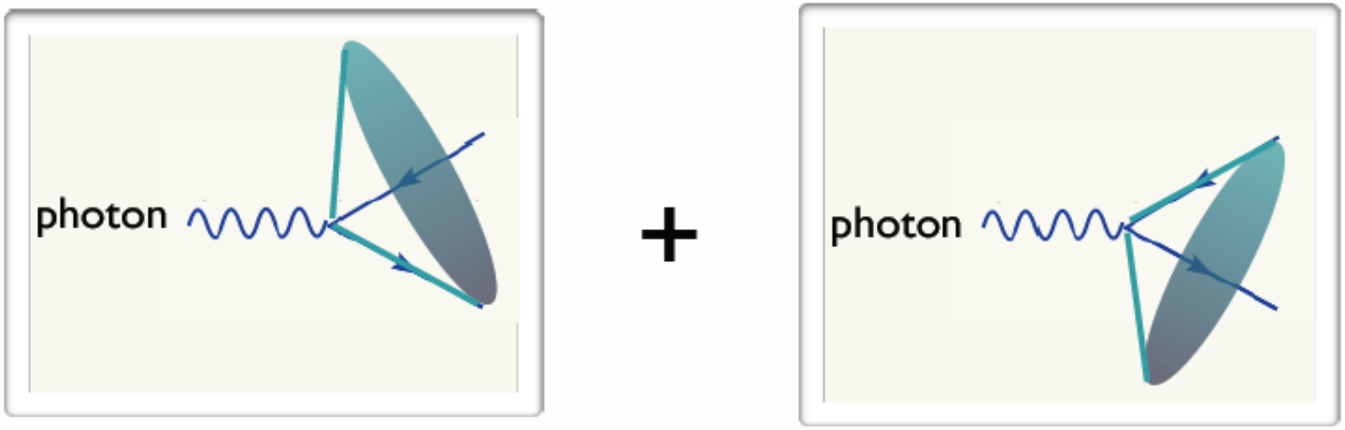


- Quantum Interference

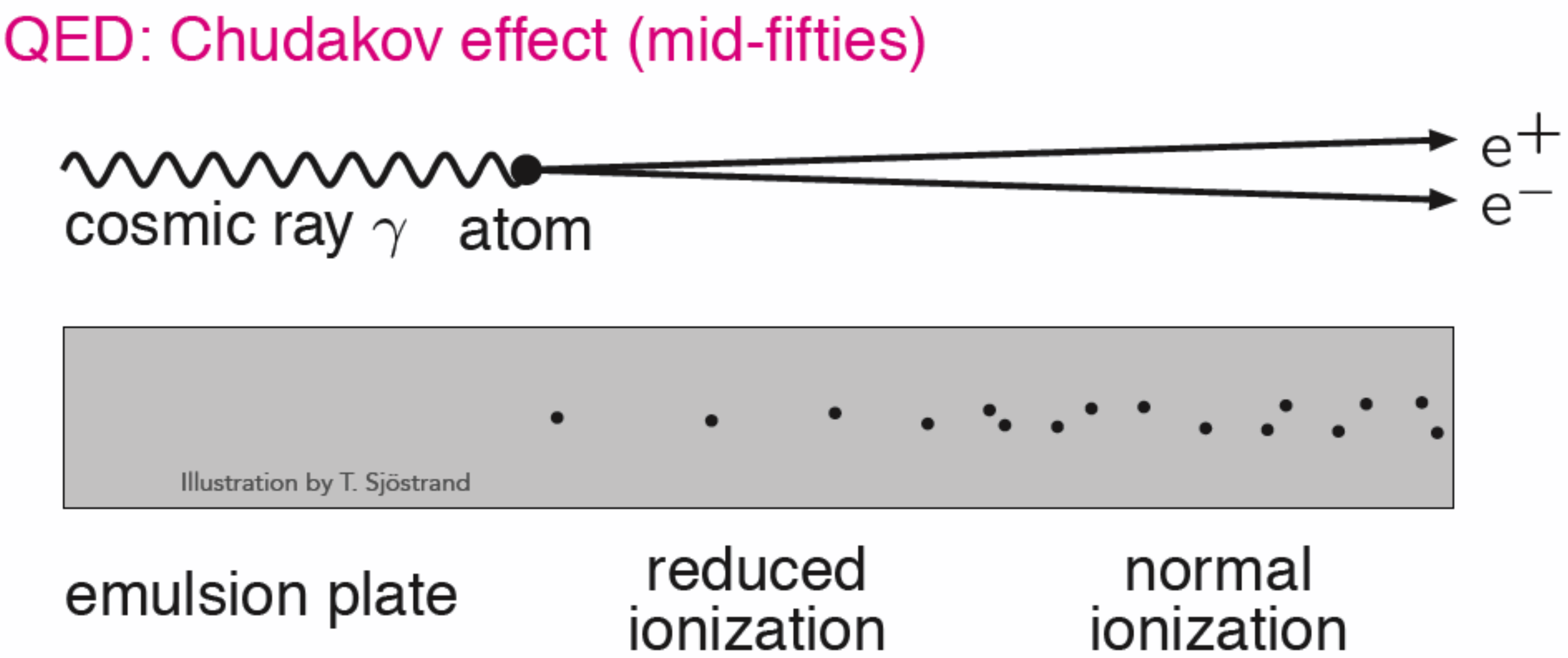
Chudakov effect



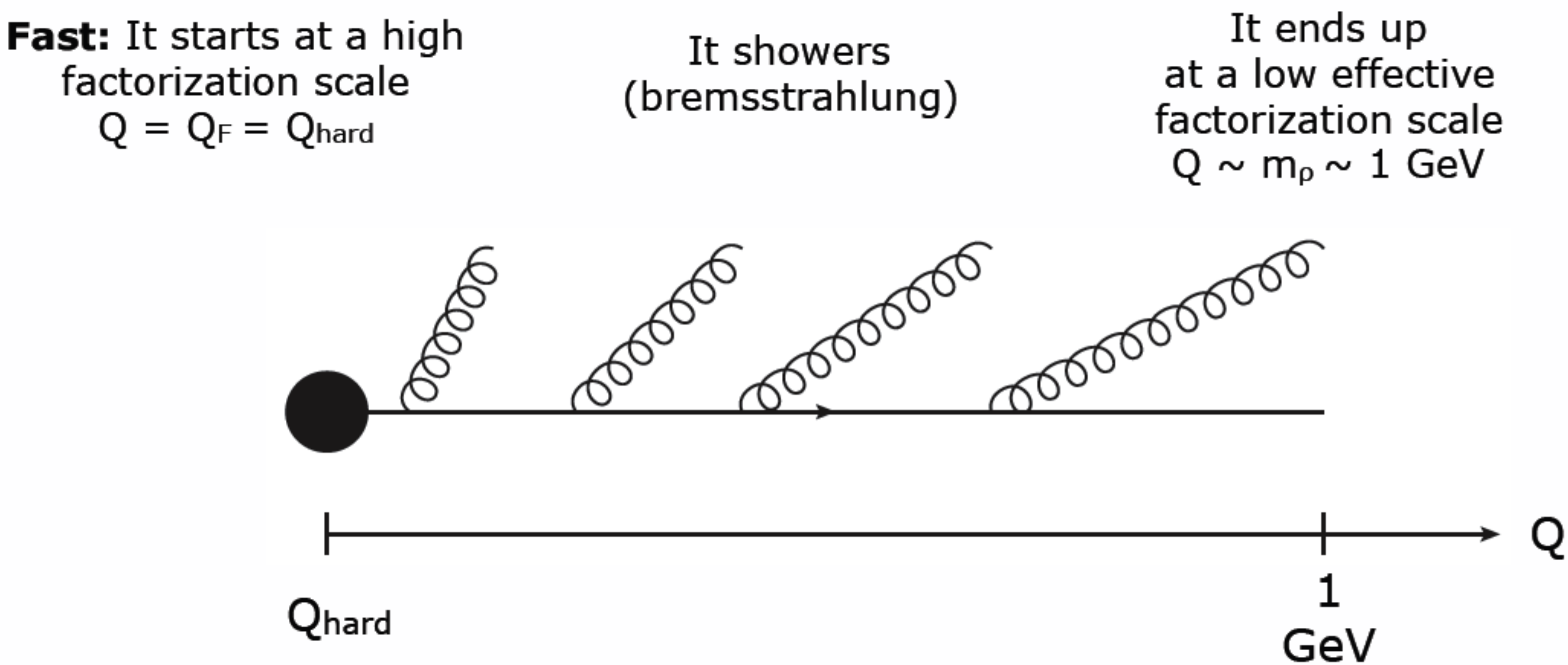
Radiation inside cones around the original partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



Chudakov effect



Angular ordering



Parton Shower Summary

- Sudakov Form-Factor: Probablility of No-emission between two scale.

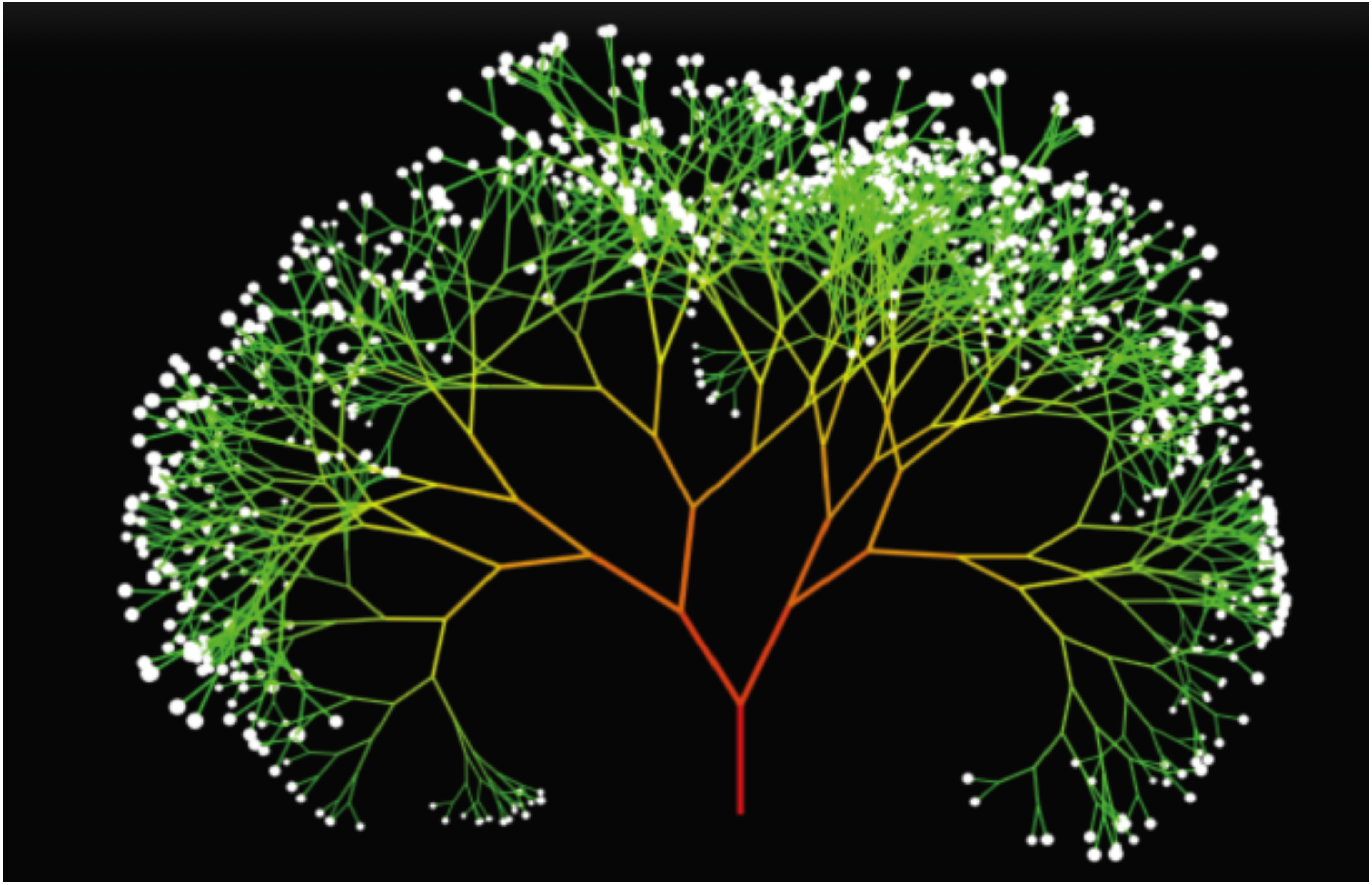
$$\Delta(Q^2, t) \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'} dz \frac{\alpha_S}{2\pi} \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

- Probalitity of K-emission

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k , \quad \forall k = 0, 1, ...$$

- Ensure that the parton shower is unitary
- Ensure cancelation of IR divergency
- Interference effect via Angular ordering

Picture of Parton Shower



Summary

- MC tools for HEP is a natural child of the marriage between HEP and computer science.
- Madgraph is a user-friendly ME generator, which has a convenient interface to NP.
- Young students are encouraged to enter and stay in the direction.

Thanks

补充材料

2. MC on the Market (1)

Some specialized and time-honored MC tools:

- ALPGEN: M.L.M, et.al. hep-ph/0206293 MLM ME-PS matching@LO
  a generator for hard multiparton processes in Hadronic Collisions.
  Powerful to generate tree-level SM background.

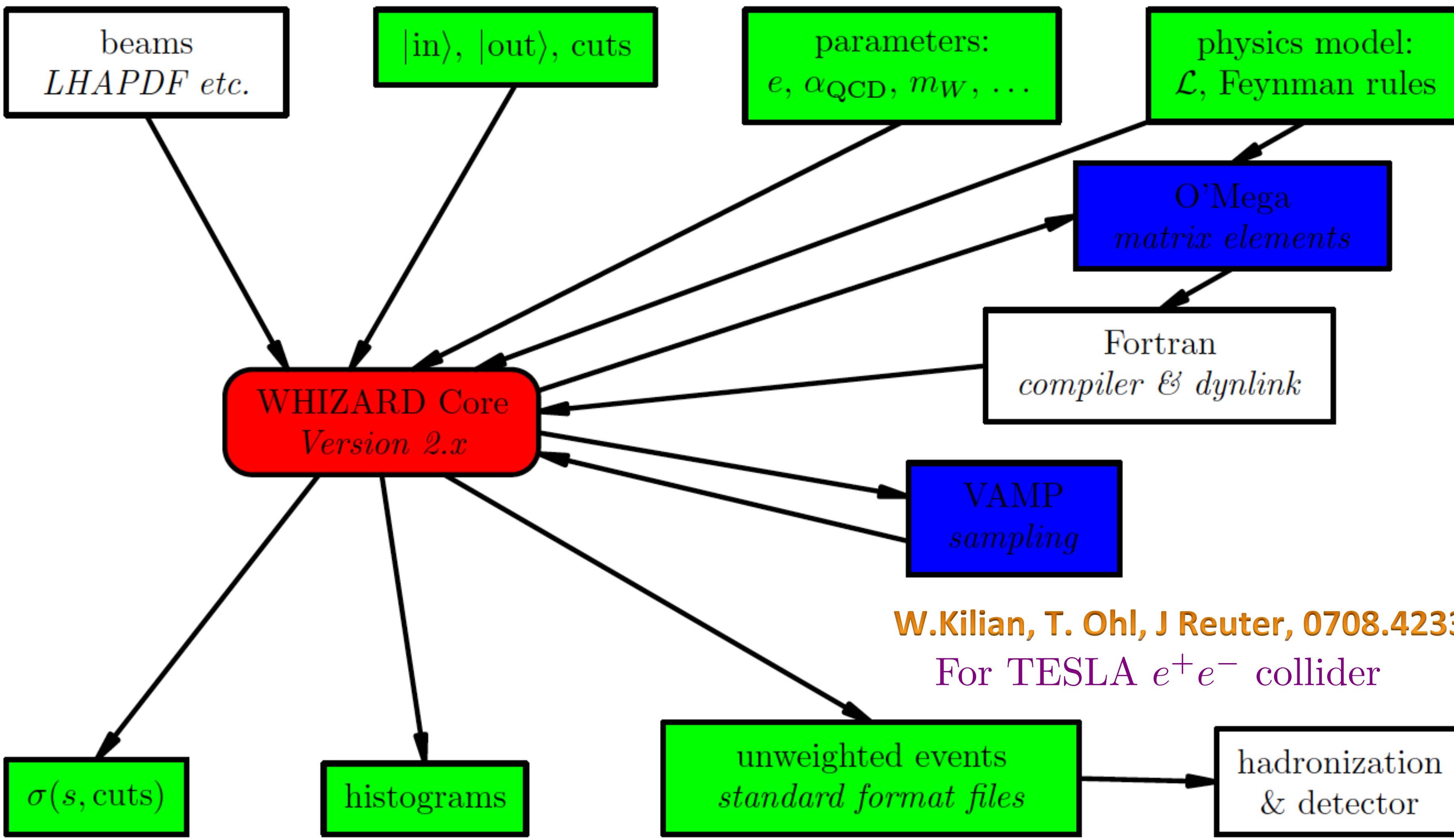
- MCFM: J.Campbell,K.Ellis,C.Williams, hep-ph/9810489
  NLO QCD $2 \to 2$, $2 \to 3$ processes, Some Gluon induced processes included, e.g. $gg \to W^+W^-$
  Parton level only, no showering, not an event generator

- POWHEG: Nason, JHEP0411:040,2004; Hamilton and Nason, JHEP1006(2010)
  NLO PS matching, Competitor of MC@NLO, Exponential of non-singular parts, universal interface to any parton shower
  No negative weight events

These specialized MC tools are useful to explore the physics potentials of future hadron colliders like SPPC.

2. MC on the Market (2)

Whizard:W, Higgs, Z and Respective Decays



W.Kilian, T. Ohl, J Reuter, 0708.4233

For TESLA e^+e^- collider

2. MC on the Market (2)

Whizard:W, Higgs, Z and Respective Decays

Basic facts:

- Helicity amplitudes with complete avoidance of redundancies
- Iterative adaptive multi-channel phase space (viable for $2 \rightarrow 10$)
- **Unweighted events** (formats: binary, HEPEVT, ATHENA, LHA, STDHEP)
- Graphical analysis tool

Able to handle multi-particle scattering processes efficiently.

Both longitudinal and transverse polarizations of beam have been realized in the package.

New physics models can be implemented by FeynRules

Whizard 2 is a tree-level ME generator [sufficient for Detector Design at the early stage of CEPC]

2. MC on the Market (3)

A few cornerstones of Madgraph: From K. Hagiwara

- In 1992, H. Murayama, I. Watanabe, and K. Hagiwara, HELAS
- In 1994, T.Stelzer and W. F. Long, Madgraph was released.
- In 2002, F. Maltoni and Tim Stelzer, MadEvent was released.
- In 2007, J. Alwall, et. al., Madgraph4 was released.
- In 2011, J. Alwall, et.al., Madgraph5 was released.
- Now, the transition to NLO is ongoing

Both Hua-Sheng Shao and Qiang Li have being involved into its development and contributed.

Remarkable features of Madgraph5:
automation and user-friendly, multiparticle processes, speedy, general interface to new physics, precision for jet description (link to pythia provided, MLM ME-PS matching)

2. MC on the Market (3)

Features of Madgraph5[1106.0522]_aMC@NLO:

- UFO (The Universal FeynRules Output) at NLO.
- ALOHA (Automatic Libraries Of Helicity Amplitudes for Feynman diagram computations) at NLO.
- MadGraph 5 framework to generate tree-like structure.
- MadEvent framework to perform phase-space integration and events generation.
- MadLoop 5 framework to calculate virtual (CC+R1+R2+UV) contributions, which uses OPP/CutTools to reduce loop integrals.
- MadFKS 5 framework to calculate real/subtraction contributions and generate hard events and counter-term events.
- MC@NLO to do matching with Parton-Shower programs and finally use Parton-Shower programs to do showering and hadronizing.
- MadSpin to calculate the spin correlations in decays.

Transition to NLO is ongoing. Provided by Hua-Sheng Shao (PKU/CERN)

2. MC on the Market (3)

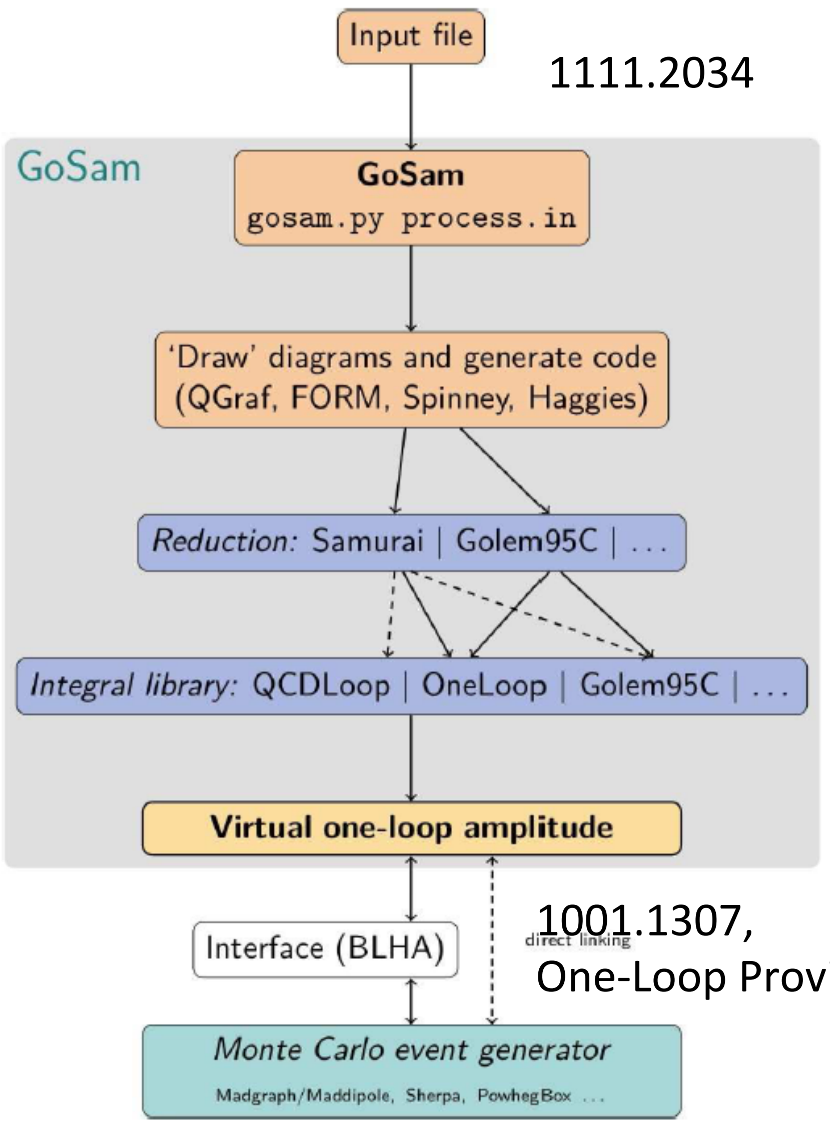
What can Madgraph5_aMC@NLO do?

- In principle, QCD corrections in Standard Model can be done,especially
  - Fixed-ordered LO and NLO corrections.
  - Unweighted events generated by matching with parton shower programs (like HY6,HY++,PY6,PY8 etc) with MC@NLO method.
  - FxFx merging can be applied to resolve the double-counting in multi-jet process.
  - MadSpin can be ultilitied to take account spin correlation in.
- EW virtual corrections (i.e. in MadLoop) in Standard Model was done.
- QCD corrections in SUSY models can be done recently.
- Many interfaces are done or are working on like GoSam, Sherpa, MadGloem.

Provided by Hua-Sheng Shao (PKU/CERN)

2. MC on the Market (3)

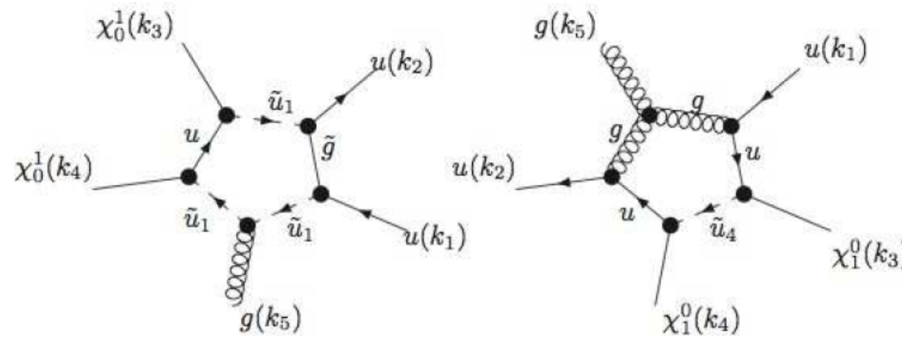
GoSam: a one-loop level VME calculator for NP



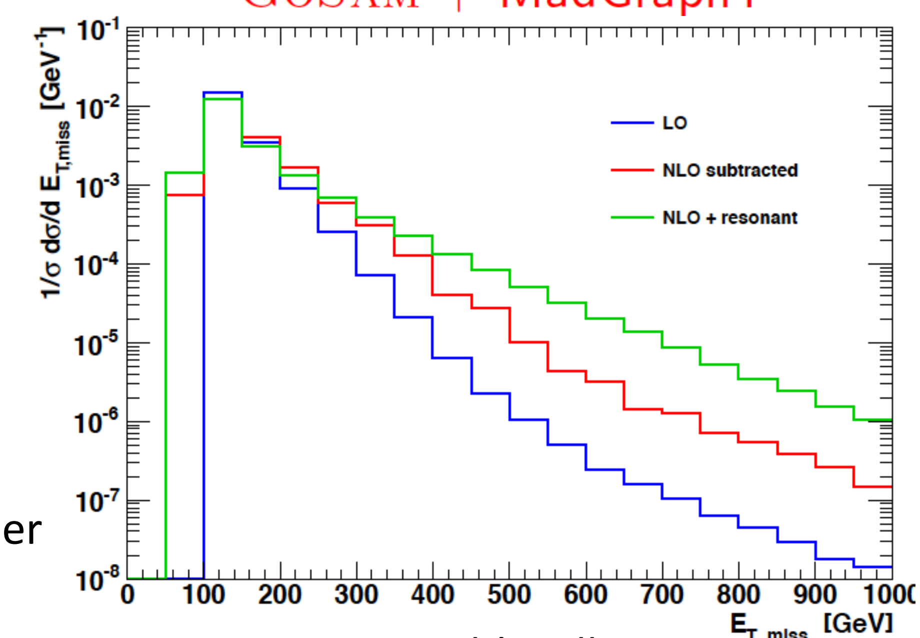
1111.2034

1001.1307,
One-Loop Provider

$\tilde{\chi}_1^0\,\tilde{\chi}_1^0 + \text{jet}$ (p19MSSM)



GoSam + MadGraph4



From G. Heinrich's talk@ACAT2013

2. MC on the Market (4)

GRACE-Loop is a generic automated program for calculating High Energy Physics processes [3].

G. Belanger, F. Boudjema, J. Fujimoto, T.Ishikawa, T. Kaneko, K. Kato, Y. Shimizu

---

- *All Feynman diagrams for a given process at fixed order of perturbation theory.*
- *A FORM or REDUCE code.*
- *A Fortran code generated for amplitude calculations.*
- *Kinematic library.*
- *The multi-dimensional integration by BASES.*
- *Event generation by SPRING.*

For GRACE system, please visit website:

Grace at tree level was released in 1993. Designed for e^+e^- colliders

http://minami-home.kek.jp/

[3] **Phys. Rept. 430 (2006) 117**

2. MC on the Market (4)

The GRACE-Loop system has also been used to calculate

- $2 \to 3$-body processes such as $e^+e^- \to ZHH$, $e^+e^- \to t\bar{t}H$, $e^+e^- \to \nu\bar{\nu}H$, etc.
- $2 \to 4$-body process as $e^+e^- \to \nu_\mu\bar{\nu}_\mu HH$.

Recently the processes:

- $e^+e^- \to t\bar{t}\gamma$ (Eur. Phys. J. C **73**, 2400 (2013)).
- $e^+e^- \to e^+e^-\gamma$ at ILC in preparation.
- $pp \to W^+W^- + 1$jet at LHC in progress.

More works may be needed to handle massless gauge bosons.

From P.H. Khiem's talk@LCWS13-Tokyo

2. MC on the Market (5)

Herwig (CERN, DESY, Durham, Karlsruhe, Manchester)

- Originated in studies of coherent QCD evolution
- Front-runner in matching of NLO QCD ME and PS
- Original framework for cluster fragmentation

Pythia (CERN, DESY, FNAL, Lund)

- Originated in hadronization studies → Lund string
- Leading in development of models for non-perturbative physics
- Extensive PS development and earliest ME+PS matching

Sherpa (CERN, Dresden, Durham, Göttingen, SLAC)

- Started with matching of LO ME and PS
- First automated framework for NLO calculations
- First automated merging of ME and PS at NLO

0311263,
JHEP02(2009)007

Herwig/Herwig++,Pythia6/Pythia8,ISAJET,Sherpa:
General Purpose MC event generator

From S. Hoeche

2. MC on the Market (5)

Special features of Sherpa

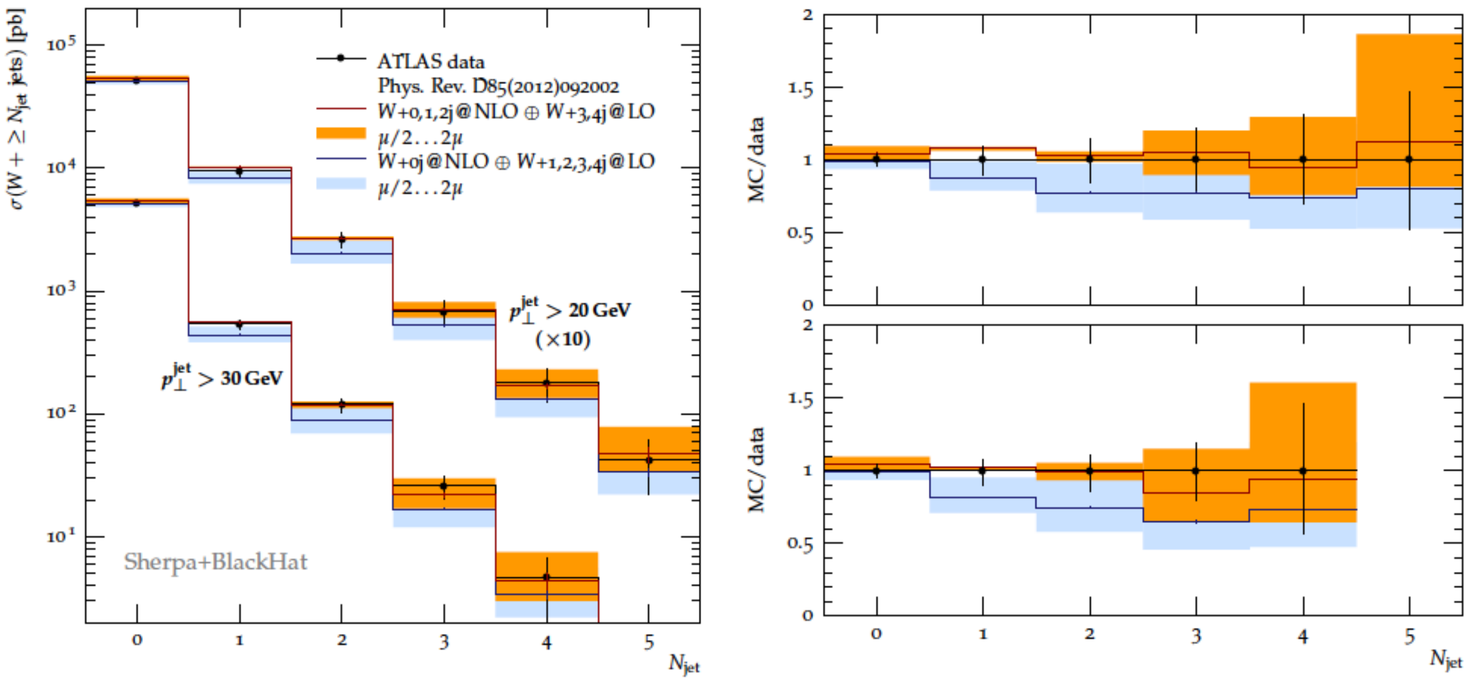
- Two matrix-element generators
  Amegic++ Kuhn,Krauss & Comix Gleisberg,Höche
- Complete framework for NLO calculations Gleisberg,Krauss
- Independent parton shower (no interface to Pythia or Herwig) based on Catani-Seymour dipole subtraction Schumann,Krauss
- Independent implementation of MC@NLO method (S-MC@NLO) based on CS dipole subtraction Höche,Krauss,Schönherr,Siegert
- ME+PS merging at LO & NLO Höche,Krauss,Schönherr,Siegert
- Complete hadron & τ decay package Krauss,Siegert
- Photon emission generator Krauss,Schönherr
- Minimum bias simulation Krauss,Zapp

From S. Hoeche

Interface to NP is provided; CKKW ME-PS matching/merging @NLO is realized.

2. MC on the Market (5)

Höche, Krauss, Schönherr, Siegert



- MEPS@NLO with W+0,1&2 jet at NLO plus W+3&4 jet at LO compared to W+0 jet at NLO plus up to W+4 jets at LO
- Better agreement with experimental data
- Largely reduced scale uncertainty

From S. Hoeche

Merging/Matching@NLO is necessary for the LHC runs and high precision MC event generators are inevitable for SPPC.