Chung-Ang University and Yantai University

Lectures on Direct Detection of Light Dark Matter

Bin Zhu
zhubin@mail.nankai.edu.cn

Integrated Mill Systems

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Contents

Motivation of Light Dark Matter

Historical Perspective

Understanding the Electroweak Sector

- Discovery of Radioactivity (1890s)
- Fermi Scale Identified (1930s)
- Non-Abelian Gauge Theory (1950s)
- Higgs Mechanism (1960s)
- W/Z Bosons Discovered (1970s)
- Higgs Discovered (2010s)

Each step required revolutionary theoretical/experimental leaps

$t \sim 100\text{years}$

Gordan Krnjaic, Brookhaven Forum 2017

Yesterday Once More

Understanding the Dark Sector?

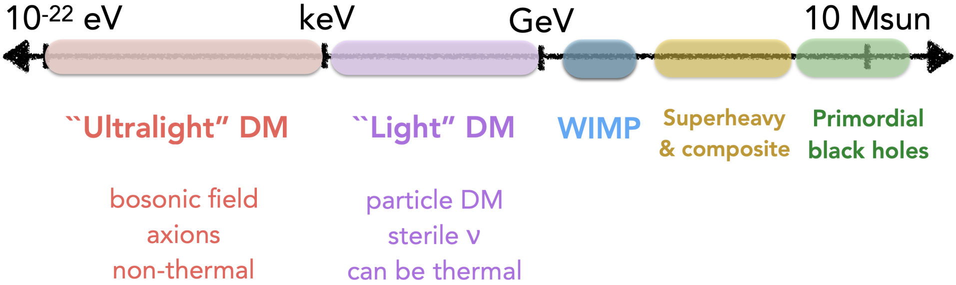
- Discovery of missing mass (1930s)
- Rotation curves (1970s)
- Precision CMB measurements (1990s)
- Dark Matter Discovery? (2030s)?

Discovery Crisis

No clear target for non-gravitational contact $\rightarrow$ Landscape of dark matter scales

Mass Scale of Dark Matter

Figure from talk by Tongyan Lin at Summer Institute 2019, Korea

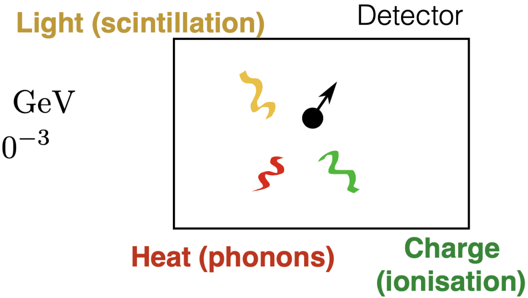


- **Bad news:** DM-SM interactions are not obligatory. If nature is unkind, we may never know the right scale.
- **Good news:** Most discoverable DM candidates are in thermal equilibrium with us in the early universe. → **WIMP + Light DM**

Direct Detection of WIMP

- Search for collisions of invisible particles with atomic nuclei $\rightarrow$ Design driver: big exposure
- Coherent elastic scattering $\rightarrow$ Big idea: Scatter coherently off all the nucleons in a nucleus: $R \sim A^2$ enhancement
- Expected low-energy of recoiling nucleus (with maximum of a few tens of keV) $\rightarrow$ Predicted signature: recoil induced ionization and scintillation

$$m_\chi \gtrsim 1 \text{ GeV}$$
$$v \sim 10^{-3}$$



Direct WIMP Detection Experiments Worldwide

Numerous underground laboratories

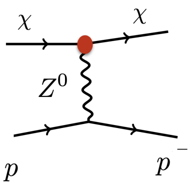
Go underground to shield detector from cosmic rays and their decay products

Direct WIMP Detection Experiments Worldwide

Variety of techniques and dedicated experiments

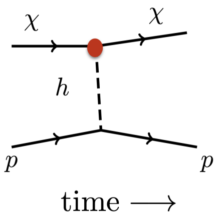
Use only radiopure materials and fabrication techniques

Classifying WIMP Interactions



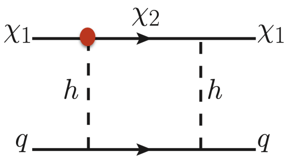
Z Exchange

$\sigma_p \sim 10^{-39}\ \text{cm}^2$



Higgs Exchange

$\sigma_p \sim 10^{-45}\text{cm}^2$



$\Delta \equiv m_2 - m_1$

Inelastic coupling EW loop

$\sigma_p \sim 10^{-47}\text{cm}^2$

Very different at low energy, despite high energy similarities

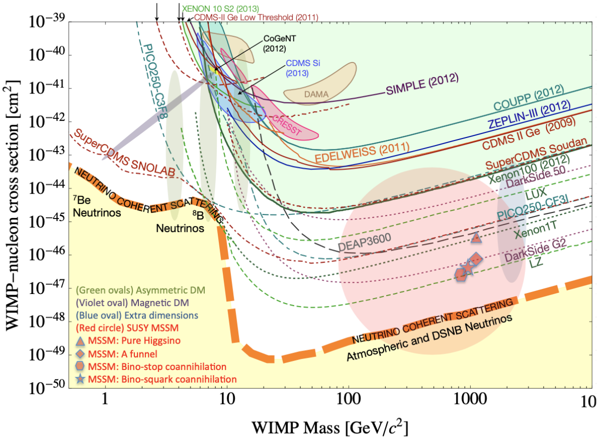
WIMP Milstones

Cushman et al. arxiv:1310.8327

Elastic Z exchange →

Higgs exchange →

1-Loop EW box →



Neutrino floor is coming for WIMP!

WIMP Search Status

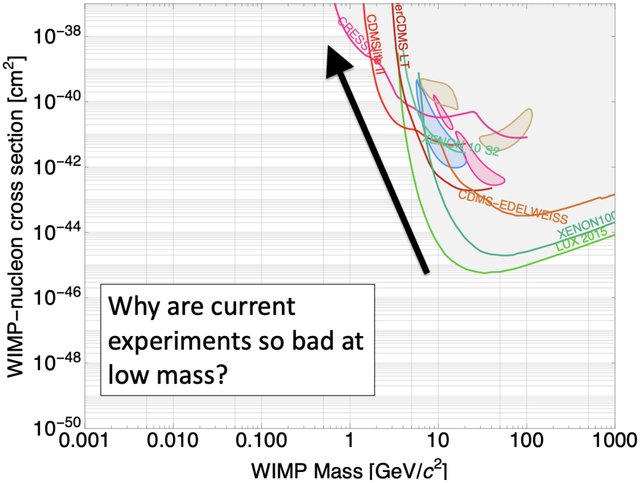
Figure from talk by Haibo Yu at CAU BSM workshop

“上穷碧落下黄泉，两处茫茫皆不见。”白居易《长恨歌》

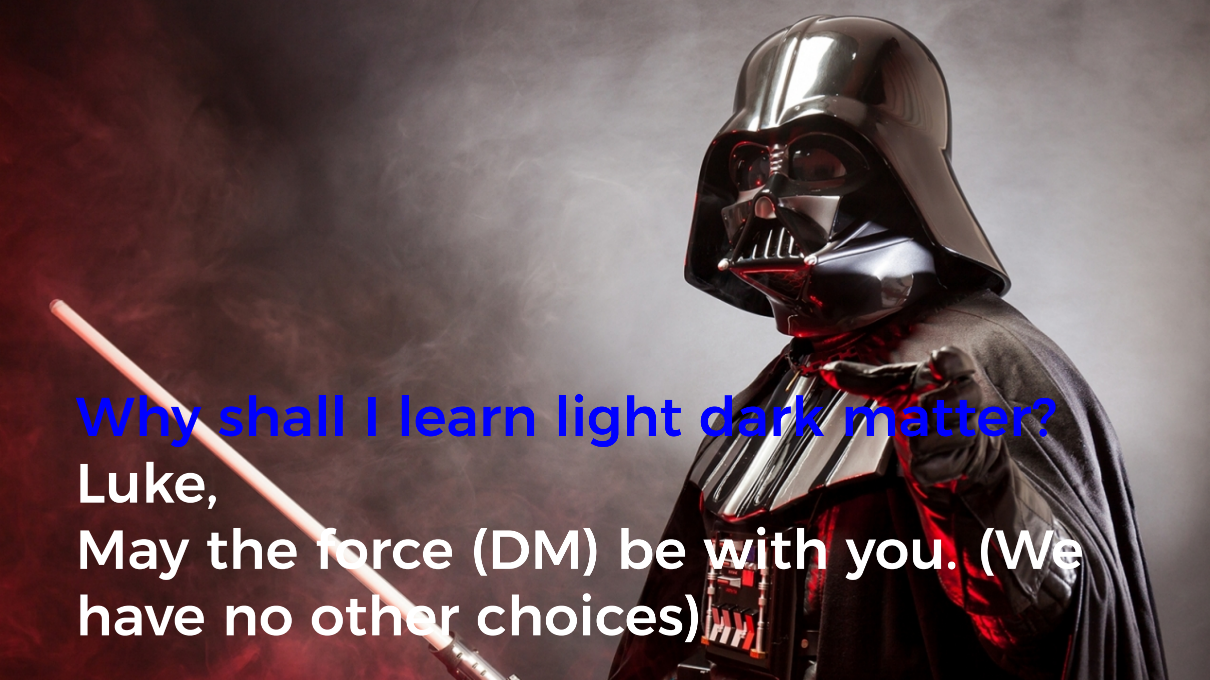
He exhausted all avenues in heaven and the nether world,
… he could not bring her existence to light.

A Song of Immortal Regret, Bai Juyi (772-846)

Opportunity or Crisis
Is Light dark matter possible target?



There is huge room for light dark matter detection → Can we go lower in DM mass?


Why shall I learn light dark matter?
Luke,
May the force (DM) be with you. (We have no other choices)

Why is nucleus bad at light dark matter?

Kinematic No-go Theorem

When dark matter is lighter than $1\mathrm{GeV}$, it resulting recoil energy is smaller than threshold 1keV

Prove that there is inefficient energy transfer from DM to nucleus → How to increase recoil energy

$$E_{\mathrm{NR}} = \frac{q^2}{2m_N} \leq \frac{2\mu_{\chi N}^2 v^2}{m_N} \simeq 1\mathrm{eV} \times \left(\frac{m_\chi}{100\mathrm{MeV}}\right)^2 \left(\frac{20\mathrm{GeV}}{m_N}\right) \quad \mathrm{vs} \quad E_{\mathrm{DM}} \sim \frac{1}{2} m_\chi v_\chi^2$$

Best nuclear recoil threshold is currently $E_R > 30\mathrm{eV}$ (CRESST-III) with DM reach of $m_\chi > 160$ MeV.
The kinematics of DM scattering against free nuclei is inefficient, and it does not always describe target response accurately.

Strategies for detecting nuclear recoils from Light DM

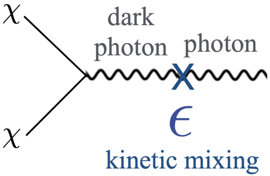
- Decreasing the heat threshold of detector - new experimental search.
  See Sec 3 and Sec ??
- Increasing the charge signal - Migdal effect.
  See Sec. ??
- Depositing the whole kinetic energy - DM absorption, Inelastic DM.
  See Sec. ??
- Add kinetic energy to light dark matter through exotic sources or processes - Accelerated DM.
  See Sec. ??

Light Dark Matter Models

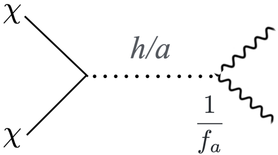
What is Light Dark Matter
$m = \mathrm{keV} - \mathrm{GeV}$

- Light dark matter needs new forces, otherwise it would be overproduced without such mediator
- Light dark matter has portal to Standard Model

vector portal

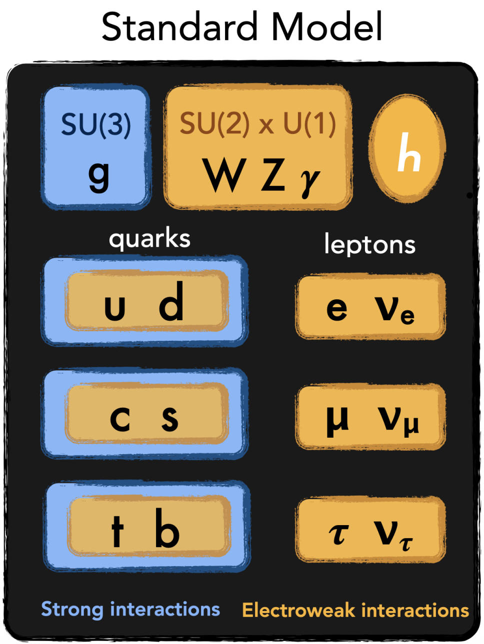


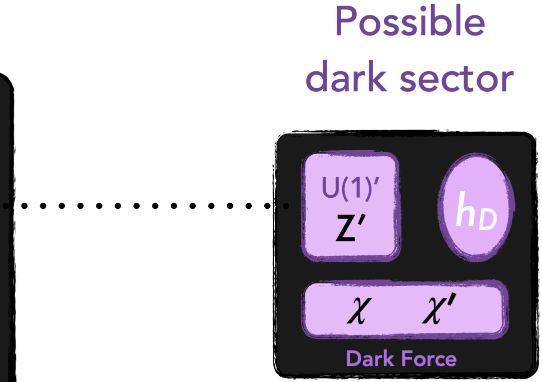
Higgs or axion portal



Model Building for Consistent Production

Vast options and constraints which can be found in Prof Hyun Min's lecture





Theory landscape includes dark gauge forces, flavor, higgs, inelastic DM, etc.

Outline

Outline

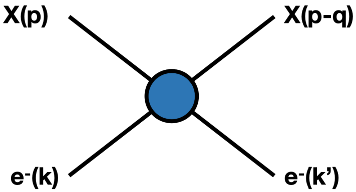
Theory of Dark Matter-Electron Scattering and Electronic Excitation

Outline

Why Electrons?

Kinematics: Just replace m_N by m_e, we can obtain a much larger electron recoil energy!

$$E_i = m_\chi + m_e + \frac{1}{2} m_\chi v^2 + E_{e,1}$$
$$E_f = m_\chi + m_e + \frac{\left| m_\chi \vec{v} - \vec{q} \right|^2}{2 m_\chi} + E_{e,2}$$



From energy and momentum conservation $E_i = E_f$, we obtain

$$\Delta E_{1\to 2} = -\frac{q^2}{2 m_\chi} + q v \cos\theta_{qv}$$

Zeroth-order Consideration

typical momentum transfer

typical size of the momentum transfer is set by the **electron's** momentum not DM.

$$q_{\rm typ} \simeq m_e v_e \sim Z_{\rm eff}\alpha m_e$$

typical energy transfer

in principle, all of the DM's kinetic energy is transferred to electron

$$\Delta E_{e,\rm typ} \simeq q_{\rm typ} v \sim 4 \text{ eV}$$

Minimal Mass

How to estimate which dark matter mass our sensitivity breaks down?

strategry

use energy and momentum conservation to derive it

- Initial dark matter energy $E_\chi = \frac{1}{2} m_\chi v_\chi^2$
- Minimal ionization energy E_{nl} (Binding energy)
- $E_\chi \geq E_{nl}$ and $v_\chi \lesssim v_{\rm esc} + v_{\rm E}$

Result: lowest bound to have ionization

$$m_\chi \gtrsim 250\mathrm{keV} \times \left(\frac{E_{nl}}{1\mathrm{eV}}\right)$$

Different target material can probe different mass range of light DM

General Formula for Free Electron

If dark matter scatters with free electron, it is just a conventional $2 \to 2$ scattering process with cross section to be

$$\sigma v_{\text{free}} = \frac{1}{4E'_\chi E'_e} \int \frac{d^3q}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{1}{4E_\chi E_e} (2\pi)^4 \delta\left(E_i - E_f\right) \delta^3\left(\vec{k} + \vec{q} - \vec{k}'\right) \left|\overline{\mathcal{M}_{\text{free}}(\vec{q})}\right|^2$$

- **momentum transfer effect is absorbed in dark matter form factor $F_{\text{DM}}(q)$. It does not mean dark matter is composite particle**

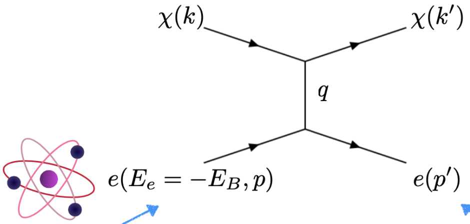
$$\overline{|\mathcal{M}_{\text{free}}(\vec{q})|^2} \equiv \overline{|\mathcal{M}_{\text{free}}\left(\alpha m_e\right)|^2} \times |F_{\text{DM}}(q)|^2$$

- **constant cross section is thus defined**

$$\overline{\sigma}_e \equiv \frac{\mu_{\chi e}^2 \overline{|\mathcal{M}_{\text{free}}\left(\alpha m_e\right)|^2}}{16\pi m_\chi^2 m_e^2}$$

Dark matter-Real electron scattering

Figure from talk by McCabe in Sixteenth Marcel Grossmann Meeting



initial electron is not a momentum eigenstate (not a plane wave)

ionised electron propagates in the Coulomb field of the ionised atom (not a plane wave)

Difference between Free Electron and Bound Electron
Different Wave-Function

for free electrons

$$\left\langle \chi_{\vec{p}-\vec{q}}, e_{\vec{k}'} \left| H_{\text{int}} \right| \chi_{\vec{p}}, e_{\vec{k}} \right\rangle = C\mathcal{M}_{\text{free}}(\vec{q}) \times (2\pi)^3 \delta^3 \left(\vec{k} - \vec{q} - \vec{k}' \right)$$

The wave-functions for electrons are just plane wave.

for bound electrons

$$\left\langle \chi_{\vec{p}-\vec{q}}, e_2 \left| H_{\text{int}} \right| \chi_{\vec{p}}, e_1 \right\rangle = C\mathcal{M}_{\textbf{free}} (\vec{q}) \int \frac{V d^3 k}{(2\pi)^3} \widetilde{\psi}_2^*(\vec{k} + \vec{q}) \widetilde{\psi}_1(\vec{k})$$

Final and initial electrons are not plane waves but to be solved by schrodinger equation. Challenge: we need to calculate bound/unbound states

Transition Probablity

$$\left| f_{1\to 2}(\vec{q}) \right|^2 = \left| \int \frac{d^3 k}{(2\pi)^3} \widetilde{\psi}_2^* \left(\vec{k}' \right) \widetilde{\psi}_1(\vec{k}) \right|^2$$

Momentum conservation is now replaced by wave-function

General Formula for Bound Electron

In terms of dark matter form factor and electron transition probability, cross-section is rewritten

$$\sigma v_{1\to 2} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} V \int \frac{d^3q}{4\pi} \frac{d^3k'}{(2\pi)^3} \delta\left(\Delta E_{1\to 2} + \frac{q^2}{2m_\chi} - qv\cos\theta_{qv}\right) \times |F_{\rm DM}(q)|^2 \, |f_{1\to 2}(\vec{q})|^2$$

- If only one final electron state, $V = 1$ and phase space d^3k', d^3q.
- Kinematics is respected by delta-function.
- Dark matter form factor $F_{\rm DM}(q)$ captures momentum transfer for specific dark matter model.
- Transition probability captures of electron response after scattering

Deal with Phase Space

- Electron recoil energy $E_e = k'^2/2m_e$

ionized electron phase space $= \sum_{l'm'} \int \frac{k'^2 dk'}{(2\pi)^3} = \frac{1}{2} \sum_{l'm'} \int \frac{k'^3 d\ln E_e}{(2\pi)^3}$

- We assume the potential is spherically symmetric and we ionize a full atomic shell therefore, sum over all initial and final angular momentum variables

$$\sigma v_{\text{ion}} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} \sum_{n'l'm'} \int \frac{d^3q}{8\pi} \frac{k'^3 d\ln E_e}{(2\pi)^3} \delta\left(\Delta E_{i\to k'l'm'} + \frac{q^2}{2m_\chi} - qv\cos\theta_{qv}\right) |F_{\text{DM}}(q)|^2 \, |f_{i\to k'l'm'}(\vec{q})|^2$$

Why using E_e

We want to have a similar behavior with DM-nucleus scattering

Ionization Factor

Absorb phase space of electron into ionization factor

$$|f_{\text{ion}}(k', q)|^2 = \frac{2k'^3}{(2\pi)^3} \sum_{n'l'm'} \left| \int d^3x \psi^*_{k'l'm'}(\vec{x}) \psi_i(\vec{x}) e^{i\vec{q}\cdot\vec{x}} \right|^2$$

- Simplified version: outgoing electron is free plane wave, initial electron is part of a spherically symmetric atom with full shells. See Essig or Ran Ding

$$\left|f^i_{ion}(k', q)\right|^2 = \frac{k'^2}{4\pi^3 q} \int_{k'-q}^{k'+q} k dk \, |\chi_{nl}(k)|^2$$

- More realistic version: solve radial Schrödinger equation for the exact unbound wavefunctions, using the effective potential extracted from the bounded wavefunctions. See Timon Emken or Zheng-Liang Liang, Lei Wu

Differential Cross-Section over Electron Recoils

Evaluate the energy conservation δ-function, and $q_{\max}$ and $q_{\min}$?

$$\frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}\ln E_e} = \frac{\overline{\sigma}_e}{8\mu_{\chi e}^2}\int_{q_{\min}}^{q_{\max}} q\,\mathrm{d}q\left|f_{\mathrm{ion}}(k',q)\right|^2\left|F_{\mathrm{DM}}(q)\right|^2\eta(v_{\min})$$

We do not know where DM comes from $\rightarrow$ Astrophysics Uncertainty

Need to perform a velocity distribution integral to get statistical result $\rightarrow$ Average

$$\eta\left(v_{\min}\right) = \int_{v_{\min}}\frac{\mathrm{d}^3 v}{v}f_{\mathrm{MB}}(v)$$

- f_{MB} **is Maxwell-Boltzmann distribution** $f_{\mathrm{MB}} = \frac{1}{N_{\mathrm{esc}}}\left(\frac{3}{2\pi\sigma_v^2}\right)^{3/2} e^{-3\mathbf{v}^2/2\sigma_v^2}$
- $v_{\min}$ **is the minimal velocity for ionziation and** $q_{\min}$, $q_{\max}$ **are determined by kinematics**

Signal Rate

Bridge to connect theory and experiment

Event rate = DM flux $\times$ particle physics $\times$ detector response

$$R = N_T \frac{\rho_\chi}{m_\chi} \int_{E_{e,cut}} \mathrm{d}\ln E_e \frac{\mathrm{d}\langle \sigma v \rangle}{\mathrm{d}\ln E_e}$$

Experiment prefers events rather than cross-section

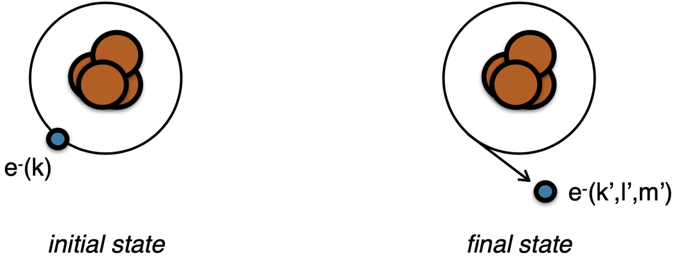
R = number of events/time/volume

- N_T is the number of target atoms $\rightarrow$ material dependent
- $\rho_\chi = 0.4 \mathrm{GeV}/\mathrm{cm}^3$ is the local DM density
- $\mathrm{R} \times \mathrm{Exposure} = \mathrm{Events}$

Simplest Target: Isolated Atom
There is no many-body correlation

Typical atom: Hydrogen, Xenon, and Argon

$$\Delta E_B \sim 10\mathrm{eV}, \quad m_\chi > 2.5\mathrm{MeV}$$



Ionization Factor for Isolated Atom

- relevant quantity is transition probability

$$f_{1\to 2}(\mathbf{q}) = \int \mathrm{d}^3 x \psi^*_{k'\ell'm'}(\mathbf{x}) e^{i\mathbf{x}\cdot\mathbf{q}} \psi_{n\ell m}(\mathbf{x})$$

- Expressed the initial and final state electron wave functions in terms of spherical coordinates

$$\psi_{n\ell m}(\mathbf{x}) = R_{n\ell}(r) Y_\ell^m(\theta, \phi)$$

- Thus transition probability is function of scalar product of radial wave function

$$f_{1\to 2}(\mathbf{q}) = \int \mathrm{d}^3 x R^*_{k'\ell'}(r) Y_{\ell'}^{m'*}(\theta,\phi) R_{n\ell}(r) Y_\ell^m(\theta,\phi) \times 4\pi \sum_{L=0}^{\infty} i^L j_L(qr) \sum_{M=-L}^{+L} Y_L^{M*}\left(\theta_q, \phi_q\right) Y_L^M(\theta,\phi)$$

$$= 4\pi \sum_{L=0}^{\infty} i^L \sum_{M=-L}^{L} I_1(q) Y_L^{M*}\left(\theta_q, \phi_q\right) \int \mathrm{d}\Omega Y_{\ell'}^{m'*}(\theta,\phi) Y_\ell^m(\theta,\phi) Y_L^M(\theta,\phi)$$

Radial Part and Angular Part

For angular part: the integral over three spherical harmonics can be re-written in terms of the Wigner $3j$ symbols

$$f_{1\to 2}(\mathbf{q}) = \sqrt{4\pi} \sum_{L=|\ell-\ell'|}^{\ell+\ell'} i^L I_1(q) \sum_{M=-L}^{+L} Y_L^{M*}(\theta_q, \phi_q)(-1)^{m'} \sqrt{(2\ell+1)(2\ell'+1)(2L+1)}$$
$$\times \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} \ell & \ell' & L \\ m & -m' & M \end{pmatrix}$$

The orthogonality of Wigner $3j$ symbols, allows us to sum over the L' and M'

$$\sum_{m=-\ell}^{\ell} \sum_{m'=-\ell'}^{\ell'} |f_{1\to 2}(\mathbf{q})|^2 = 4\pi \sum_{L} I_1(q)^2 \sum_{M} Y_L^{M*}(\theta_q, \phi_q)$$

Radial part: the core is wavefunction

$I_1(q) \equiv \int \mathrm{d}r r^2 R^*_{k'\ell'}(r) R_{n\ell}(r) j_L(qr)$

Initial and Final State Wave Functions

Initial state wave-function is Roothaan-Hartree-Fock (RHF) ground state wave function

It is just a linear combination of Slater-type orbitals

$$R_{n\ell}(r) = a_0^{-3/2} \sum_j C_{j\ell n} \frac{\left(2Z_{j\ell}\right)^{n'_{j\ell}+1/2}}{\sqrt{\left(2n'_{j\ell}\right)!}} \left(\frac{r}{a_0}\right)^{n'_{j\ell}-1} \exp\left(-Z_{j\ell}\frac{r}{a_0}\right)$$

Final state wave function is similar with hydrogen wave function except energy is positive and spectra is continuum

It is solved by the Schrodinger equation with a hydrogenic potential $-Z_{eff}/r$

$$R_{k'\ell'}(r) = \frac{(2\pi)^{3/2}}{\sqrt{V}} (2k'r)^{\ell'} \frac{\sqrt{\frac{2}{\pi}} \left|\Gamma\left(\ell' + 1 - \frac{iZ_{\mathrm{eff}}}{k'a_0}\right)\right| e^{\frac{\pi Z_{\mathrm{eff}}}{2k'a_0}}}{(2\ell'+1)!} e^{-ik'r} {}_1F_1\left(\ell' + 1 + \frac{iZ_{\mathrm{eff}}}{k'a_0}, 2\ell' + 2, 2ik'r\right)$$

Scattering Kinematics

In terms of energy conservation

$$\mathbf{v} \cdot \mathbf{q} = \Delta E_{1\to 2} + \frac{q^2}{2m_\chi}$$

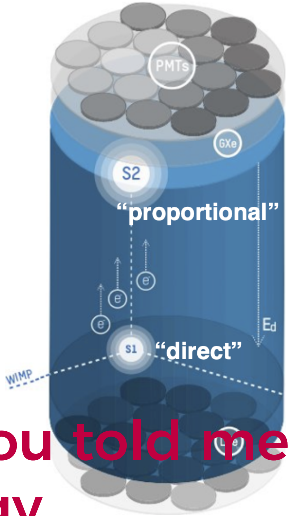
- **Minimal velocity is obtained by setting** $\cos\theta_{qv} = 1$

$$v_{\min}(k', q) = \frac{E_B + k'^2/(2m_e)}{q} + \frac{q}{2m_\chi}$$

- **Taking** $\cos\theta_{qv} = 1$, $E_e = 0$ **and** $v = v_{\max}$, **the range of** q **is**

$$\begin{aligned} q_{\min} &= m_\chi v_{\max} - \sqrt{m_\chi^2 v_{\max}^2 - 2m_\chi E_B} \\ &= \frac{E_B}{v_{\max}}, \quad \text{for } m_\chi \to \infty \\ q_{\max} &= m_\chi v_{\max} + \sqrt{m_\chi^2 v_{\max}^2 - 2m_\chi E_B} \end{aligned}$$

a XENON detector



https://commons.wikimedia.org/wiki/File:2phaseTPC_b.jpg

i.e. XENON10, XENON100, XENON1T, LUX

DM-electron scattering

=

S2 only signal

measures **P**hoto**E**lectrons

PE, you told me, we measure recoil energy

*can also do this with LAr detectors like DarkSide

Real Event Rate

Frome Recoil energy E_e to PE

$$\frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{dS2}} = \int d\ln E_e \epsilon(\mathrm{S2}) P\left(\mathrm{S2} \mid \Delta E_e\right) \frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{d}\ln E_e}$$

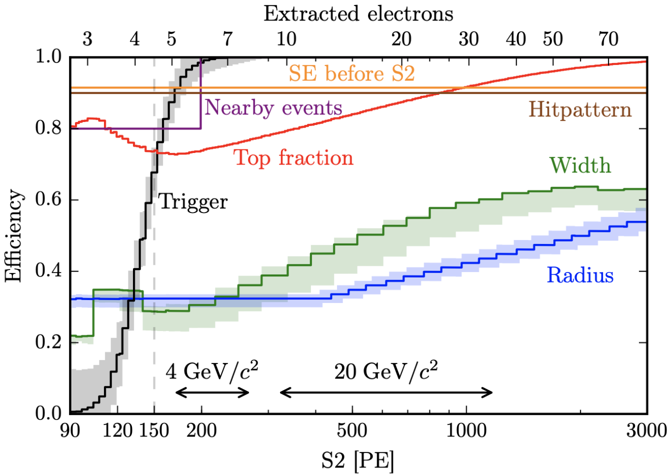
- $\epsilon(\mathrm{S2})$ **is the detector efficiency,** $S2 = \mathrm{PE}$
- **The probability function** P **that converts energy transfer into the photoelectron (PE) in S2**
- $\Delta E_e = E_e + E_{nl}$

True Signal Rate

We can compare our signal rate $dR/dS2$ **to data directly to obtain exclusion limit**

Detector Efficiency

Take XENON1T as example



Probability Function $P\left(\mathrm{S2} \mid \Delta E_e\right)$

$$P\left(\mathrm{S2} \mid \Delta E_e\right)=\sum_{n_e^{\mathrm{s}}, n_e} P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right) \cdot P\left(n_e^{\mathrm{s}} \mid n_e\right) \cdot P\left(n_e \mid \langle n_e\rangle\right)$$

- $P\left(n_e \mid \langle n_e\rangle\right)$ **is the number of electrons escaping the interaction point, which follows a binomial distribution**

$$P\left(n_e \mid \langle n_e\rangle\right)=\operatorname{binom}\left(n_e \mid N_Q, f_e\right)=C_{N_Q}^{n_e} f_e^{n_e}\left(1-f_e\right)^{N_Q-n_e}, N_Q=\Delta E_e / 13.8 \mathrm{eV}$$

- $P\left(n_e^{\mathrm{s}} \mid n_e\right)=80\%$ **is possibility of electrons surviving the drift in Xenon1T**
- **PE transformation probability** $P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right)$ **is Gaussian distribution**

$$P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right)=\operatorname{Gauss}\left(\mathrm{S2} \mid g_2 n_e^{\mathrm{s}}, \sigma_{\mathrm{S2}}\right)$$

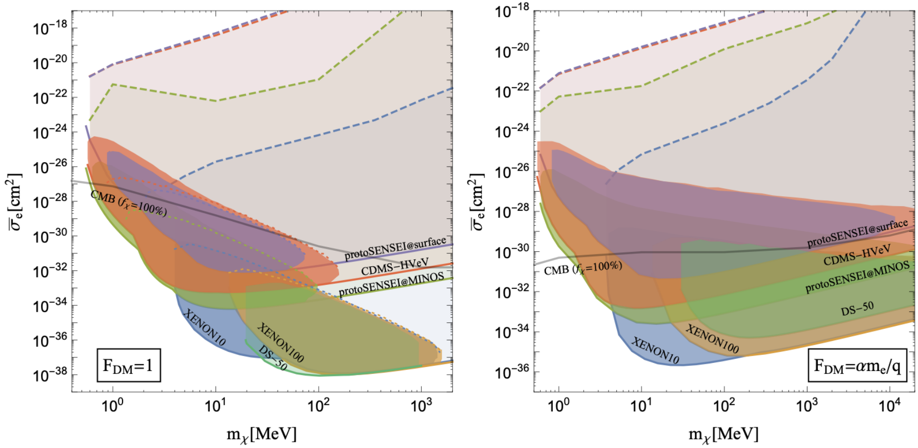
XENON10 and XENON1T Data

XENON10	
bin [S2]	obs. events
[14,41)	126
[41,68)	60
[68,95)	12
[95,122)	3
[122,149)	2
[149,176)	0
[176,203)	2

XENON1T	
bin [S2]	obs. events
[150,200)	8
[200,250)	7
[250,300)	2
[300,350)	1
-	-
-	-
-	-

Find Limits

Signal + Backgrounds < Number of observed events



Upper bound comes from the Earth attenuation effect. See Sec ??

Brief Summary on DM Induced Electron Ionizations

- Complication: Target electrons are bound states.
- Electrons are not in a momentum eigenstates
- Example: Ionization spectrum for isolated atom:

$$\frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{d}E_e} = \frac{1}{m_N}\frac{\rho_\chi}{m_\chi}\sum_{nl}\frac{\left\langle \mathrm{d}\sigma_{\mathrm{ion}}^{nl} v\right\rangle}{\mathrm{d}E_e}$$

$$\frac{\mathrm{d}\left\langle \sigma_{\mathrm{ion}}^{nl} v\right\rangle}{\mathrm{d}E_e} = \frac{\sigma_e}{8\mu_{\chi e}^2 E_e}\int \mathrm{d}q q\,|F_{\mathrm{DM}}(q)|^2\left|f_{\mathrm{ion}}^{nl}(k',q)\right|^2\eta\left(v_{\mathrm{min}}\left(\Delta E_e,q\right)\right)$$

- Predictions require the precise evaluation of an ionization form factor.
- There is still theoretical uncertainty in the evaluation of the ionization form factors. See 1904.07127.
- For crystals, this requires methods from condensed matter physics. form factors.

Challenge for Isolated Atom

- detector specific backgrounds i.e. e^- gets trapped in liquid-gas interface and is later released
  Need a better detector setup
- ionization energy $(12.1\mathrm{eV})$ limits DM mass reach to few MeV
  Find a material with smaller ionization energy