Neutrino oscillation, CP asymmetry, matter effect and sterile neutrino

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Outline

1. Overview on the neutrinos and their properties
2. Discussions about neutrino oscillation probabilities and CP asymmetries in vacuum
3. Derivations of matter effects on the neutrino oscillation parameters
4. Introduction to sterile neutrino
5. Sterile neutrino imprints in the long-baseline oscillations

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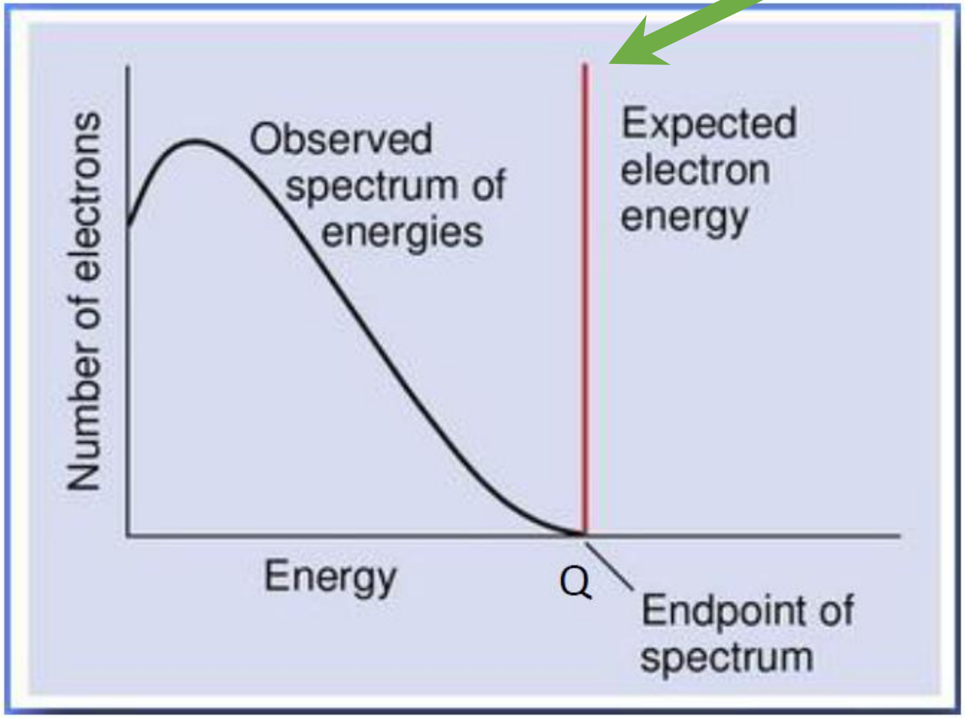
Neutrinos in the SM and beyond

A bit of history

$${}^{A}_{Z}N \rightarrow {}^{A}_{Z+1}N' + e^-$$

$$E_e = (m_N - m_{N'})c^2 = \text{constant.}$$

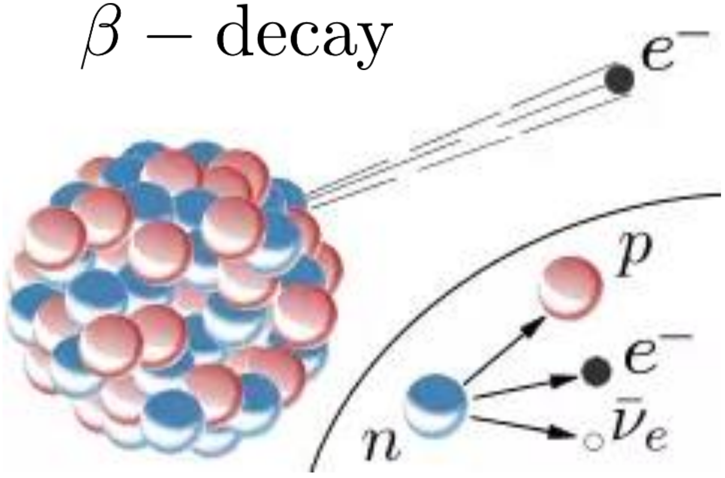
Why it is a continuous spectrum?



Energy maybe conserved in a statistical sense

N. Bohr

$\beta-$decay



Dec. 4, 1930: A desperate remedy

Zürich, Dec. 4, 1930

Physics Institute of the ETH
Gloriastrasse

Zürich

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton mass. - The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.

Now it is also a question of which forces act upon neutrons. For me, the most likely model for the neutron seems to be, for wave-mechanical reasons (the bearer of these lines knows more), that the neutron at rest is a magnetic dipole with a certain moment μ. The exp that the ionizing effect of such a neutron can not be bigger than the one o μ is probably not allowed to be larger than e • (10 -13cm).

But so far I do not dare to publish anything about this idea, and trustful radioactive people, with the question of how likely it is to find experime neutron if it would have the same or perhaps a 10 times larger ability to ge a gamma-ray.

I admit that my remedy may seem almost improbable because one probab neutrons, if they exist, for a long time. But nothing ventured, nothing gaine the situation, due to the continuous structure of the beta spectrum, is il my honored predecessor, Mr Debye, who told me recently in Bruxelles: "O about this at all, like new taxes." Therefore one should seriously discuss ev dear radioactive people, scrutinize and judge. - Unfortunately, I cann Tübingen since I amindispensable here in Zürich because of a ball on the n 7. With my best regards to you, and also to Mr. Back, your humble servant

W. Pauli

- 1932 Chadwick discovers the neutron

Neutron ≠ Pauli's neutron = Neutrino (Fermi)

Little neutron

- 1956 ν_e detected by Reines and Cowan (Nobel prize 1995)

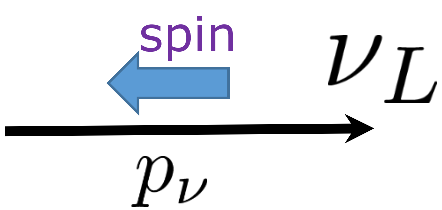
- 1962 ν_μ discovered at Brookhaven by Ledermann, Schwartz and Steinberger (Nobel prize 1988)

- 1995 LEP measurement of Z decay width found 3 active neutrino flavors: $N_\nu = 3.00 \pm 0.06$ $(\nu_e, \nu_\mu, \nu_\tau)$

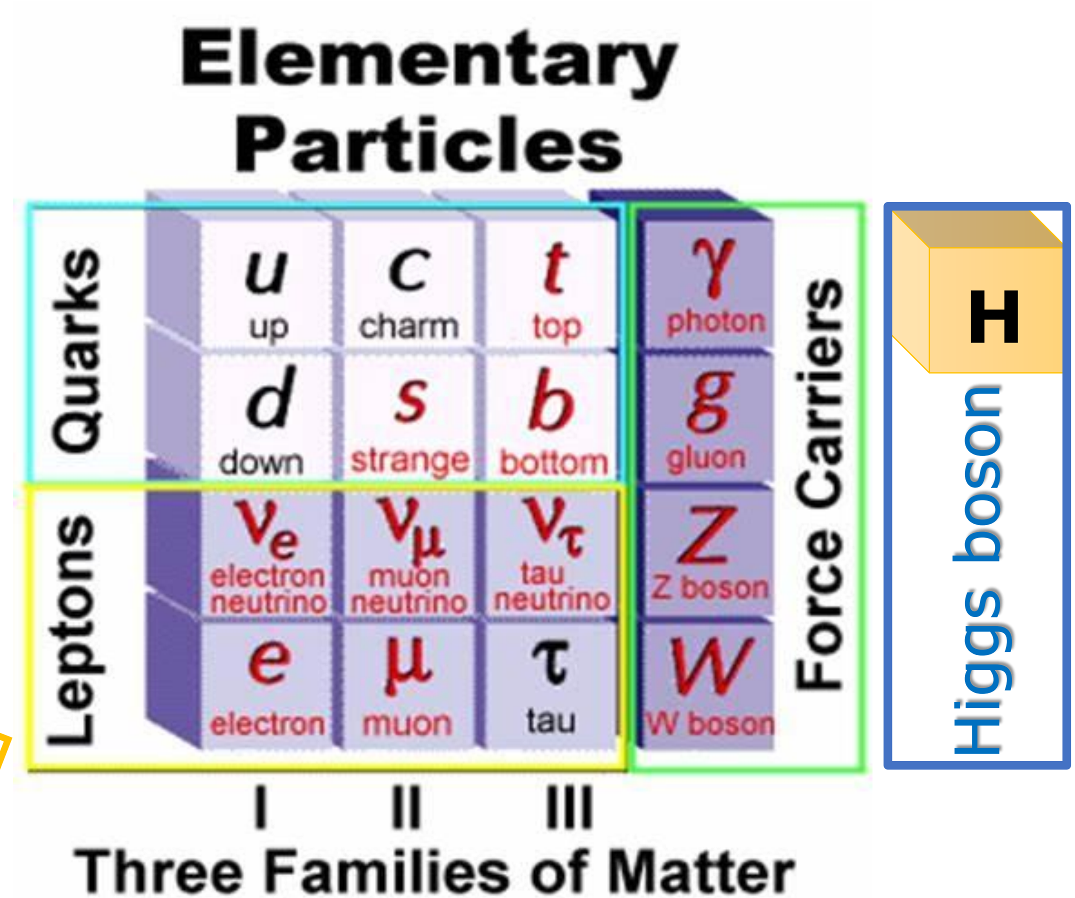
- 2000 detection of ν_τ by the DONUT experiment (Fermilab)

Neutrinos in the SM are:

- Electrically neutral
- Spin ½ (fermion)
- Massless
- Stable
- Left-handed

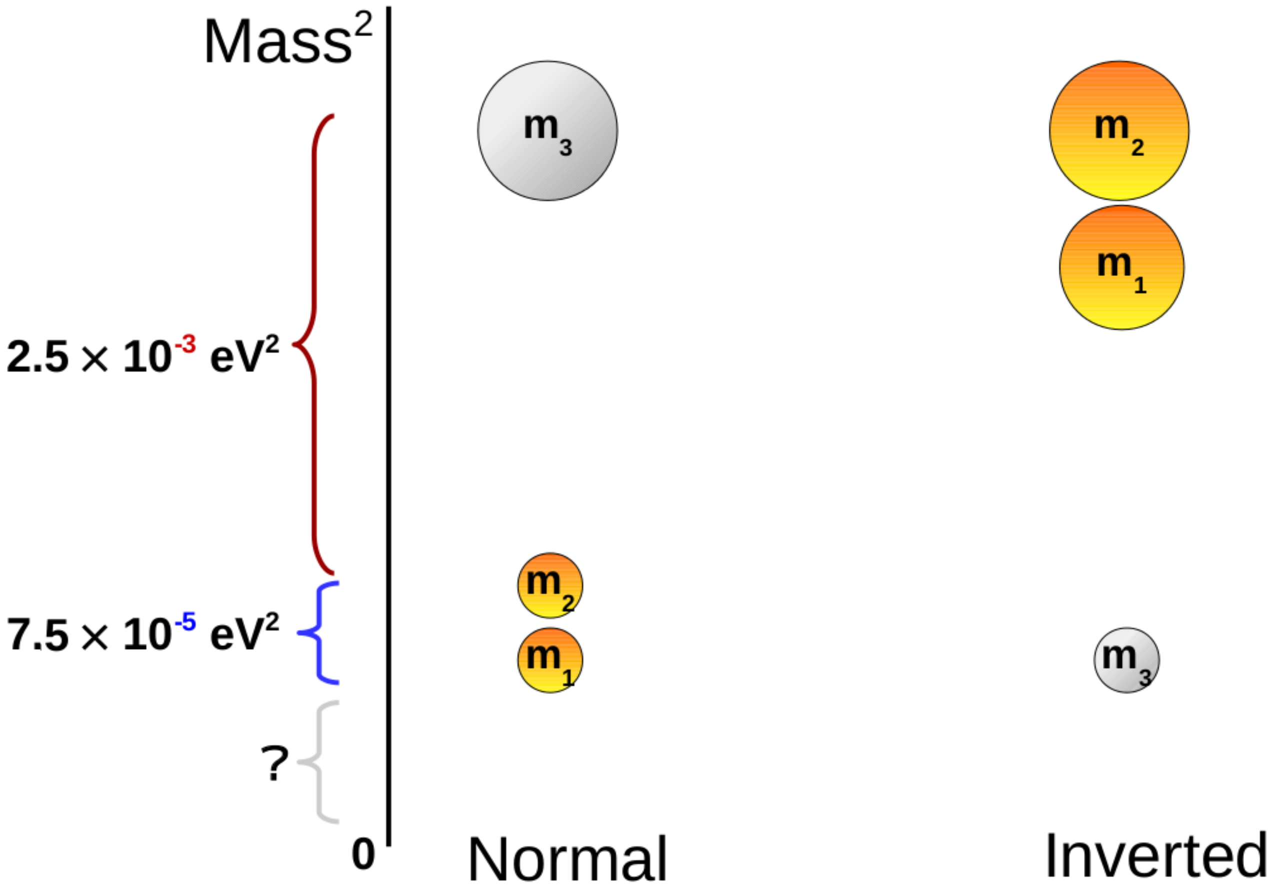


- Associate with charged lepton



But neutrinos are massive

Two possible mass ordering



Neutrinos mass generation

- Dirac neutrino: Higgs mechanism

$$\mathcal{L} \supset y^N_{ij} \nu^c_i l_j h + h.c. \quad \Rightarrow m^N_{ij} \nu^c_i \nu_j \quad m_N = y^N v.$$

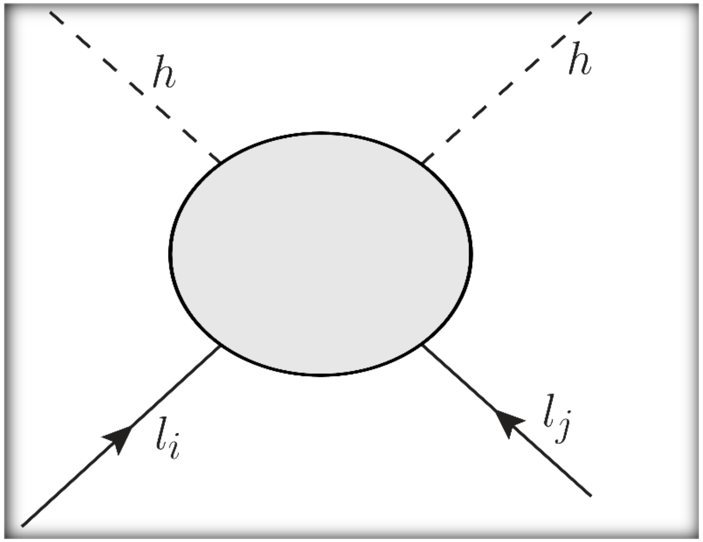
$$y^N \sim 10^{-12} \text{ to have } m_\nu \sim 0.05\,\text{eV}$$

- Majorana neutrino: Weinberg operator

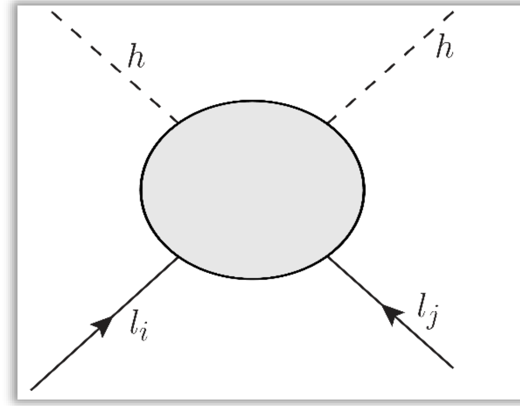
$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \frac{c_{ij}}{2\Lambda}(l_i h)(l_j h) \quad .$$

$$\Rightarrow \frac{1}{2} m^\nu_{ij} \nu_i \nu_j \quad m_\nu = (cv)\frac{v}{\Lambda}$$

$$\Lambda \simeq 0.6 \times 10^{15} c \left(\frac{0.05\ \text{eV}}{m_\nu}\right) \text{GeV}.$$

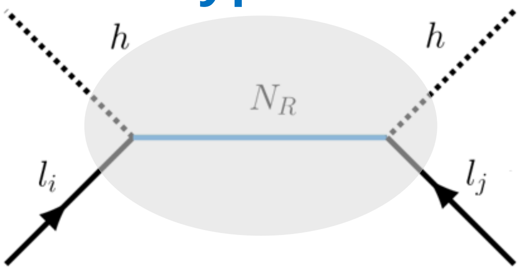


Tree-level realisations

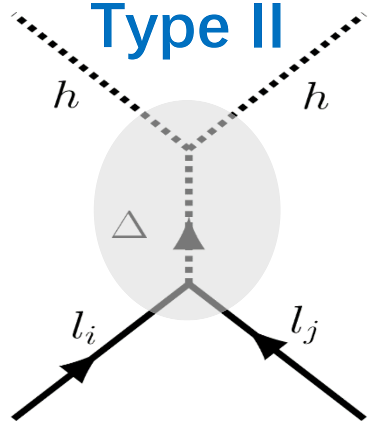


Seesaw mechanism (type I, II, III)

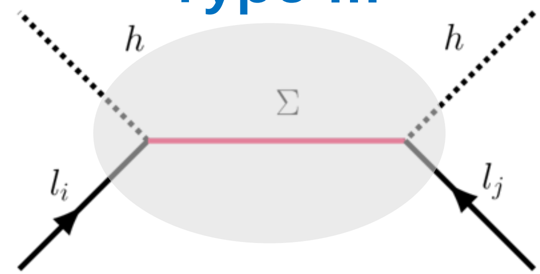
Type I



Type II



Type III



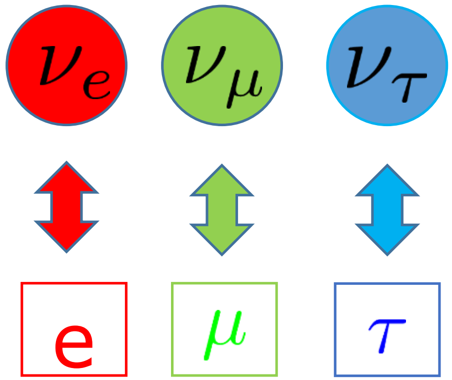
Type I seesaw

$$\mathcal{L} \supset y^N_{aj} N_a l_j h + \frac{1}{2} M_{ab} \overline{N^c}_a N_b + h.c.$$

$$\Rightarrow \boxed{m_\nu = -m_N^T M^{-1} m_N} \quad m_N = y^N v$$

Neutrino flavor mixing

Flavor eigenstate



Mass eigenstate

ν_1 ν_2 ν_3

Flavor sate $\neq$ Mass eigenstate

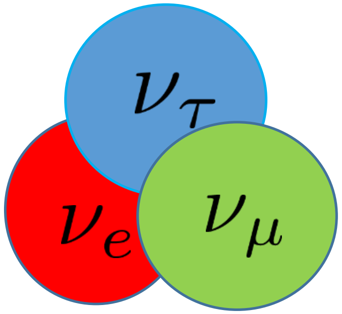
Flavor: characteristic of interactions

Mixing

characteristic of propagations

Mixing matrix: $\nu_f = U_{\mathrm{PMNS}}\, \nu_m$

$\mathbf{U}_{\mathrm{PMNS}}$ is a unitary matrix



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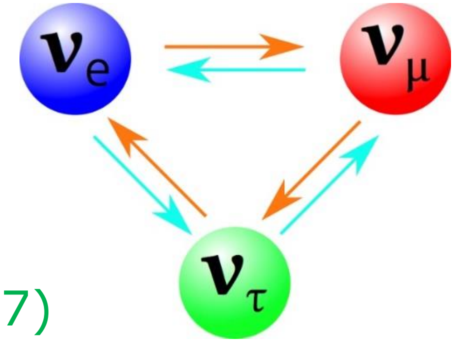
Neutrino oscillations and CP asymmetries in vacuum

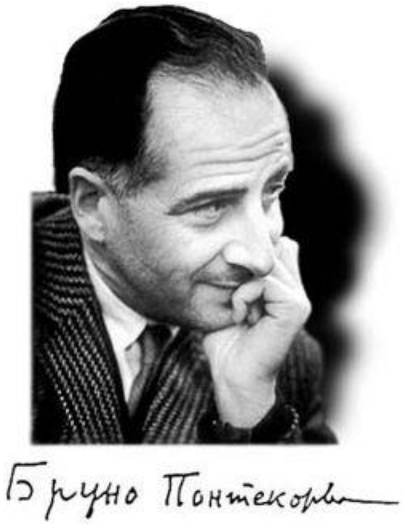
Neutrino oscillation (in vacuum)

Oscillation: periodic transformation of neutrino flasvors during the propagation

B. Pontecorvo mentioned a possibility of neutrino mixing and oscillation

"Mesonium and antimesonium"
Zh. Eksp. Teor. Fiz. 33, 549 (1957)
[Sov. Phys. JETP 6, 429 (1957)] translation





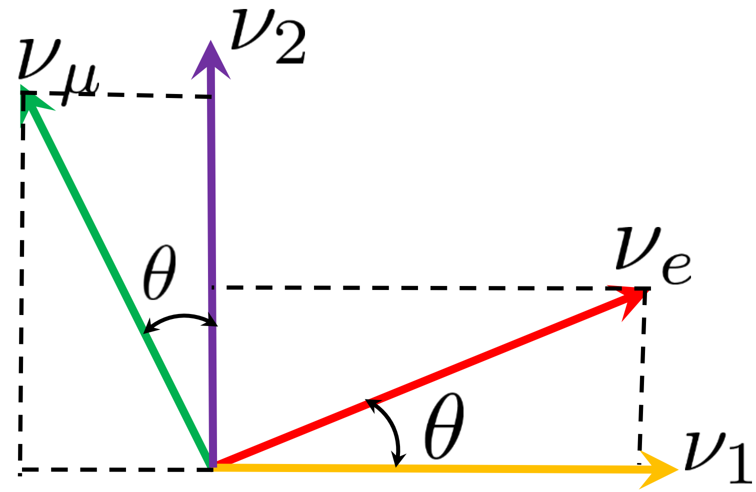
Neutrino oscillation: theoretial description

Exampel:2 flavor oscillations

Mixing matrix

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$



$$H|\nu_k\rangle = E_k|\nu_k\rangle, \qquad E_k = \sqrt{\mathbf{p}^2 + m_k^2}.$$

Evolution equation $i\frac{\mathrm{d}}{\mathrm{d}t}|\nu_k(t)\rangle = H|\nu_k(t)\rangle$, $\quad |\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle$

$$|\nu_\alpha(t)\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t}|\nu_k\rangle$$

Transition amplitude

$$
\begin{aligned}
A\left(\nu_\alpha(0) \rightarrow \nu_\beta(t)\right) &= \langle \nu_\beta(t) | \nu_\alpha(0) \rangle \\
&= \sum_\gamma \sum_k U_{\gamma k} e^{iE_k t} U_{\beta k}^* \langle \nu_\gamma(0) \mid \nu_\alpha(0) \rangle \\
&= \sum_k U_{\alpha k} e^{iE_k t} U_{\beta k}^*, \qquad \alpha, \beta = e, \mu
\end{aligned}
$$

Oscillation probability

$$
P\left(\nu_\alpha \rightarrow \nu_\beta\right) = \left|A\left(\nu_\alpha(0) \rightarrow \nu_\beta(t)\right)\right|^2 = \sum_j \sum_k U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} e^{-i(E_j - E_k)t}
$$

Taking approximations $E_k \simeq p, \;\; E_k \simeq E + \frac{1}{2E} m_k^2, \; t \simeq L$

$$
P\left(\nu_\alpha \rightarrow \nu_\beta\right) = \left(\left|U_{\beta 1}\right|^2 \left|U_{\alpha 1}\right|^2 + \left|U_{\beta 2}\right|^2 \left|U_{\alpha 2}\right|^2\right) + 2 U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2} U_{\beta 2}^* \cos\left(E_2 - E_1\right) L.
$$

Specifically,

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) = \underbrace{\sin^2(2\theta)}_{\text{Amplitude}} \sin^2\left(\underbrace{\frac{\Delta m^2}{4E}}_{\text{frequency}} L\right),$$

$$P(\nu_e \to \nu_e) = P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right).$$

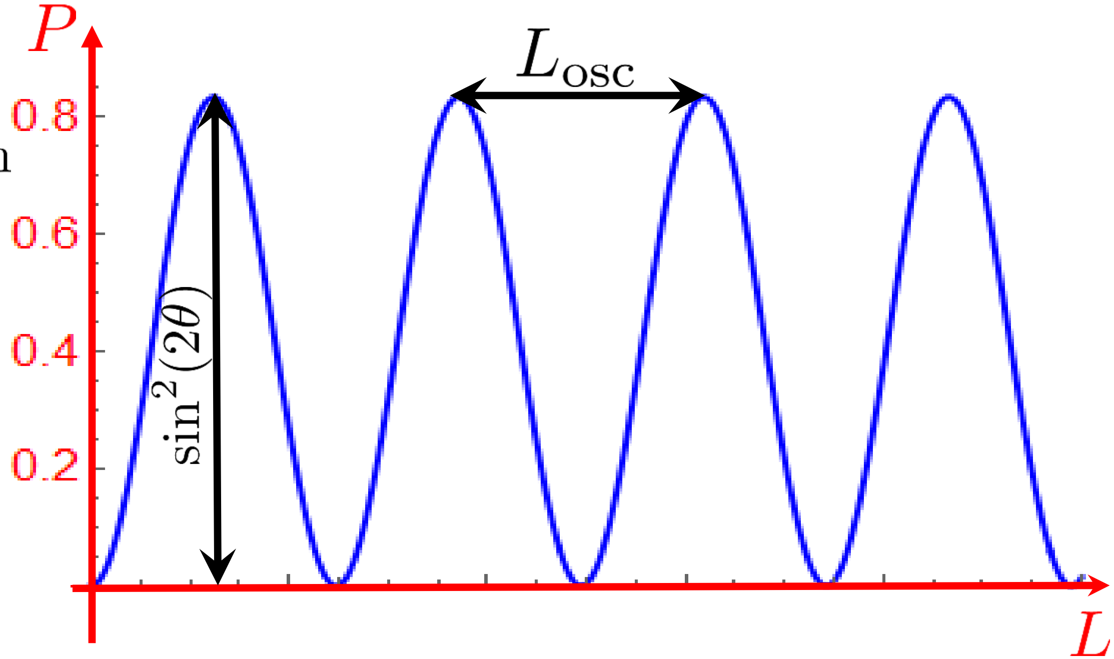
$\Delta m = m_2^2 - m_1^2$. In natural unit, $\frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m^2}{\text{eV}^2}\frac{L}{\text{km}}\frac{\text{GeV}}{E}$

Oscillation length:

$$L_{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}\text{m}$$

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$



3 neutrino oscillation probabilities

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathrm{Re}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$
$$+ 2\sum_{k>j} \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta y}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right).$$

$P(\nu_\alpha \to \nu_\beta) = 0$ if $\Delta m^2_{kj} \equiv m^2_k - m^2_j = 0$ or no mixing

- Survival probability, $\alpha = \beta$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{k>j} |U_{\alpha k}|^2 |U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$

- Unitarity, $\sum_\alpha P(\nu_\alpha \to \nu_\beta) = \sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$
- Unlike 2 flavors, $P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha)$ if $\alpha \neq \beta$

What is the form of mixing matrix U?

Standard parametrization

3 mixing angles and 1 Dierac phase

Atmospheric | Reactor | Solar | Majorana phases

$$U=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}e^{i\alpha_1/2}&0&0\\0&e^{i\alpha_2/2}&0\\0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}c_{12}c_{13}&s_{12}c_{13}&s_{13}e^{-i\delta}\\-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}&c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}&s_{23}c_{13}\\s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}&-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta}&c_{23}c_{13}\end{pmatrix}\begin{pmatrix}e^{i\alpha_1/2}&0&0\\0&e^{i\alpha_2/2}&0\\0&0&1\end{pmatrix},$$

$$\tan^2\theta_{12}=|U_{e2}|^2/|U_{e1}|^2$$
$$\sin^2\theta_{13}=|U_{e3}|^2$$
$$\tan^2\theta_{23}=|U_{\mu3}|^2/|U_{\tau3}|^2$$

If $\delta\neq 0,\pi$, then

$$P(\nu_\alpha\to\nu_\beta)\neq P(\nu_\beta\to\nu_\alpha)$$

δ is a CP violating phase

Different neutrino sources and oscillation properties

Source	Type of ν	$\bar{E}$[MeV]	L[km]	$\min(\Delta m^2)$[eV^2]
Reactor	$\bar{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\bar{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \bar{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

CP asymmetries

CPT transformations: $\psi(t,x) \xrightarrow{C} \psi^c(t,x) \equiv \mathcal{C}\bar{\psi}^T,$

$$\boldsymbol{x} \xrightarrow{P} -\boldsymbol{x},\ t \xrightarrow{T} -t$$

$$\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu$$
$$\mathcal{C}^\dagger = \mathcal{C}^{-1},\ \mathcal{C}^T = -\mathcal{C}$$
$$\mathcal{C} = i\gamma_2\gamma_0$$

Oscillation probability transform as

$$P(\nu_\alpha \to \nu_\beta) \xrightarrow{CPT} P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) = P(\nu_\alpha \to \nu_\beta)$$ **CPT ✓**

$$P(\nu_\alpha \to \nu_\beta) \xrightarrow{CP} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$ **CP ✓** If they are equal

Since $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta)(U \to U^*)$, we define mesure of CP aymmetry

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 4\sum_{i<j} \mathrm{Im}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right) \sin\frac{\Delta m^2_{ji}L}{2E}$$

$\Delta P_{\alpha\beta} = 0$ ⟺ CP is conserved

$$\Delta P_{\alpha\beta} = 4\sum_{i<j} \mathrm{Im}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E}$$

If $\delta = 0,\ \pi$, then U is real, thus $\Delta P_{\alpha\beta} = 0$, so δ is Dirac CP violating phase.

$$\Delta P_{\alpha\beta} = -\Delta P_{\beta\alpha},$$

$$\Downarrow$$

$$\Delta P_{\alpha\alpha} = 0$$

$$\Downarrow$$

No CP violation in survival probability

CP violation is contained in

$$J^{\alpha\beta}_{ij} = \mathrm{Im}\left[U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right]$$

Jarlskog invariant.

$$J_{ij}^{\alpha\beta} = \mathrm{Im}\left[U_{\alpha i}U_{\beta j}U_{\alpha j}^{*}U_{\beta i}^{*}\right]$$

Properties:

1. Rephasing invariant $U_{\alpha j} \to e^{i\theta_\alpha}U_{\alpha j}\, e^{-i\theta'_j}$

2. $J_{ij}^{\alpha\beta} = -J_{ij}^{\beta\alpha} = -J_{ji}^{\alpha\beta} = J_{ji}^{\beta\alpha}.$

3. $\sum_{\alpha} J_{ij}^{\alpha\beta} = \sum_{i} J_{ij}^{\alpha\beta} = 0$

4. Only one is idependent for three fanilies

$$J_{ij}^{\alpha\beta} = J\sum_{\gamma,k}\epsilon_{\alpha\beta\gamma}\epsilon_{ijk},$$

$$J = \mathrm{Im}\left[U_{e2}U_{\mu 3}U_{e3}^{*}U_{\mu 2}^{*}\right]$$

$$= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\cos\theta_{13}\sin 2\theta_{13}\sin\delta,$$

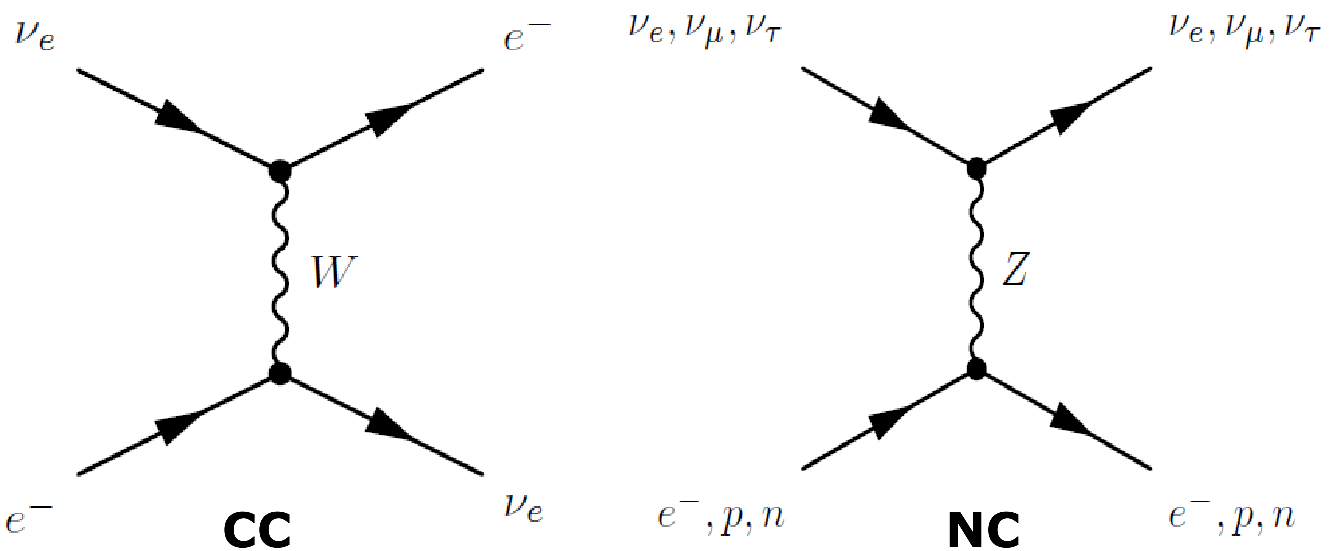
$J = 0$ if $\delta = 0, \pi$

3

Matter effects

Neutrino oscillation (in matter)

Neutrino interacts with matter



Aain 2 neutrino case

Effective Hamiltonian

$$U^T m_\nu U = m_D = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$U^T = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix}$$

$$H = H_0 + \sqrt{2} N_e G_F \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \boxed{\cdots} \rightarrow \propto \mathbb{I}$$

$$H_0 = \frac{m_\nu^\dagger m_\nu}{2E} = \frac{U^\dagger m_D^2 U}{2E} = \frac{\Delta m_0^2}{4E} \begin{pmatrix} -\cos 2\theta_0 & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 \end{pmatrix} + \boxed{\cdots}$$

$$H = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \cdots$$

$$A \equiv 2\sqrt{2} E G_F N_e$$

Masses and mixing angle are effected

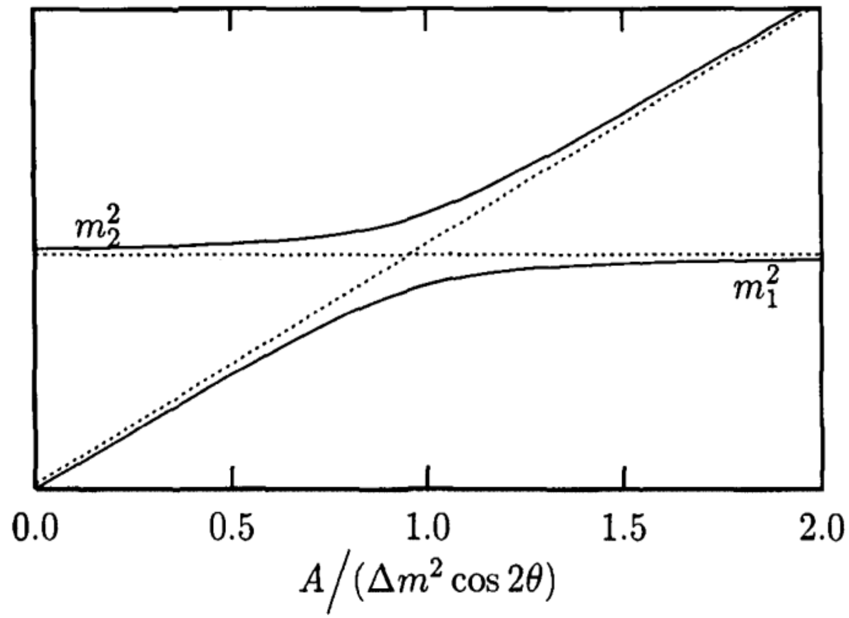
$$\Delta m^2 = \Delta m_0^2 \sqrt{\sin^2 2\theta_0 + \left(\cos 2\theta_0 - \frac{A}{\Delta m_0^2}\right)^2}$$

Masses and mixing angle are effected

$$\Delta m^2 = \Delta m_0^2 \sqrt{\sin^2 2\theta_0 + \left(\cos 2\theta_0 - \frac{A}{\Delta m_0^2}\right)^2}$$

$$\sin^2 2\theta = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + \left(\cos 2\theta_0 - \frac{A}{\Delta m_0^2}\right)^2}$$

$$A \equiv 2\sqrt{2} E G_F N_e$$



Resonance condition: $A = \Delta m_0^2 \cos 2\theta_0$

$$\sin^2 2\theta = 1$$ ⇒ **Amplitude is maximum, independent of vacuum mixing angle**

$$\Delta m^2 = \Delta m_0^2 |\sin^2 2\theta_0|$$

Minimum value between the two eigenvalues of effective Hamiltonian H

Summary: neutrino parameters relevant for oscillations

Nuber of flavors	Number of parameters	Total
2	1 Δm^2 , 1 angle, 0 phase	2
3	2 Δm^2, 3 angles, 1 phase	6
4	3 Δm^2, 6 angles, 3 phases	12
n	n-1 Δm^2 ,$\frac{n(n-1)}{2}$ angles, $\frac{(n-1)(n-2)}{2}$ phases	$n(n-1)$

Determination of neutrino oscillation parameters

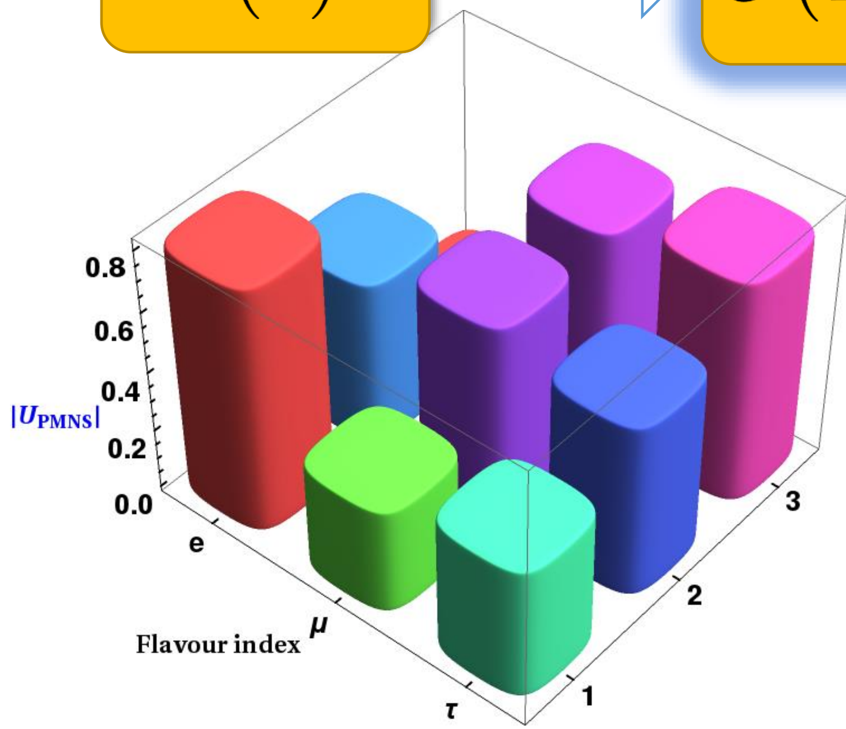
Relative 1σ uncetainity $\leq 10\%$

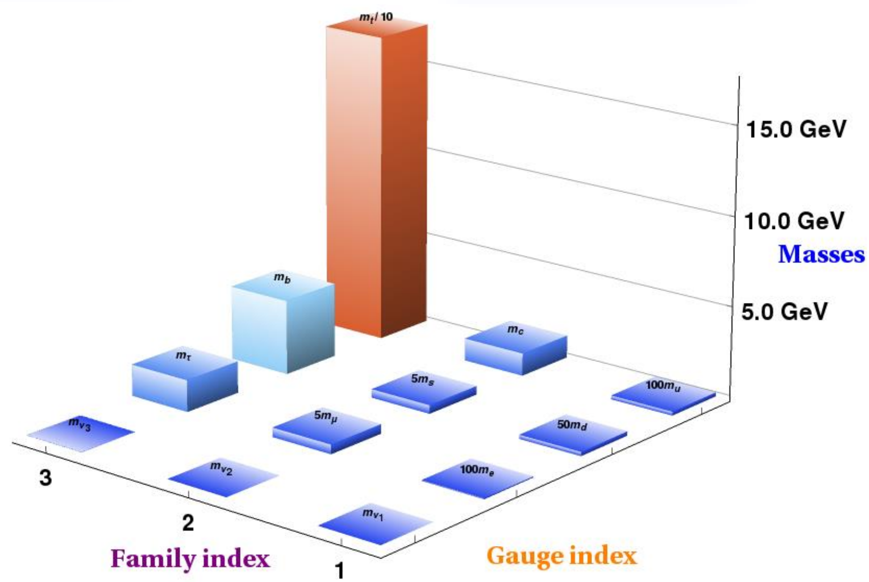
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

Ivan Esteban et. al JHEP 09 (2020) 178

20 years ago → Now → Future

$\mathcal{O}(1)$ → $\mathcal{O}(10\%)$ → $\mathcal{O}(1\%)$





$$|U|_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

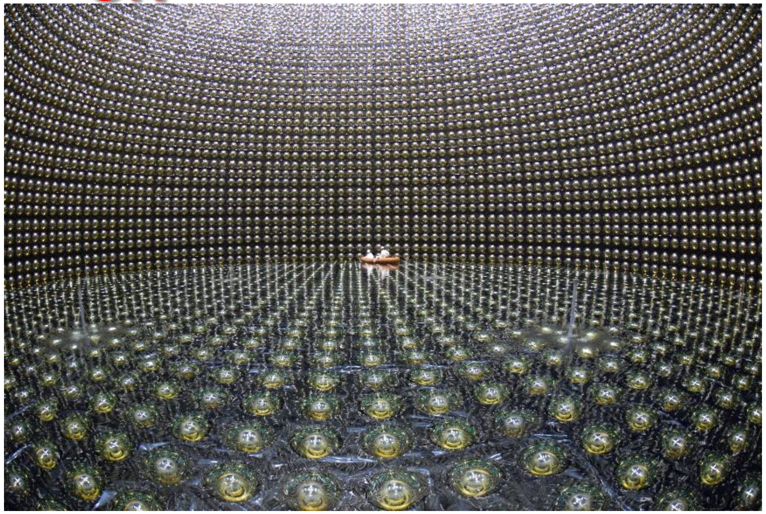
Ivan Esteban et. al JHEP 09 (2020) 178

The Nobel Prize in Physics 2015 for the discovery of neutrino oscillations

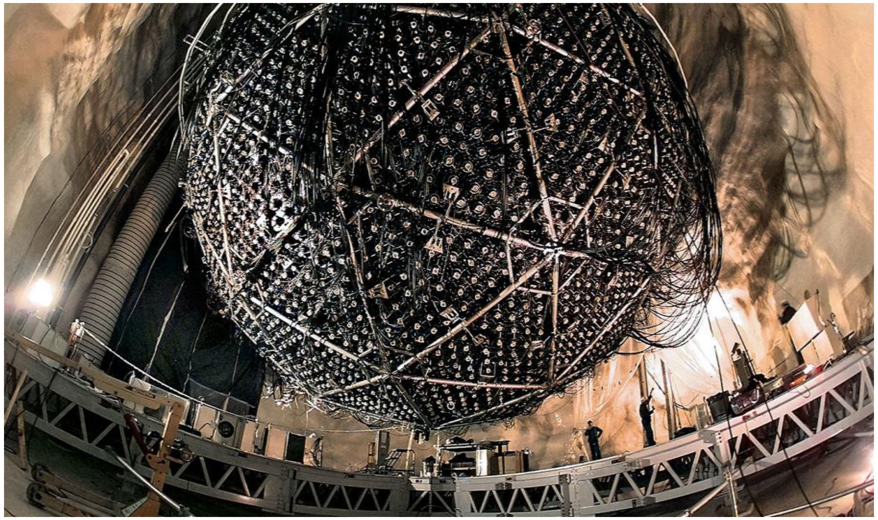
Takaaki Kajita

Arthur B. McDonald

SK



SNO



4

Introduction to eV-mass sterile neutrino

Sterile neutrino

- **Sterile neutrinos** (or **inert neutrinos**) is a neutrino without the SM interactions
- 1968 proposed by Bruno Pontecorvo
- Only attend gravitational interaction
- Mixes with active neutrinos

Sterile Neutrinos: the Ghost Particle's Ghost!

Currently, no strict bound on the sterile neutrino mass.

Why eV-mass sterile neutrino?

1 Short baseline exp.

LSND (a):

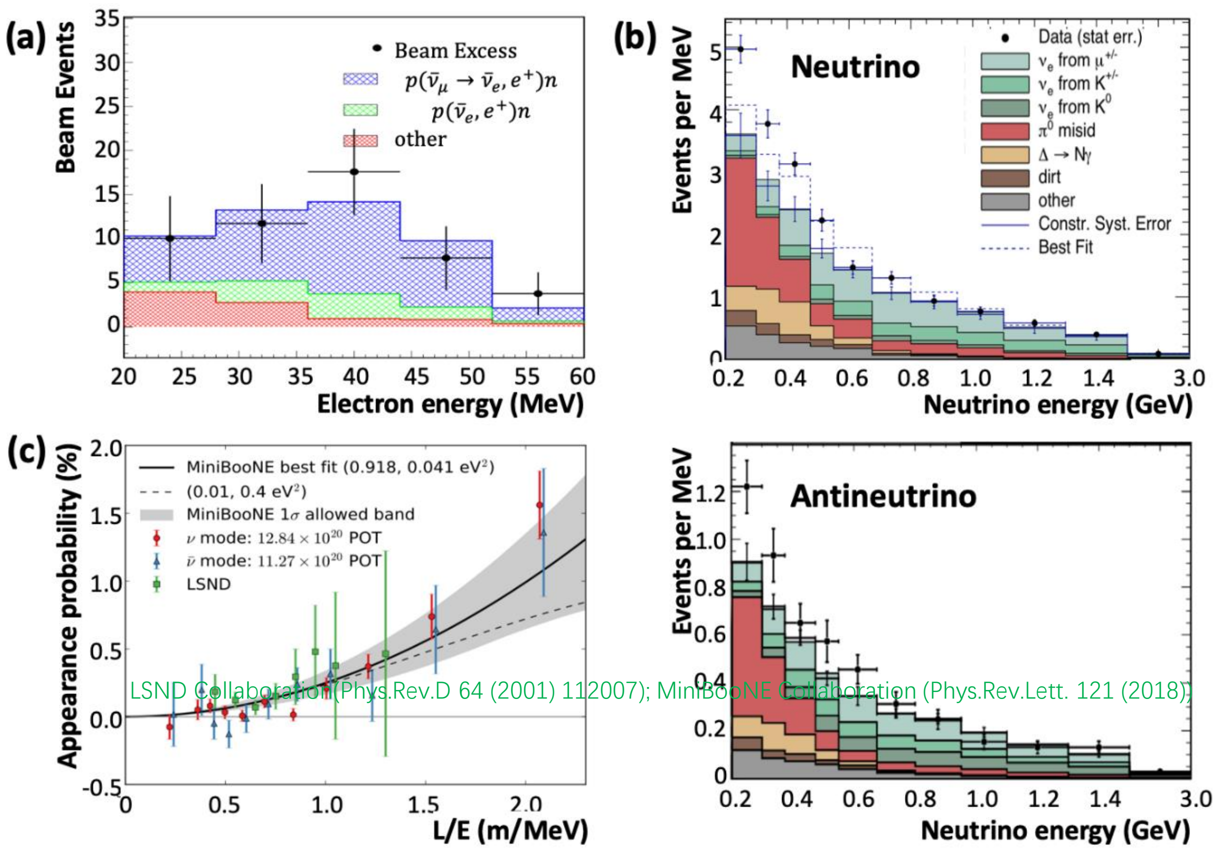
$\bar{\nu}_\mu \to \bar{\nu}_e$ excess at 3.8σ

MiniBooNE (b):

$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ excess at 4.7σ

Combined (c):

excess at 6.0σ



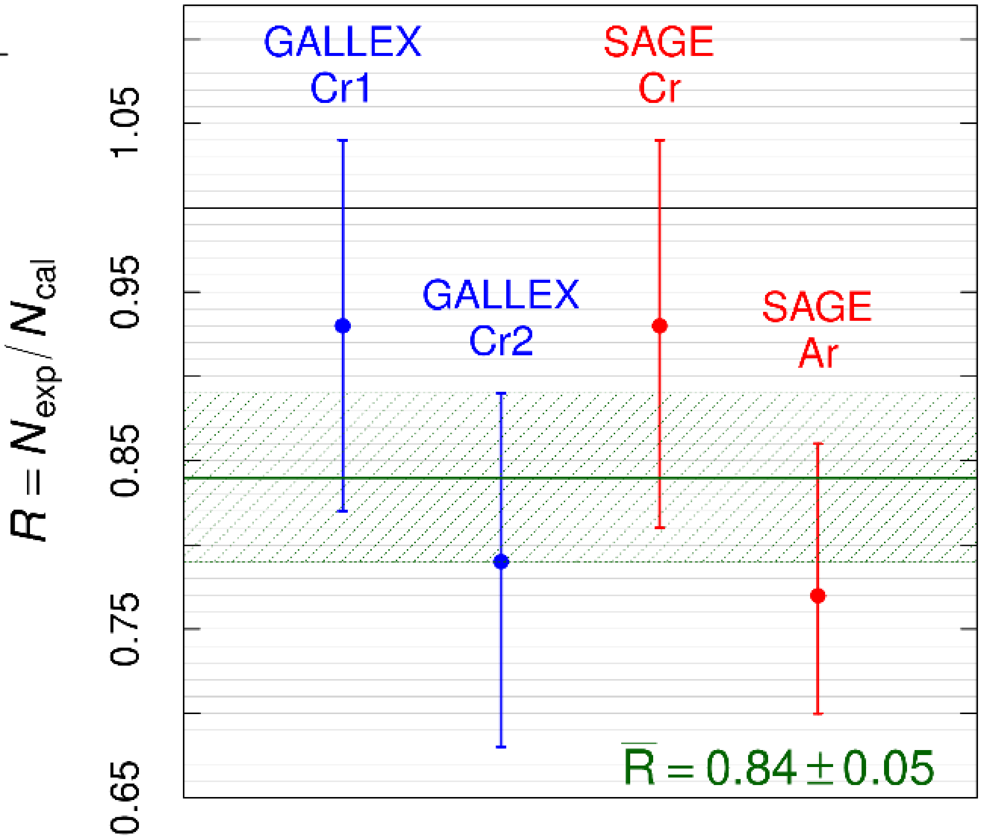
LSND Collaboration (Phys.Rev.D 64 (2001) 112007); MiniBooNE Collaboration (Phys.Rev.Lett. 121 (2018))

Phys. Rev. Lett. **121** (2018) 221801 [arXiv:1805.12028].

2 Gallium anomaly

$$\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$$

- ν_e rate deficit $\sim 15\%$
- significance 3σ

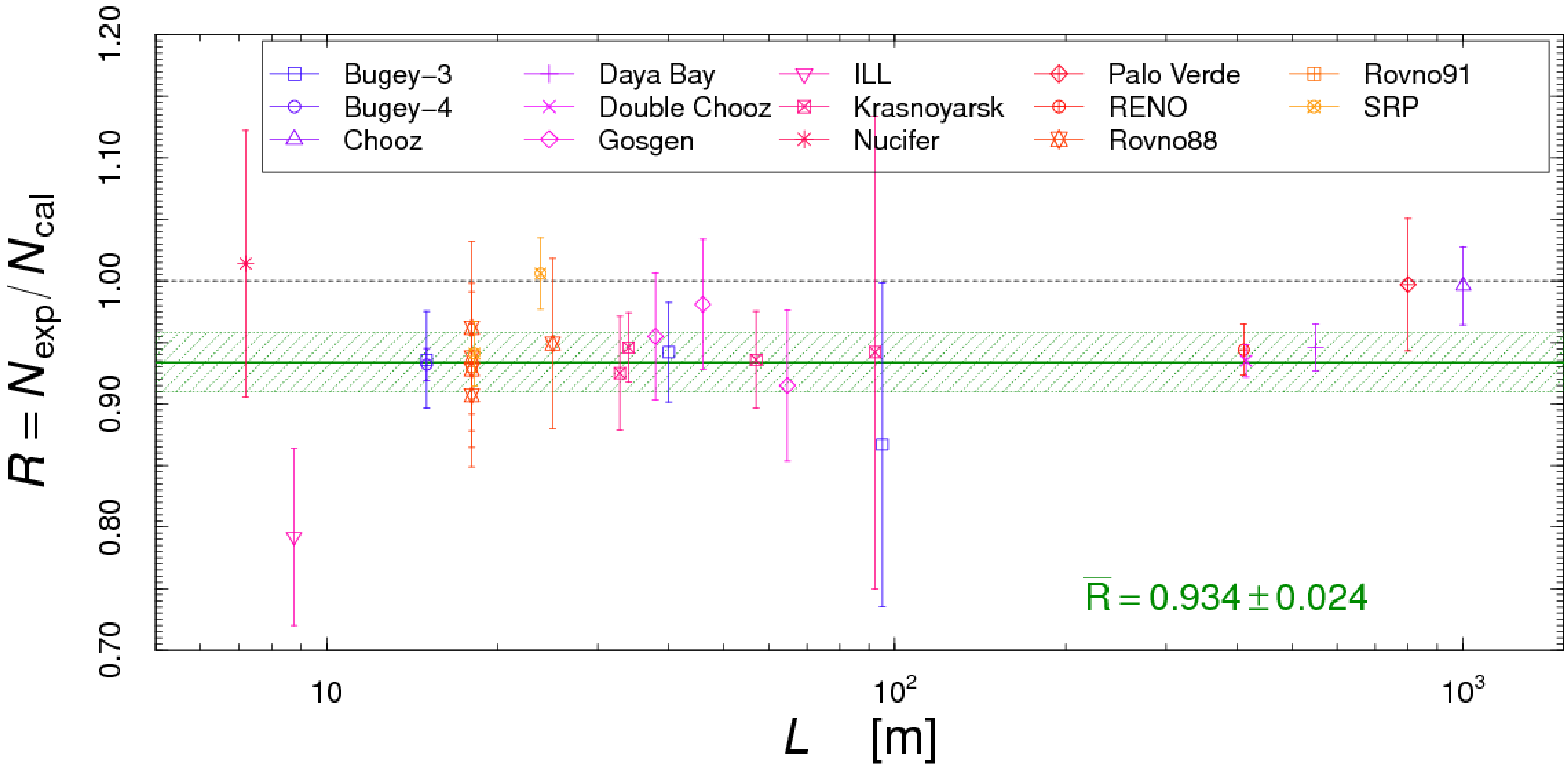


S. Gariazzo et. al., (J.Phys.G 43 (2016) 033001)

3 Reactor anomaly

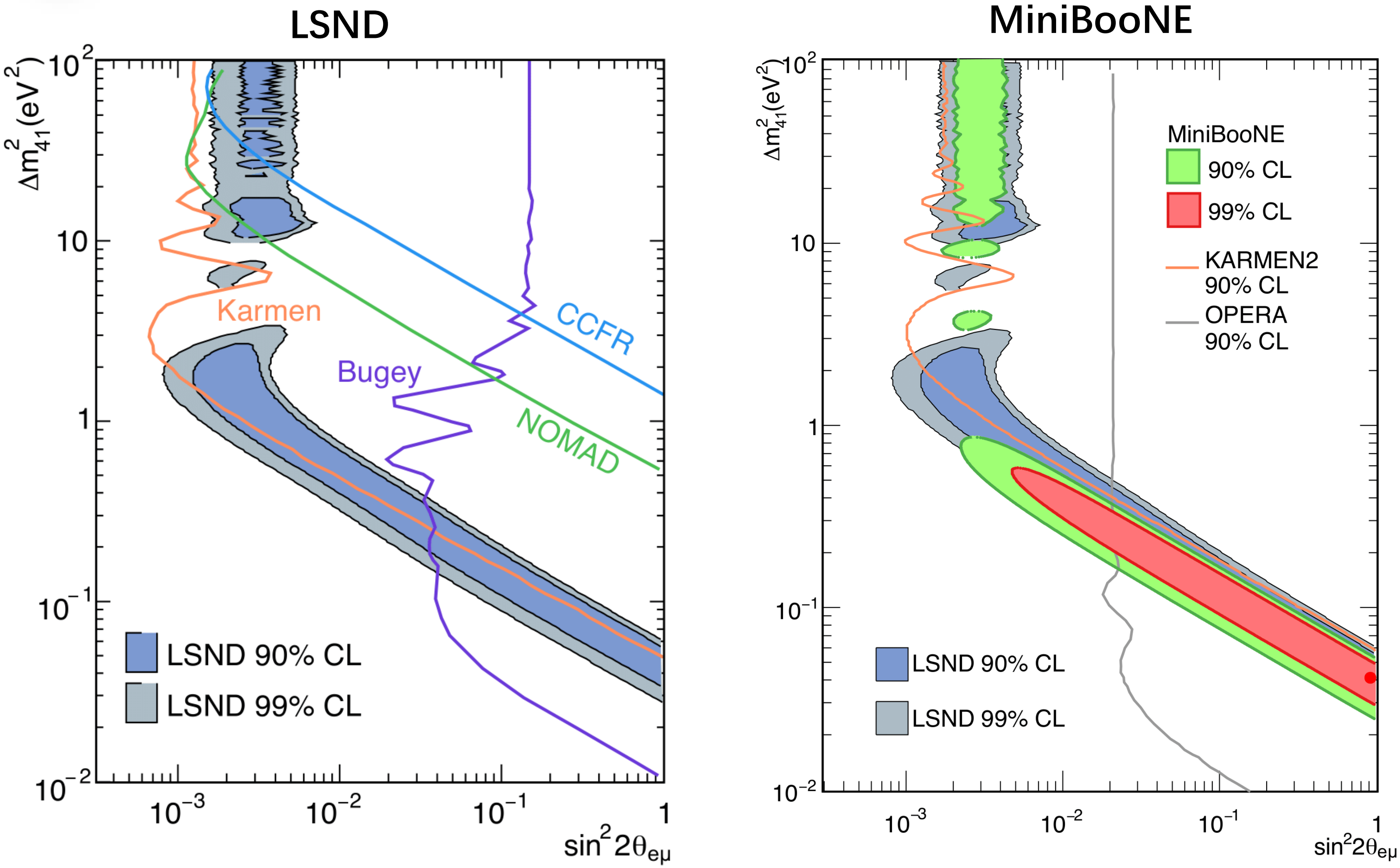
- $\bar{\nu}_e$ rate deficit $> 6\%$
- significance 2.8σ

$\bar{R} = 0.934 \pm 0.024$



S. Gariazzo, C. Giunti, M. Laveder and Y. F. Li (JHEP 06 (2017) 135)

Experimental hints

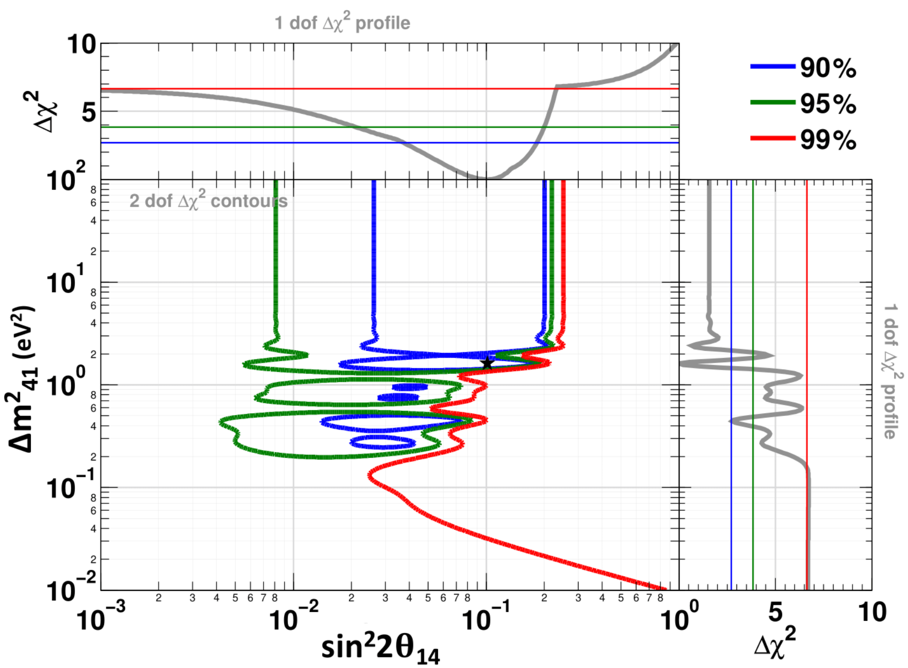


$$\sin^2(2\theta_{\alpha\beta}) \equiv 4\,|U_{\alpha4}|^2\,|\delta_{\alpha\beta} - U_{\beta4}|^2$$

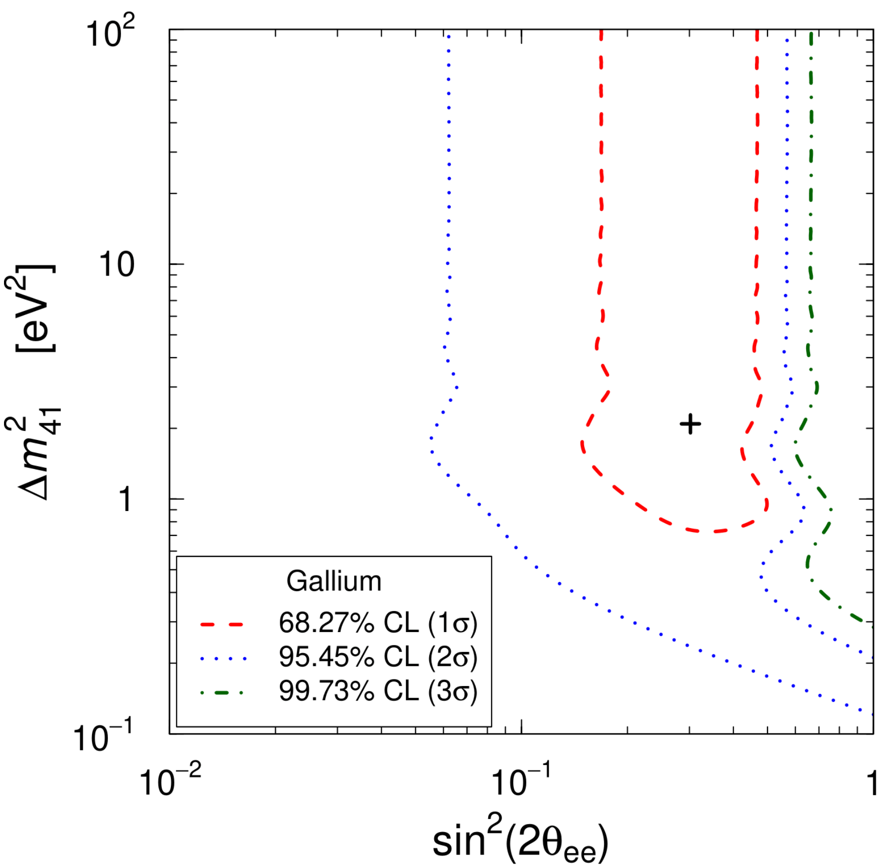
S. Böser et al, Prog.Part.Nucl.Phys. 111 (2020) 103736

$$\sin^2(2\theta_{ee}) = \sin^2(2\theta_{14}); \qquad \sin^2(2\theta_{e\mu}) = \sin^2(2\theta_{14})\sin^2(2\theta_{24})$$

RAA



GA



$$\sin^2(2\theta_{ee}) = \sin^2(2\theta_{14});$$

S. Böser et al, Prog.Part.Nucl.Phys. 111 (2020) 103736

All of these results imply a sterile neutrino with a mass about 1 eV and mixing with active neutrino is about 0.1 might exist.

5

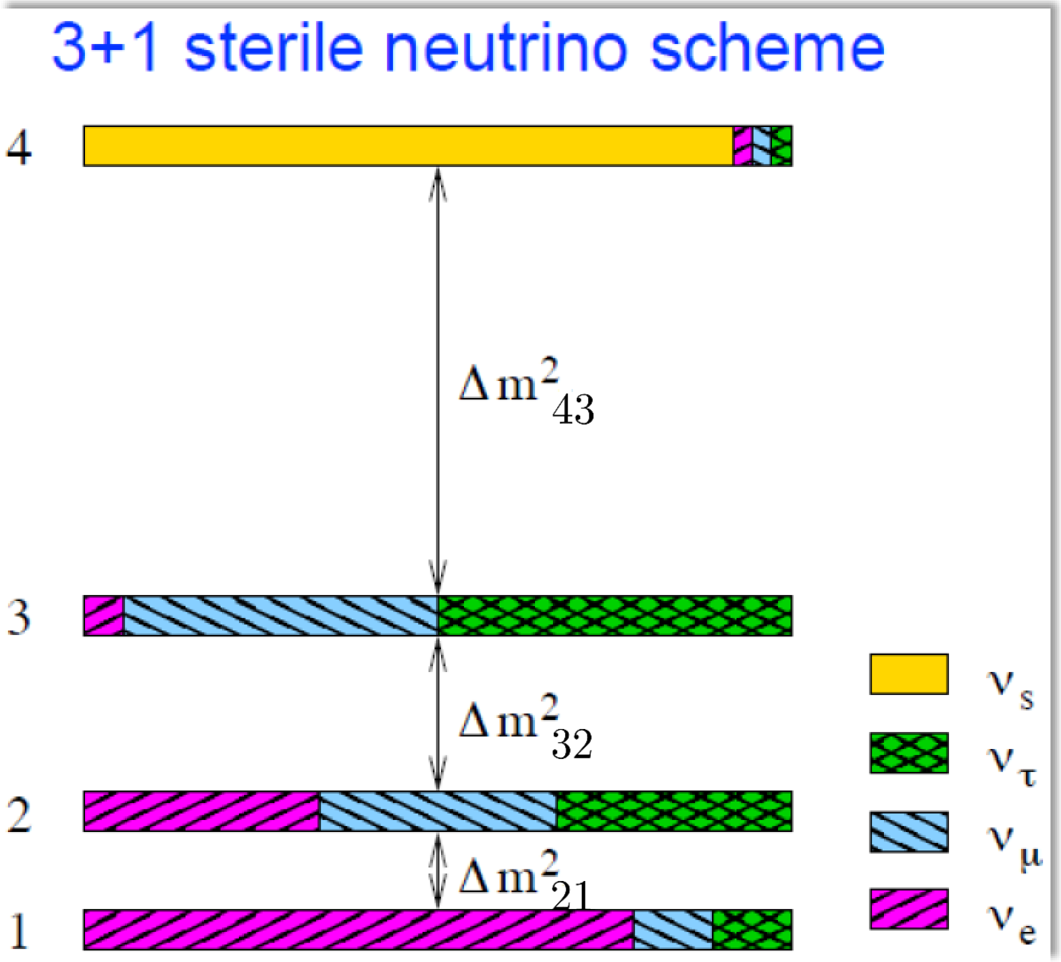
Light sterile neutrino in long baseline oscillations

Based on the work with prof. Chun Liu

Based on: JHEP 06 (2020) 094
[arXiv:1911.12524]

Description of 3+1 neutrino scheme

- **Neutrino masses**



Description of 3+1 neutrino scheme

- **Neutrino mixing**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

U_{PMNS} (upper-left 3×3 block)

$U = U_s U_3$

$$U_s = R_{34}(\theta_{34},0)R_{24}\left(\theta_{24},\delta_{24}^{CP}\right)R_{14}\left(\theta_{14},\delta_{14}^{CP}\right) \;,$$
$$U_3 = R_{23}(\theta_{23},0)R_{13}\left(\theta_{13},\delta_{13}^{CP}\right)R_{12}(\theta_{12},0)\,,$$

Matter effects

Effects on masses

Effective Hamiltonian $H_{\rm eff} = H_0 + V$

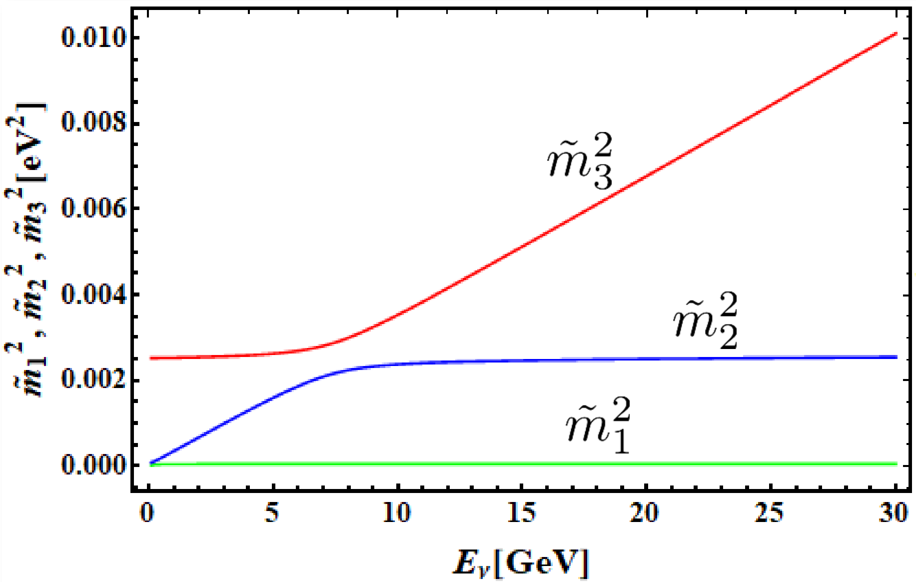
$$H_{\rm eff} = \frac{1}{2E}\left[U\,{\rm diag}\left(m_1^2, m_2^2, m_3^2, m_4^2\right)U^\dagger + 2E\,{\rm diag}\left(V_{\rm CC}, 0, 0, -V_{\rm NC}\right)\right]$$
$$= \frac{1}{2E}\tilde{U}\,{\rm diag}\left(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2, \tilde{m}_4^2\right)\tilde{U}^\dagger.$$

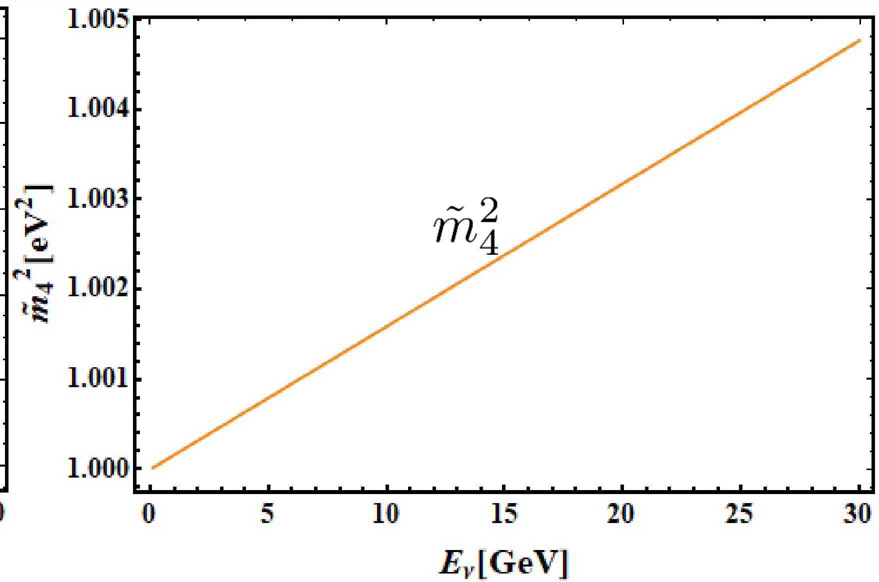
Weak charged current and neutral current potentials

$$V_{\rm CC} = \sqrt{2}\,G_{\rm F}\,N_e,$$
$$V_{\rm NC} = -\frac{1}{\sqrt{2}}\,G_{\rm F}\,N_n.$$

Effective mass squares satisfy

$$(\tilde{m}_i^2)^4 - c_3(\tilde{m}_i^2)^3 + c_2(\tilde{m}_i^2)^2 - c_1\tilde{m}_i^2 + c_0 = 0$$





$$\tilde{m}_1^2 = \frac{c_3}{4} - \frac{1}{2}\left[\sqrt{2z} + \sqrt{-2z - 2p + \sqrt{\frac{2}{z}}\,q}\,\right],$$

$$\tilde{m}_2^2 = \frac{c_3}{4} - \frac{1}{2}\left[\sqrt{2z} - \sqrt{-2z - 2p + \sqrt{\frac{2}{z}}\,q}\,\right],$$

$$\tilde{m}_3^2 = \frac{c_3}{4} + \frac{1}{2}\left[\sqrt{2z} - \sqrt{-2z - 2p - \sqrt{\frac{2}{z}}\,q}\,\right],$$

$$\tilde{m}_4^2 = \frac{c_3}{4} + \frac{1}{2}\left[\sqrt{2z} + \sqrt{-2z - 2p - \sqrt{\frac{2}{z}}\,q}\,\right],$$

$$p = \frac{8c_2 - 3c_3^2}{8},$$

$$q = -\frac{c_3^3 - 4c_2c_3 + 8c_1}{8},$$

$$r = \frac{-3c_3^4 + 256c_0 - 64c_1c_3 + 16c_2c_3^2}{256}$$

$$z^3 + pz^2 + \tfrac{1}{4}\left(p^2 - 4r\right)z - \tfrac{1}{8}q^2 = 0$$

Effects mixing matrix

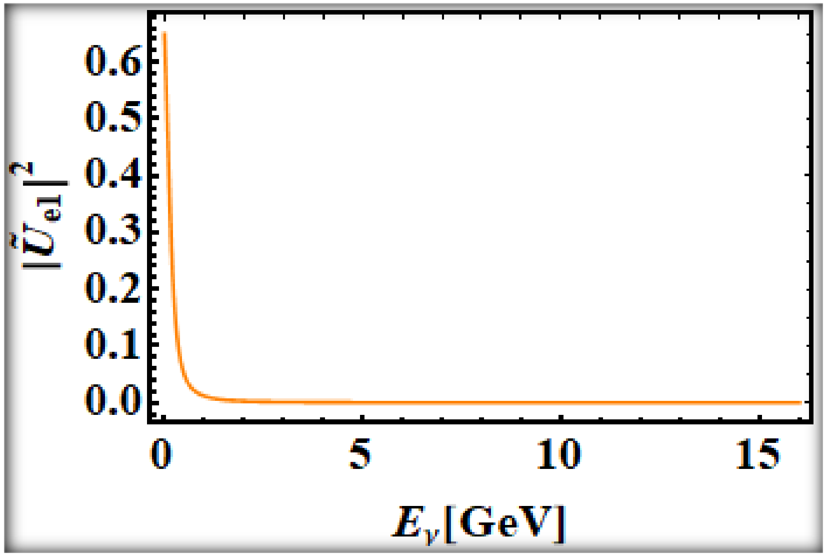
$\alpha = \beta$

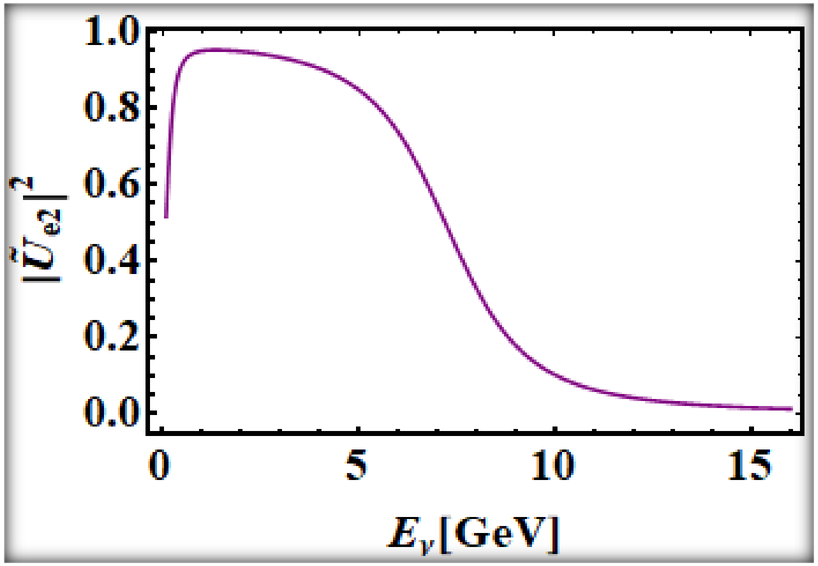
$$\left|\tilde{U}_{\alpha i}\right|^2 = \left(\prod_{k\neq i}^{4} \Delta\tilde{m}^2_{ik}\right)^{-1} \left\{\sum_{j=1}^{4}\left[\prod_{r\neq i}^{4}\left(\Phi_{\alpha\alpha} - \delta m^2_{rj}\right)\right]\left|U_{\alpha j}\right|^2 - \frac{1}{2}\sum_{mn\gamma}\left(\Delta m^2_{mn}\right)^2 U_{\alpha m}U^*_{\alpha n}U^*_{\gamma m}U_{\gamma n}\Phi_{\gamma\gamma}\right\}$$

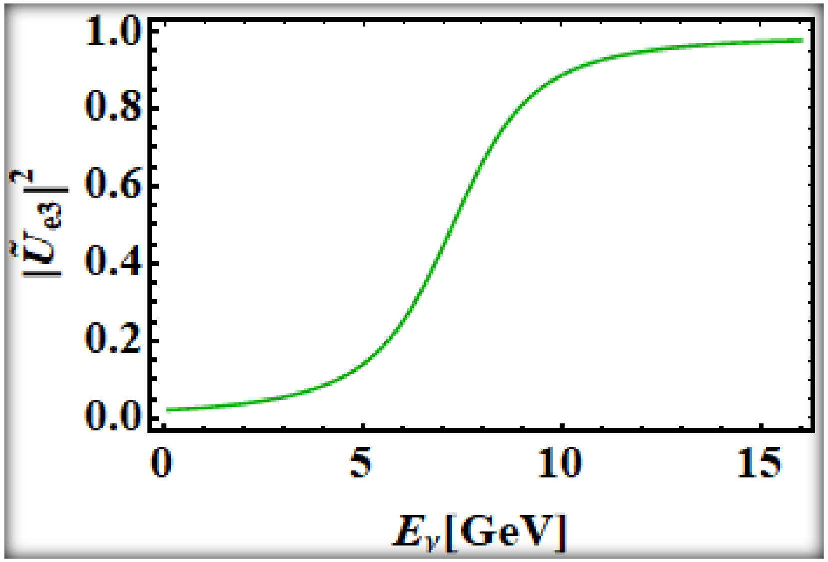
$\alpha \neq \beta$

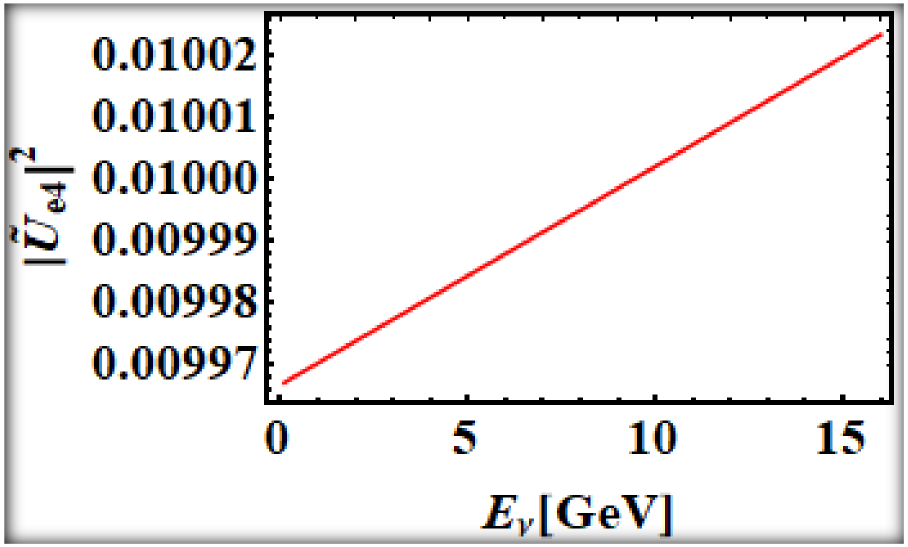
$$\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} = \left(\prod_{k\neq i}^{4}\Delta\tilde{m}^2_{ik}\right)^{-1}\left\{\sum_{j=1}^{4}\left[\prod_{r\neq i}^{4}\left(\Phi_{\alpha\alpha} + \Phi_{\beta\beta} - \delta m^2_{rj}\right) - \frac{3}{2}\left(\delta m^2_{ij}\right)^2\left(\Phi_{\alpha\alpha} + \Phi_{\beta\beta}\right) - \delta m^2_{ij}\left[\left(\Phi_{\alpha\alpha} + \Phi_{\beta\beta}\right)\sum_{l=1}^{4}\Delta\tilde{m}^2_{li} - 2\left(\Phi_{\alpha\alpha} + \Phi_{\beta\beta}\right)^2 - \Phi_{\alpha\alpha}\Phi_{\beta\beta}\right]\right]U_{\alpha j}U^*_{\beta j} - \frac{1}{2}\sum_{mn\gamma}\left(\Delta m^2_{mn}\right)^2 U_{\alpha m}U^*_{\beta n}U^*_{\gamma m}U_{\gamma n}\Phi_{\gamma\gamma}\right\}.$$

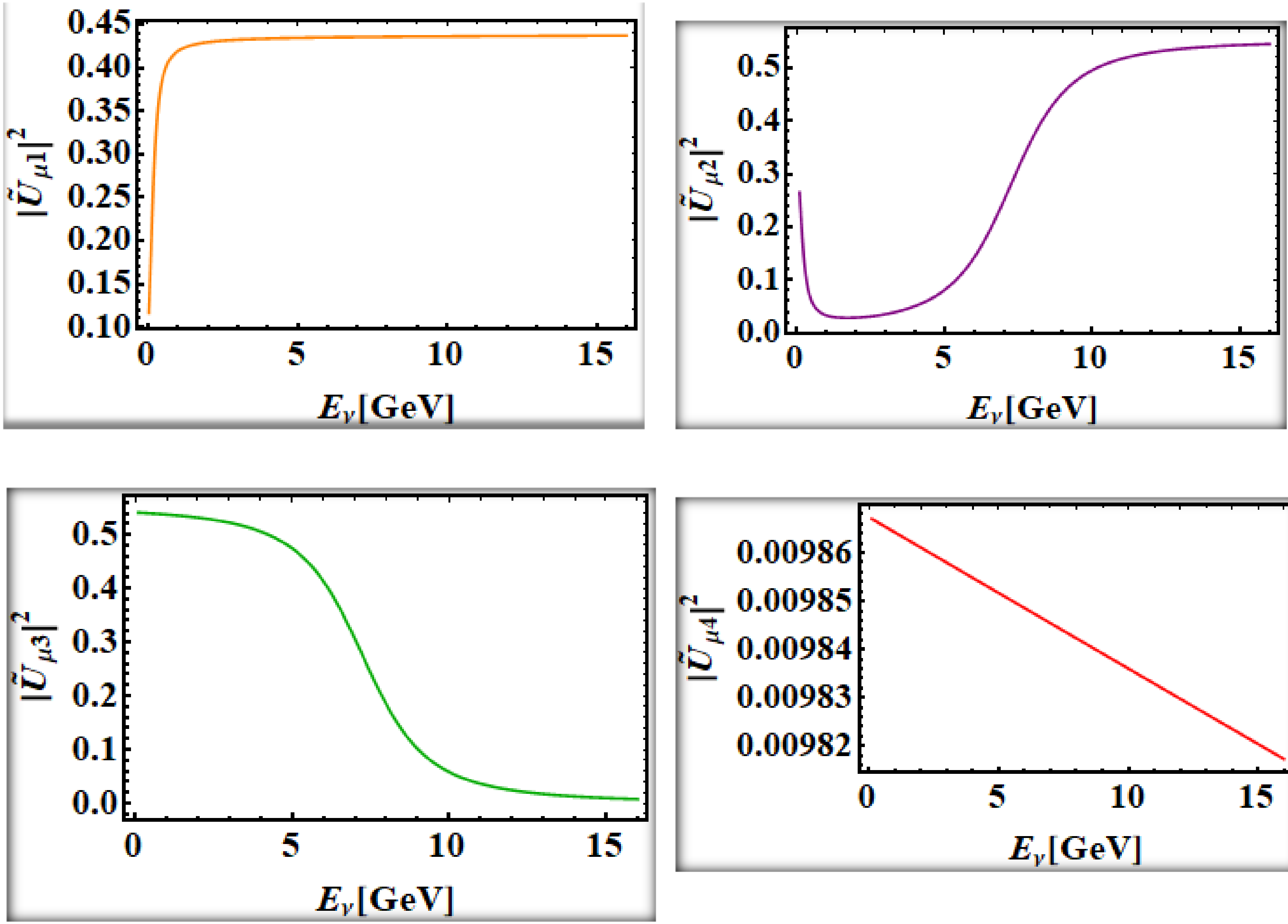
$$\delta m^2_{ij} = \tilde{m}^2_i - m^2_j \qquad \Phi = 2E\,\mathrm{diag}\left(V_{\mathrm{CC}}, 0, 0, -V_{\mathrm{NC}}\right)$$


0.6
0.5
0.4
0.3
0.2
0.1
0.0
$|\tilde{U}_{e1}|^2$
0
5
10
15
E_ν[GeV]


1.0
0.8
0.6
0.4
0.2
0.0
$|\tilde{U}_{e2}|^2$
0
5
10
15
E_ν[GeV]


1.0
0.8
0.6
0.4
0.2
0.0
$|\tilde{U}_{e3}|^2$
0
5
10
15
E_ν[GeV]


0.01002
0.01001
0.01000
0.00999
0.00998
0.00997
$|\tilde{U}_{e4}|^2$
0
5
10
15
E_ν[GeV]


0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
$|\tilde{U}_{\mu 1}|^2$
0 5 10 15
E_ν[GeV]
0.5
0.4
0.3
0.2
0.1
0.0
$|\tilde{U}_{\mu 2}|^2$
0 5 10 15
E_ν[GeV]
0.5
0.4
0.3
0.2
0.1
0.0
$|\tilde{U}_{\mu 3}|^2$
0 5 10 15
E_ν[GeV]
0.00986
0.00985
0.00984
0.00983
0.00982
$|\tilde{U}_{\mu 4}|^2$
0 5 10 15
E_ν[GeV]

Effects on neutrino oscillation probabilities

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(\tilde{U}_{\alpha i}\tilde{U}^*_{\alpha j}\tilde{U}^*_{\beta i}\tilde{U}_{\beta j}\right)\sin^2\frac{\Delta\tilde{m}^2_{ji}L}{4E}$$

$$+ 2\sum_{i<j} \mathrm{Im}\left(\tilde{U}_{\alpha i}\tilde{U}^*_{\alpha j}\tilde{U}^*_{\beta i}\tilde{U}_{\beta j}\right)\sin\frac{\Delta\tilde{m}^2_{ji}L}{2E}$$

Effects on CP asymmetries

$$\Delta\tilde{P}_{\alpha\beta} = 4\sum_{i<j}\tilde{J}^{\alpha\beta}_{ij}\sin\frac{\Delta\tilde{m}^2_{ji}L}{2E}$$

$$\tilde{J}^{\alpha\beta}_{ij} \equiv \mathrm{Im}\left(\tilde{U}_{\alpha i}\tilde{U}_{\beta j}\tilde{U}^*_{\alpha j}\tilde{U}^*_{\beta i}\right)$$

Independent asymmetry variables

n	$\Delta P_{\alpha\beta}$	$J_{ij}^{\alpha\beta}$	n	$\Delta P_{\alpha\beta}$	$J_{ij}^{\alpha\beta}$
1	0	0	2	0	0
3	1	1	4	3	9
5	6	36	6	10	100
7	15	225	8	21	441
9	28	784	10	36	1296

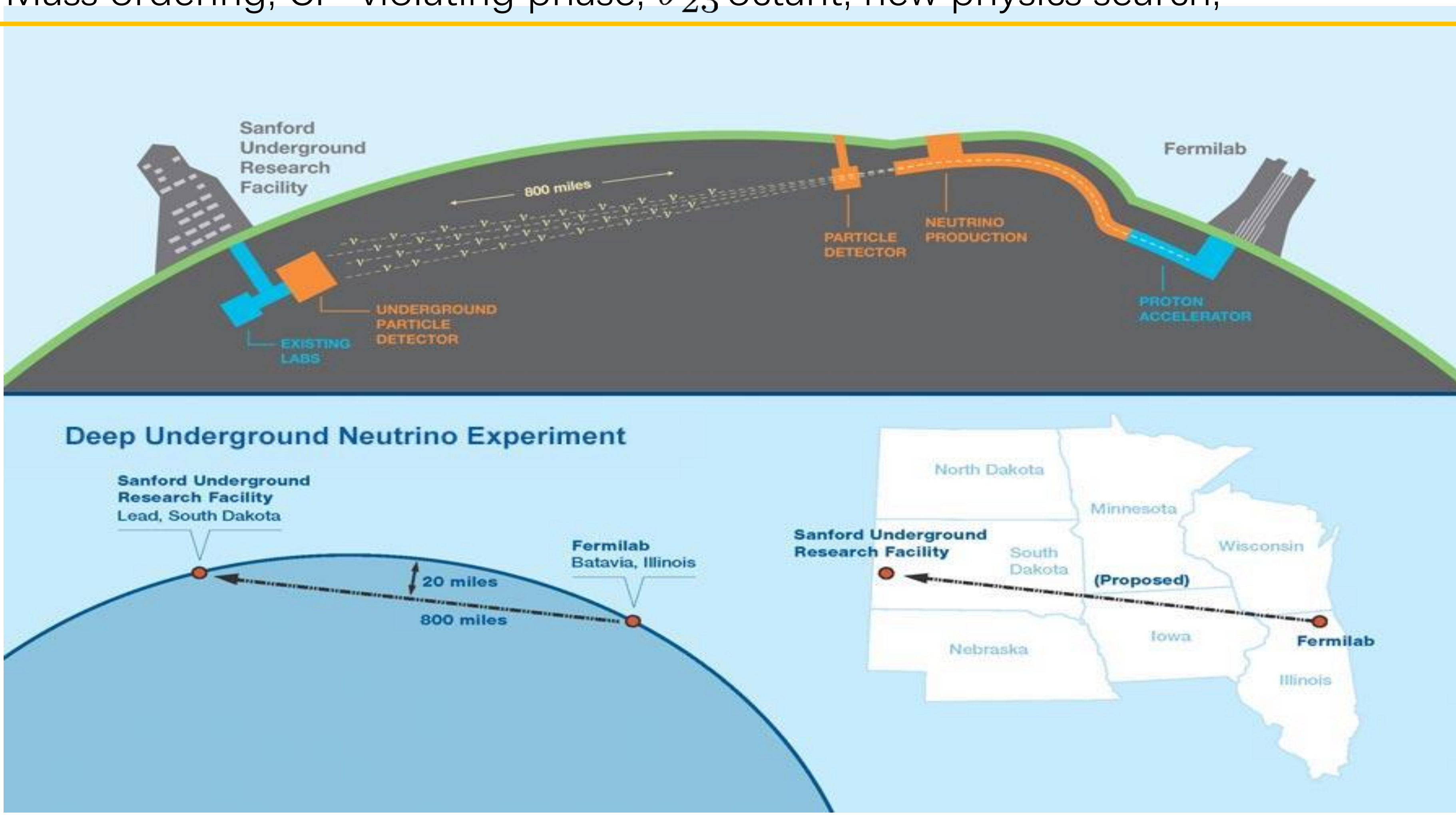
$$\Delta P_{\alpha\beta} : (n-1)(n-2)/2 \quad J_{ij}^{\alpha\beta} : [(n-1)(n-2)/2]^2$$

With 3 neutrinos, there is only one $\Delta P_{\alpha\beta}$ and $J_{ij}^{\alpha\beta}$.

With 4 neutrinos, there are 3 $\Delta P_{\alpha\beta}$ and 9 $J_{ij}^{\alpha\beta}$.

Implications for 3 long-baseline experiments

DUNE LArTPC, 4×10Kt modules, baseline is 1300 Km.
Mass ordering, CP-violating phase, θ_{23} octant, new physics search,⋯



NOvA

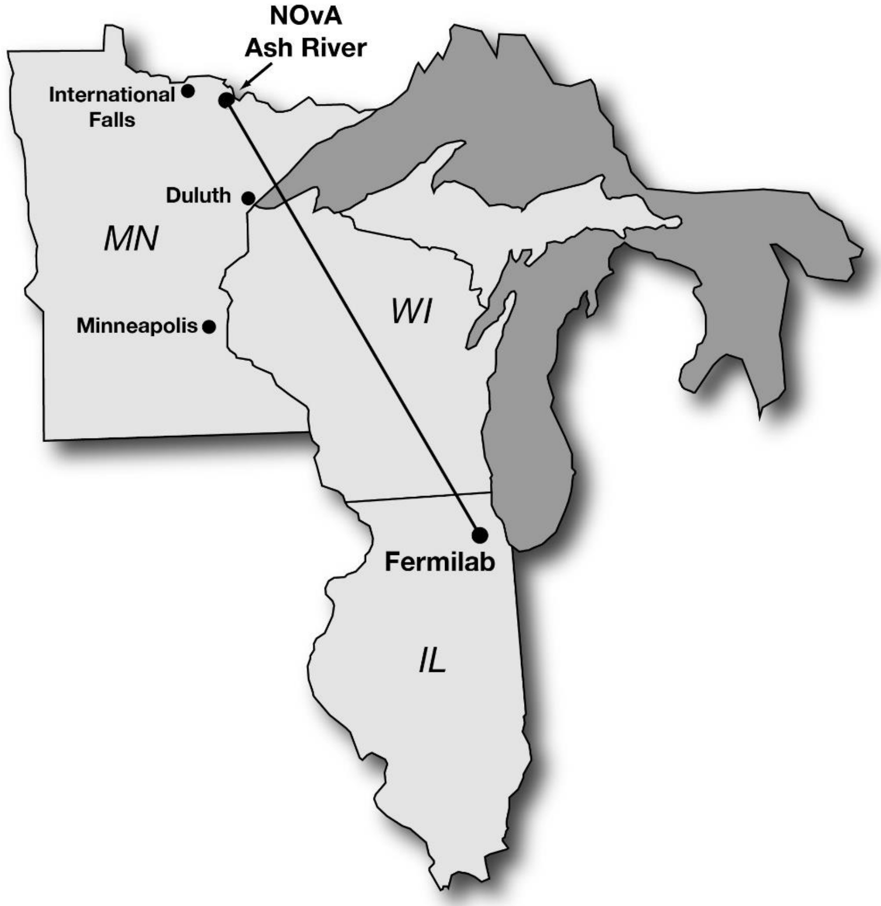
14 Kt liquid scintillator detector

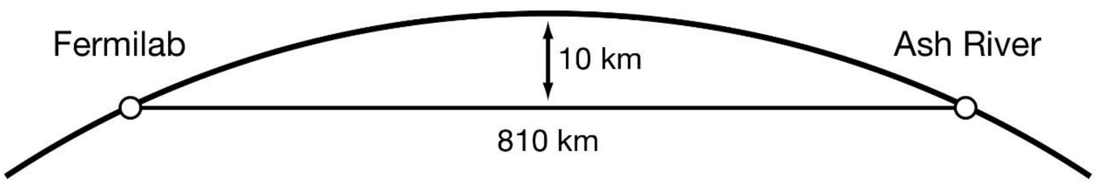
$$\nu_\mu \rightarrow \nu_e$$

Mass ordering

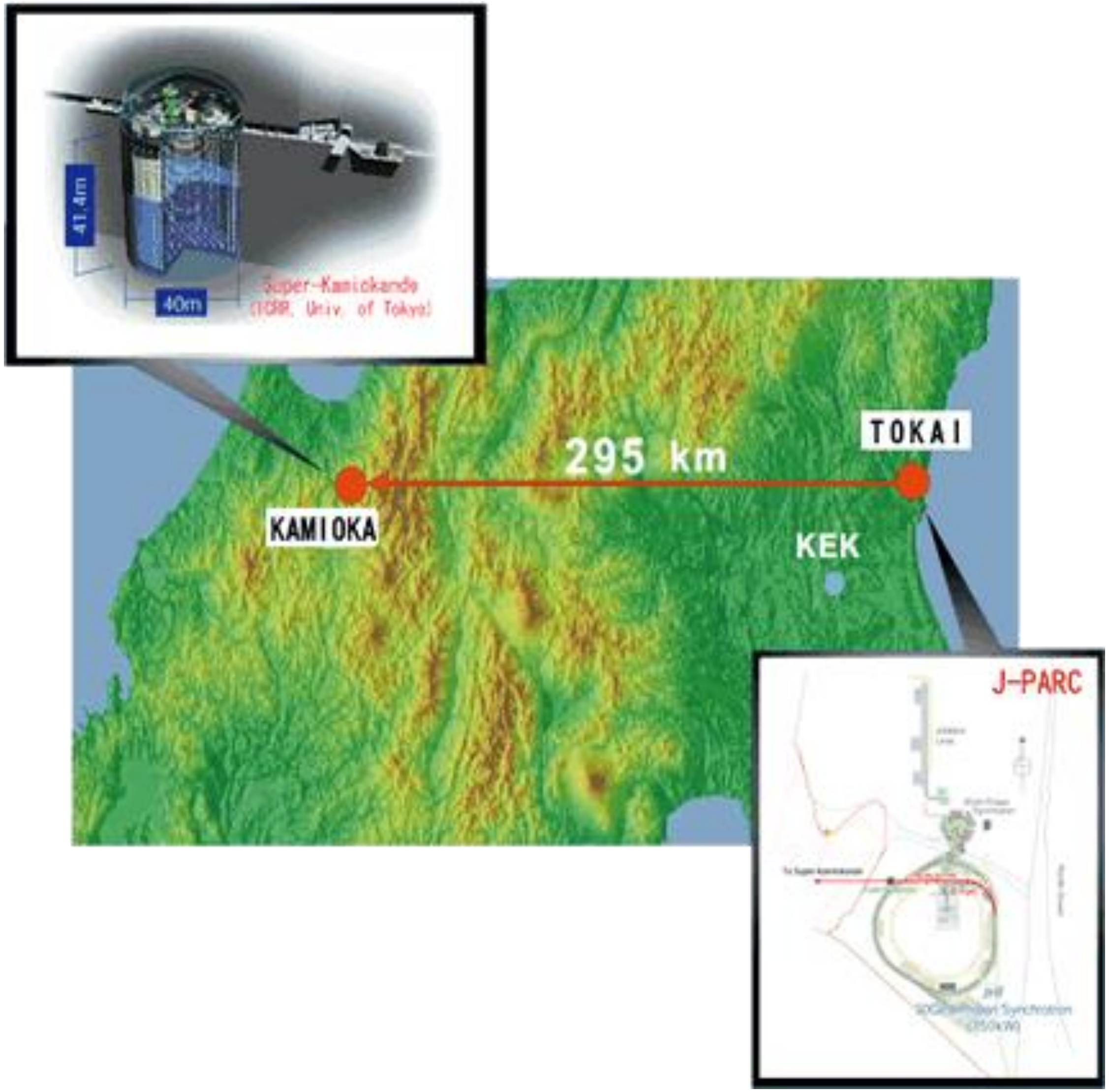
Octent of θ_{23}

CP-violating phase





T2HK



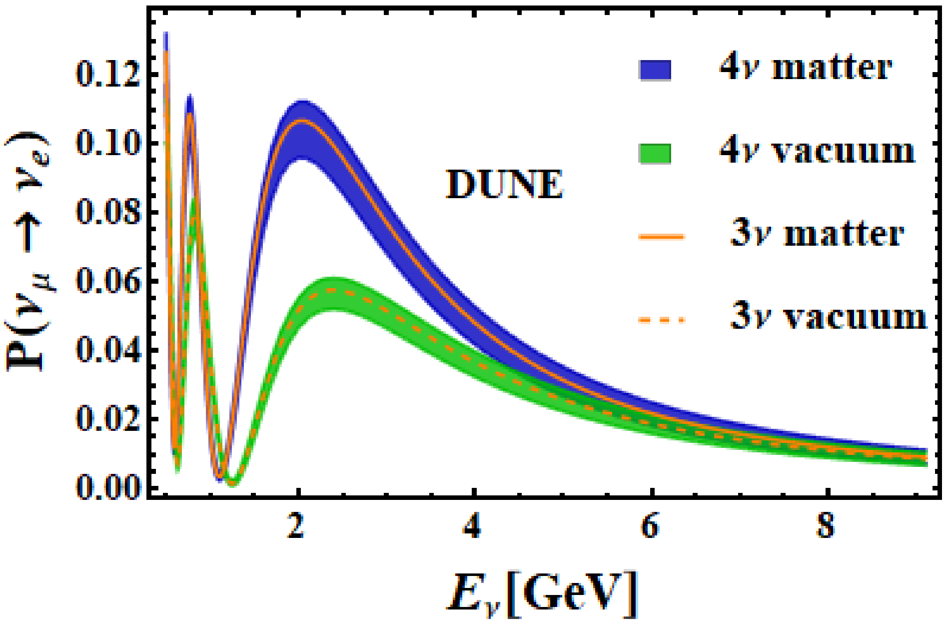
1 Mt Cherenkov detector (20 x SuperK)

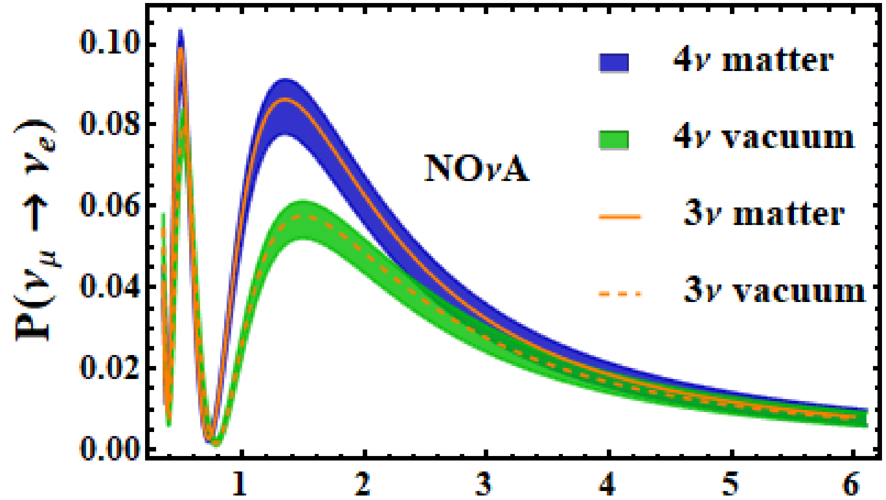
$$\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$$

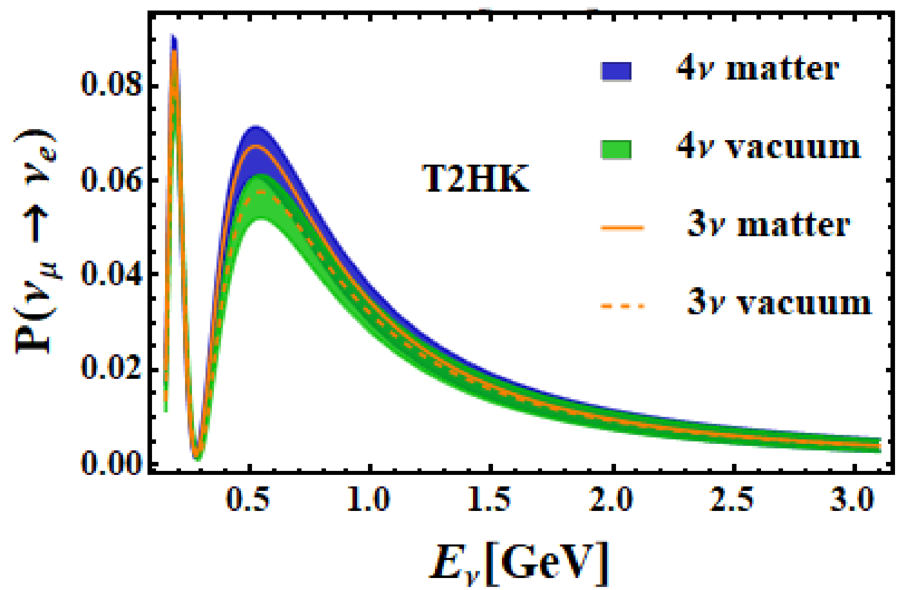
CP asymmetry

Neutrino oscillation probabilities

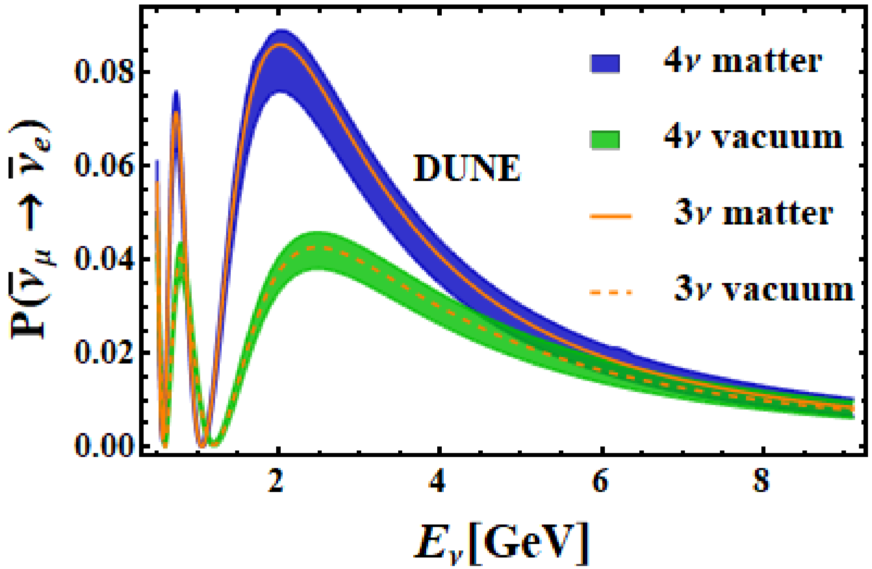
$\nu_\mu \to \nu_e$ channel

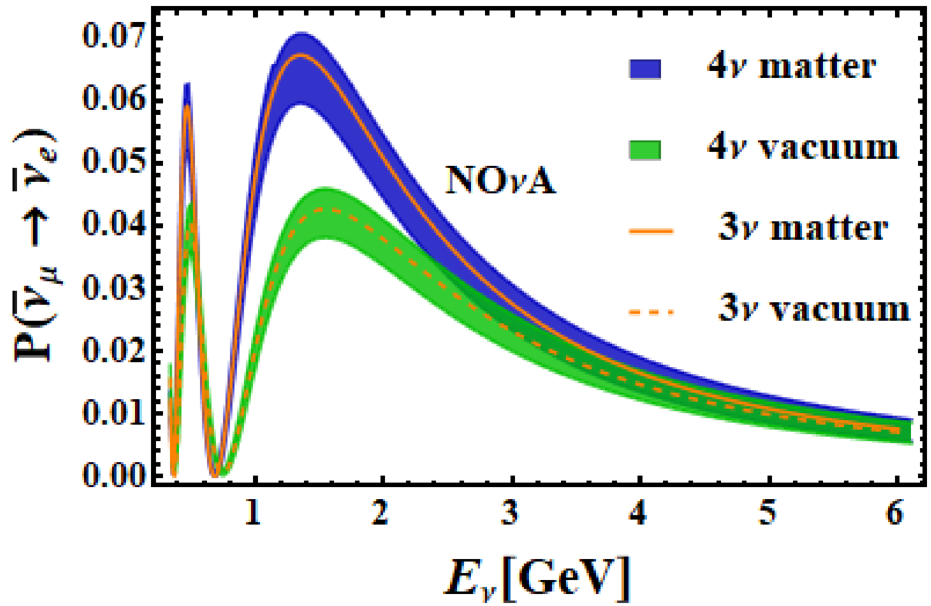


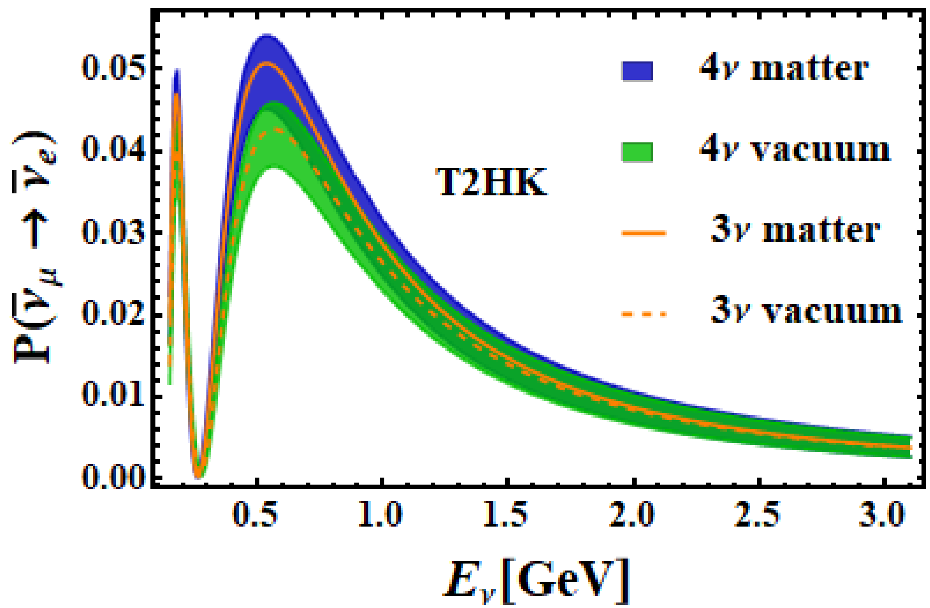




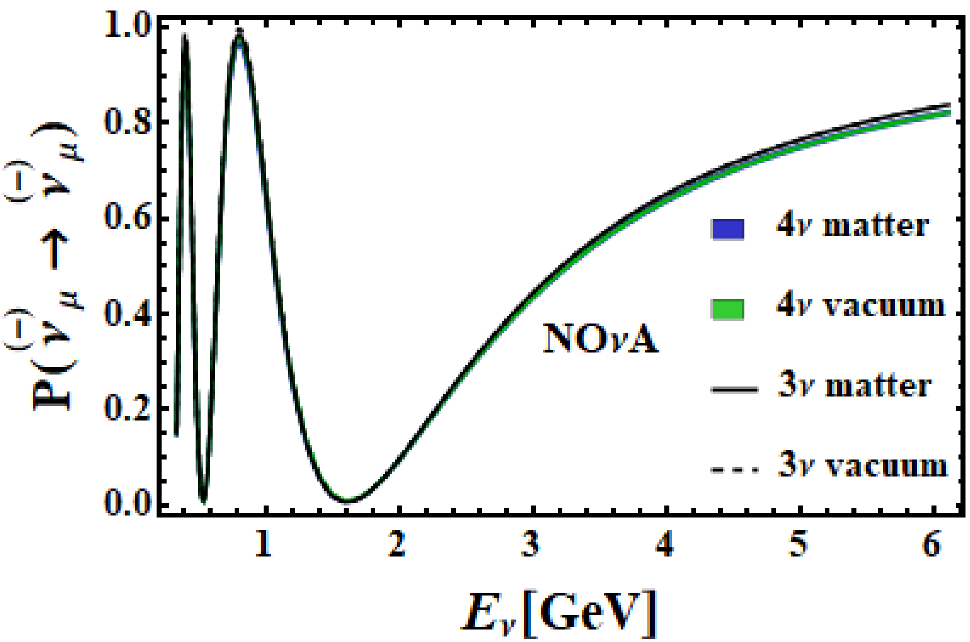
$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ channel

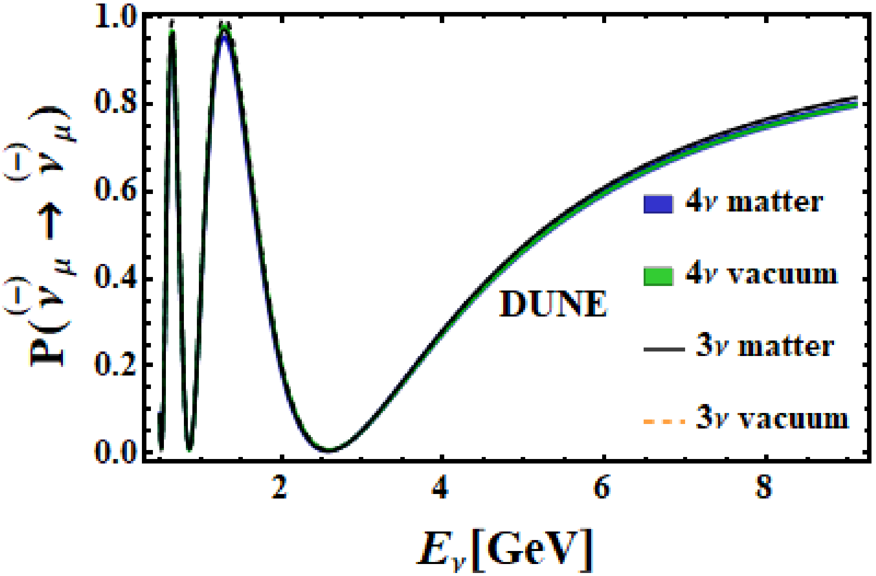


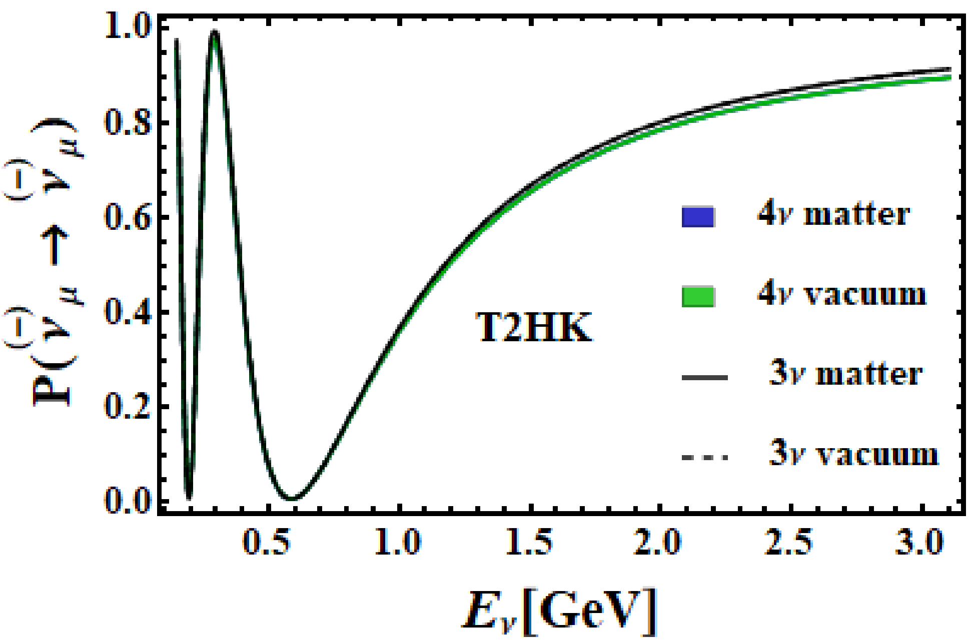




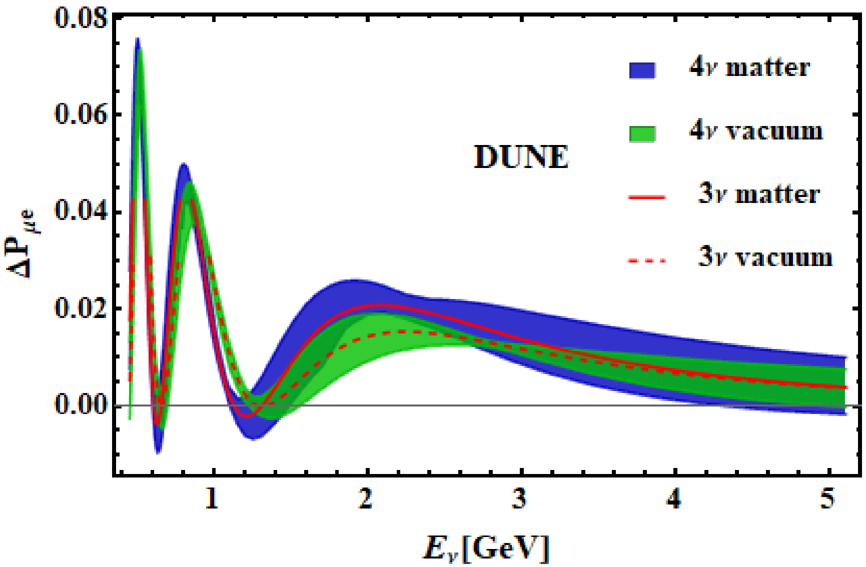
$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ channel

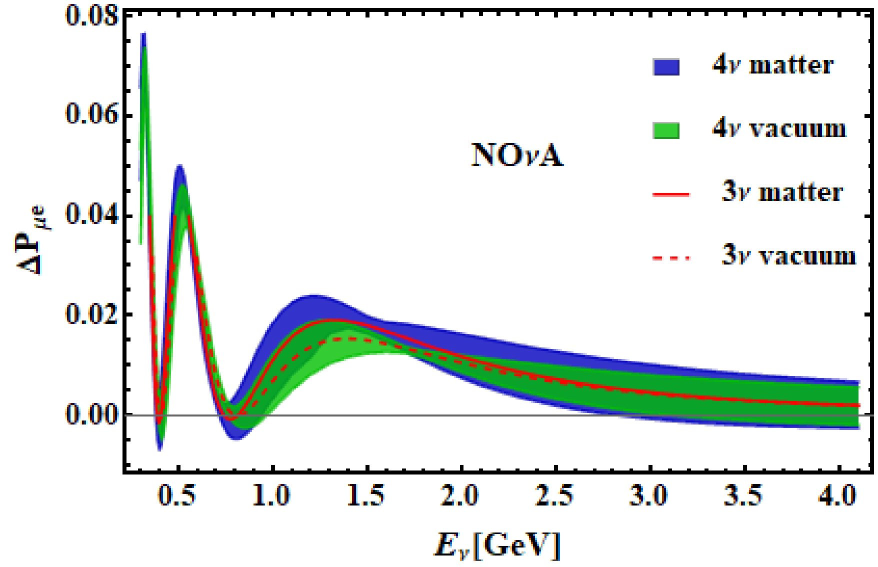


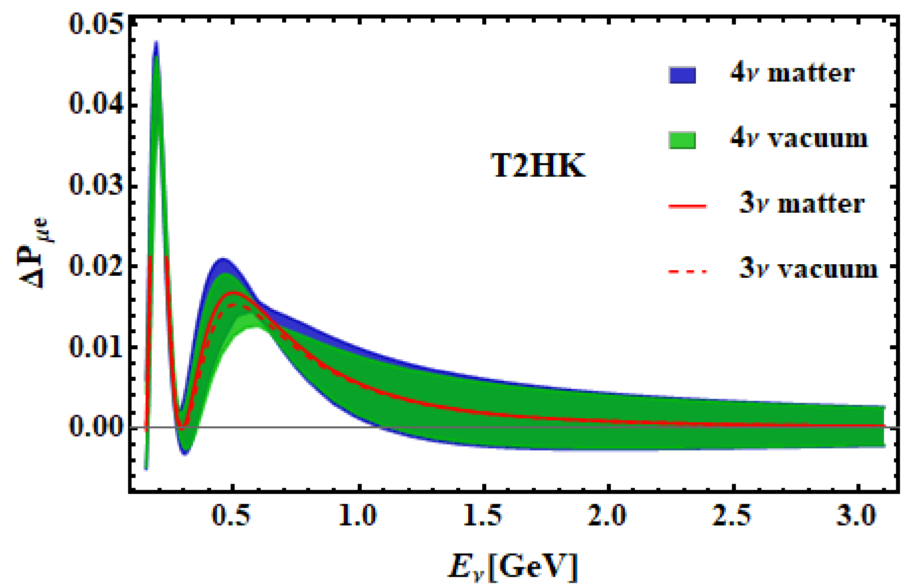




CP asymmetries







Summary

1. Neutrinos are massive, neutrino flavors mix, and they oscillate during the propagation

2. During the oscillation CP asymmetries may appear, and rephasing invariants quantify the asymmetry

3. Matter affects neutrino masses and mixing parameters and enhances the oscillation probabilities

4. There are some hints of the sterile neutrino and discription of 3+1 neutrino scheme

5 Matter affects on 3+1 neutrino oscillations in long baseline experiments

6 The most probable channels to search for the sterile neutrino signal in DUNE, NOvA and T2HK are $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$

Thank you!