Cosmological Collider Physics

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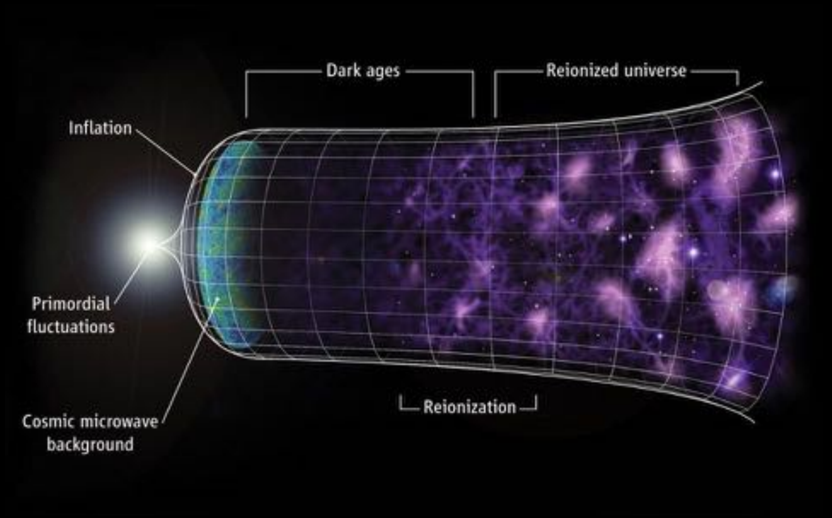
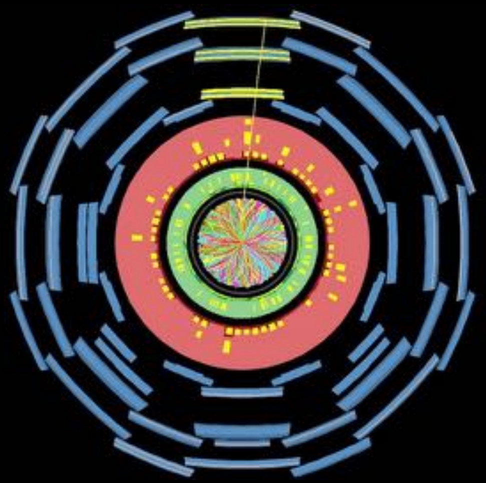


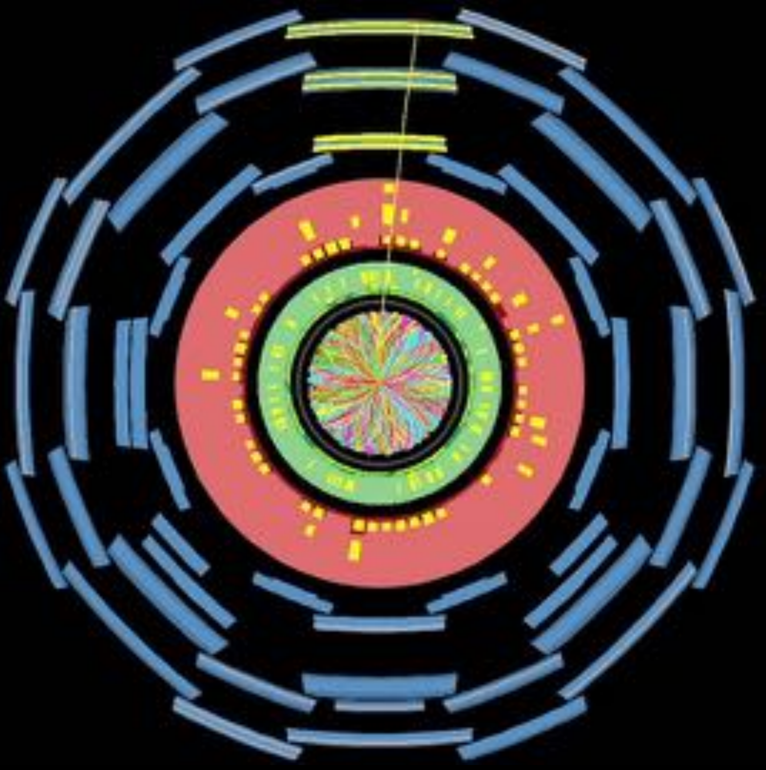
We’ve learnt cosmology

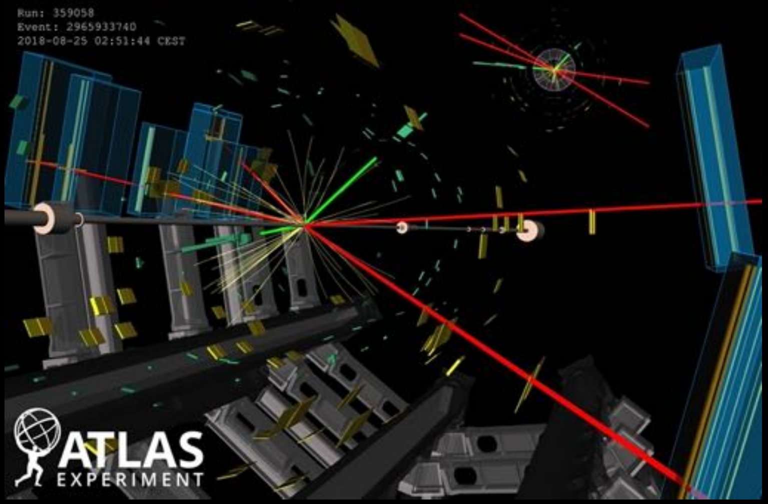


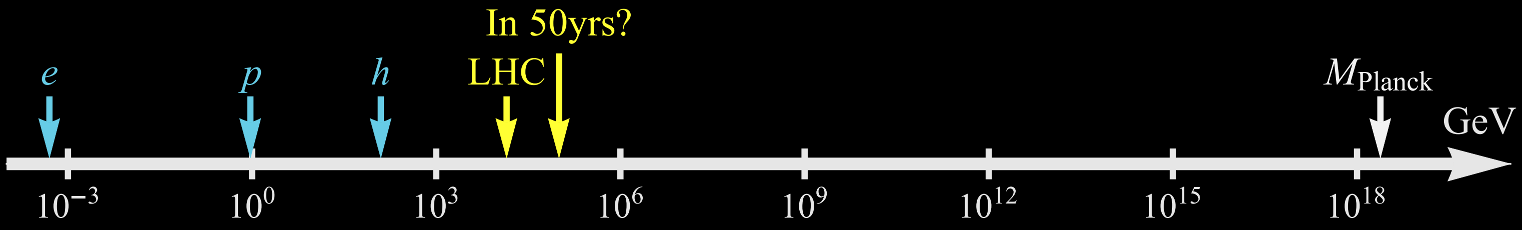
We’ve learnt collider physics

Now, what is a cosmological collider?

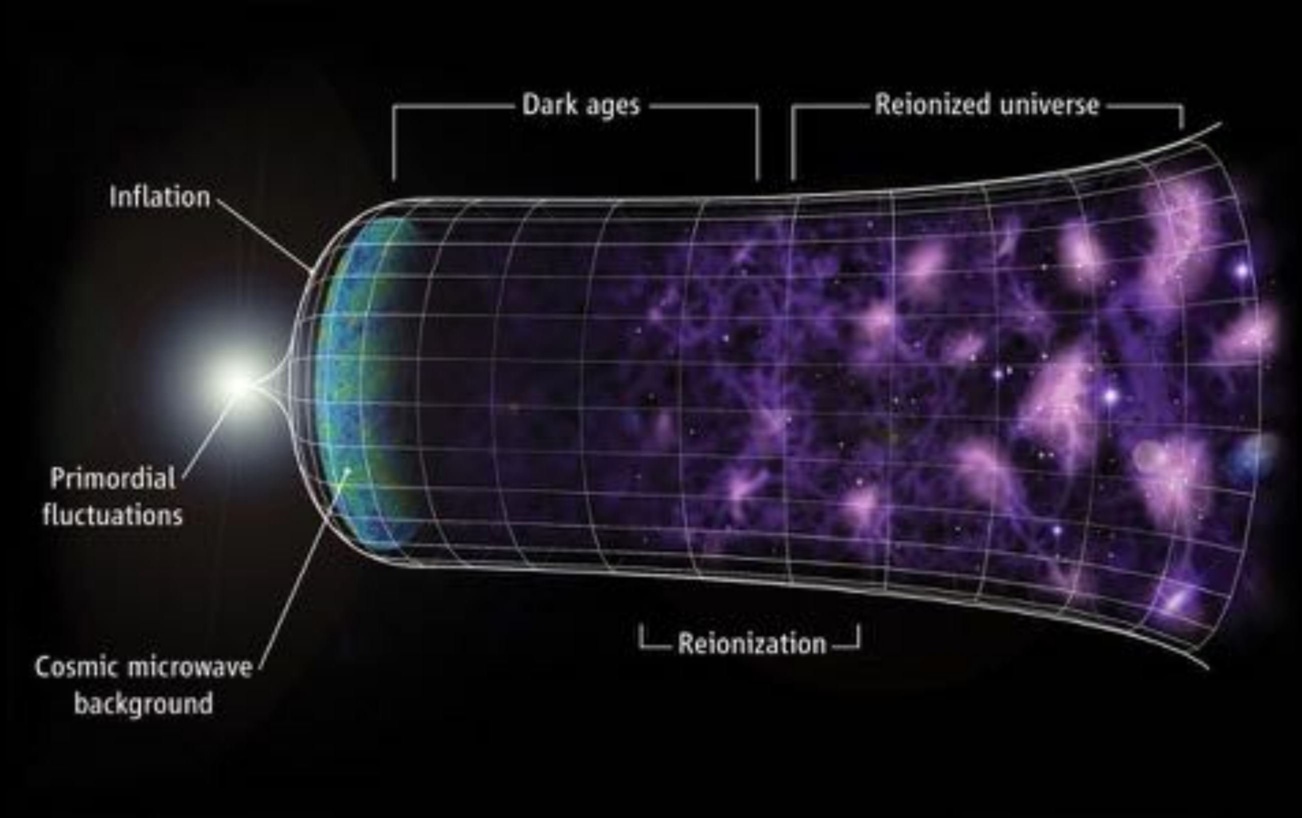
Collider physics







Cosmic inflation



The scale factor $a(t)$

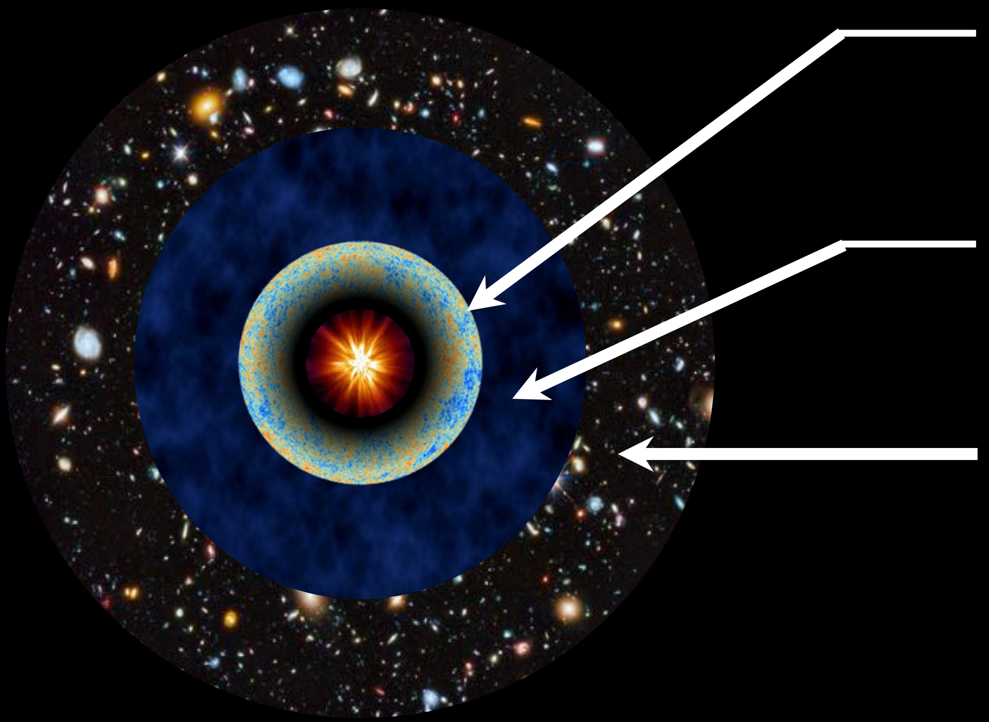
e^{Ht} inflation $H \sim 10^{14}\mathrm{GeV}$

$t^{1/2}$ radiation domination

$t^{2/3}$ matter domination

前方高能

Cosmological Collider

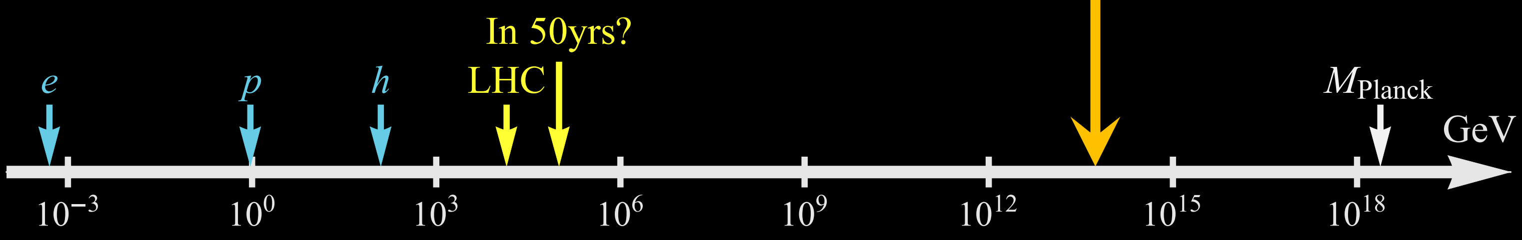


Photon decoupling
CMB

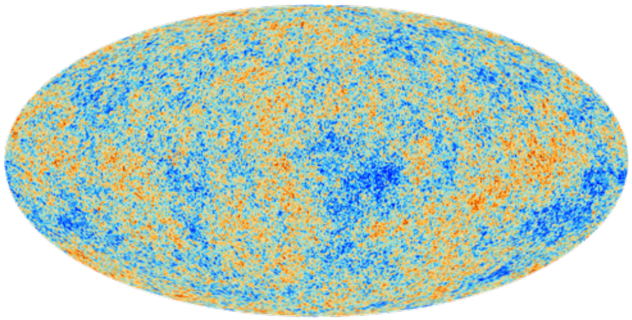
Dark ages
21cm tomography

Galaxies formed
LSS survey

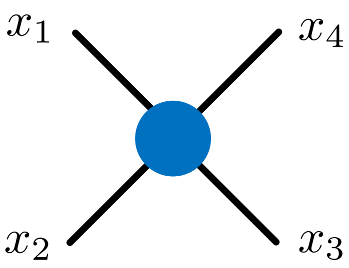
Inflation scale

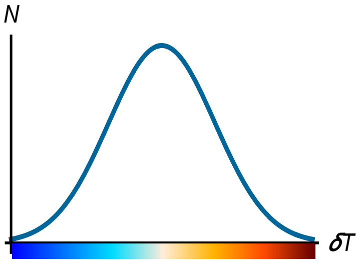


How to extract information from CMB map?

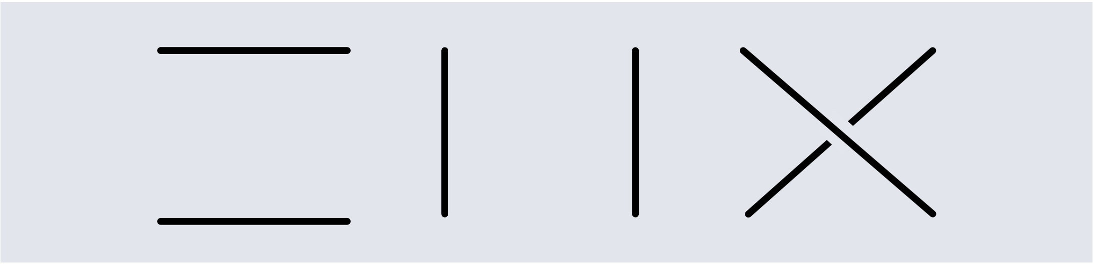


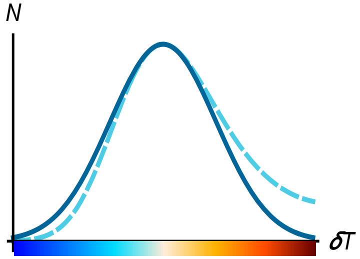
$$\langle \delta T(x_1) \cdots \delta T(x_n) \rangle$$



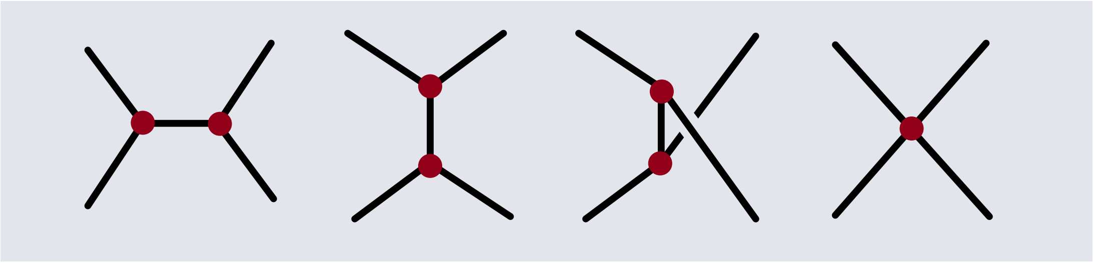


Gaussian

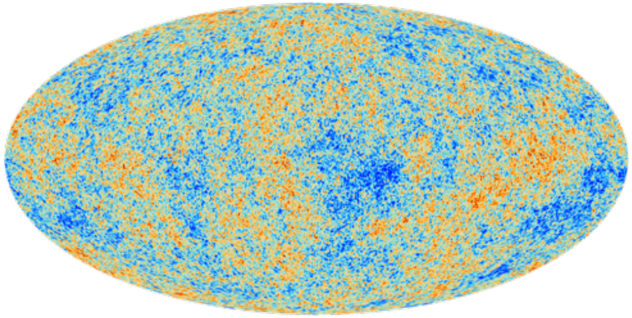




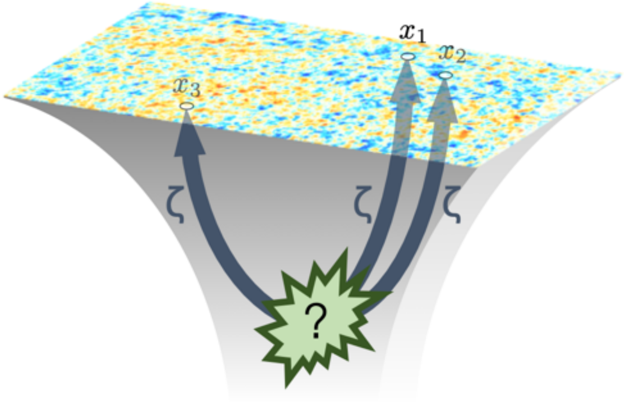
Non-Gaussian



What’s the physics behind?



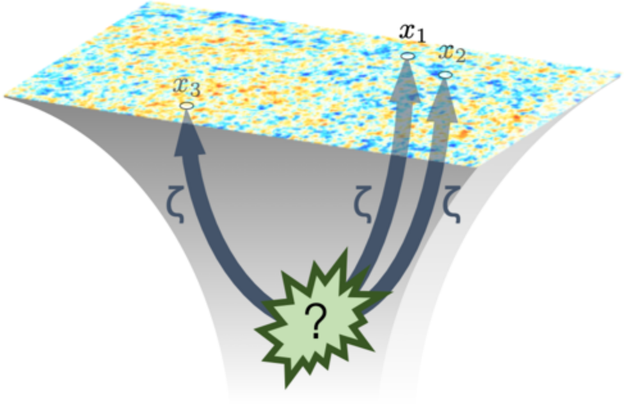
During the inflation, the expansion rate is fast enough to produce particles in pairs purely from quantum fluctuations.



The fluctuations were then redshifted and “frozen” at large scales and seeded the density perturbation that we see today.

Non-Gaussianity ~ interaction

Discover new heavy particles



When massive particles are produced, the inflation did two things:

1. Dilute the number density
2. Exhaust the momentum, so that the particle quickly becomes nonrelativistic

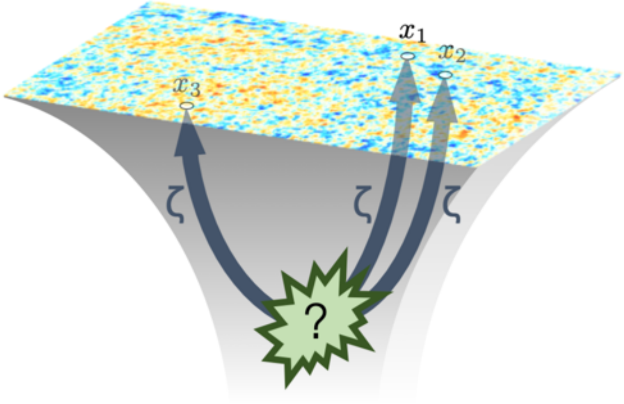
$$\sigma(t) \sim (Ae^{+imt} + Be^{-imt})e^{-\frac{3}{2}Ht}$$

We would be able to measure the mass if we can trace the time dependence

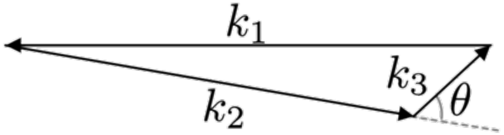
But we can't. We observe only the final state

[e.g. through CMB]

Discover new heavy particles



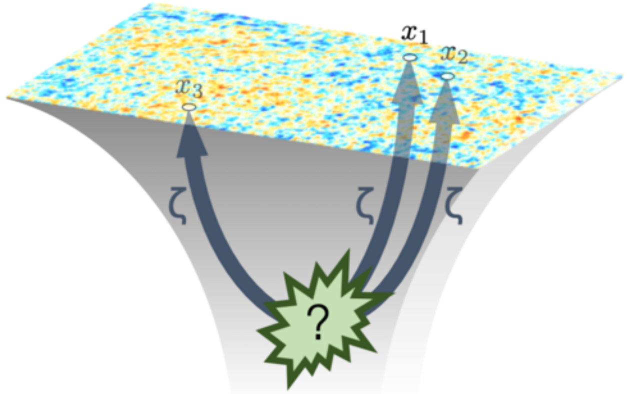
A solution: we try to measure the 3-point correlation in the squeezed limit



Small-momentum mode redshifts earlier, and oscillates like a nonrelativistic particle when the other two large-momentum modes are still deeply inside the horizon

The ratio of long and short momenta is actually a measure of time difference => Measure the 3pt function at different k ratio ~ measure the mode at different time

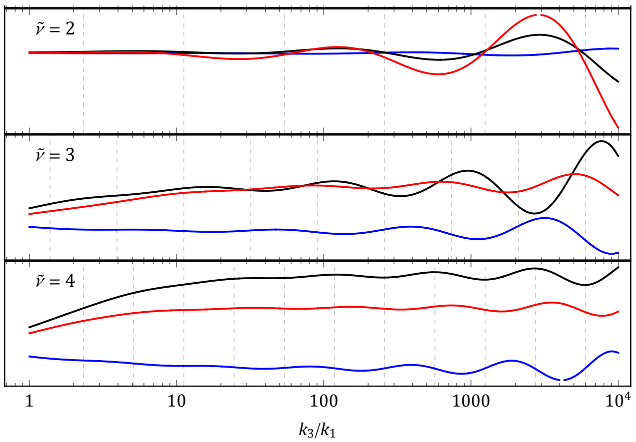
Discover new heavy particles



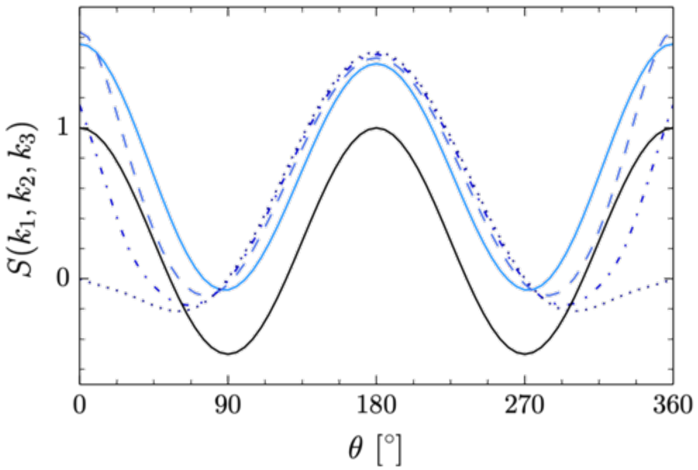
Chen, Wang, 0911.3380;1205.0160
Arkani-Hamed, Maldacena, 1503.08043

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m)\left(\frac{k_3}{k_1}\right)^{1/2\pm\nu} P_s(\cos\theta)$$

$$\nu = \begin{cases} \sqrt{\dfrac{9}{4} - \dfrac{m^2}{H^2}} & s = 0 \\ \sqrt{\left(s - \dfrac{1}{2}\right)^2 - \dfrac{m^2}{H^2}} & s \neq 0 \end{cases}$$



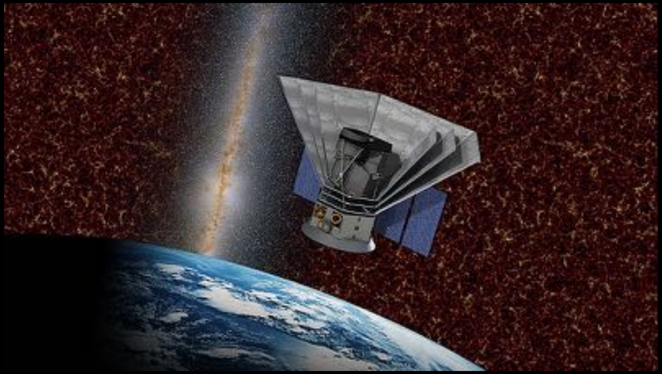
Chen, Chua, Guo, Wang, ZZX, Xie, 1803.04412



Lee, Baumann, Pimentel, 1607.03735

Why now?

Planck: final data release in 2018



SPHEREx: selected by NASA in 2019, launching in ~2024

Planck 2018

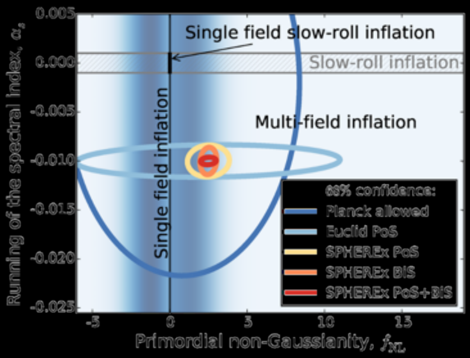
1905.05697

$$f_{\rm NL}^{\rm (local)} = -0.9 \pm 5.1$$

$$f_{\rm NL}^{\rm (equil)} = -26 \pm 47$$

$$f_{\rm NL}^{\rm (ortho)} = -38 \pm 24$$

O(1) in ~10yrs?

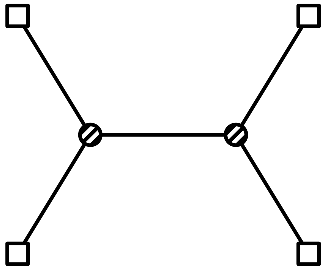


SPHEREx, 1412.4872

O(0.01) ultimately
21cm tomography

Meerburg, Muñoz, Ali-Haïmoud, Kamionkowski, 1506.04152; Münchmeyer, Muñoz, Chen, 1610.06559; Dizgah, Lee, Muñoz, Dvorkin 1801.07265;

Why now?



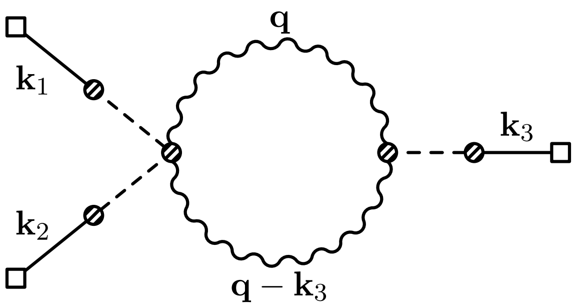
Computing cosmic correlators are challenging!

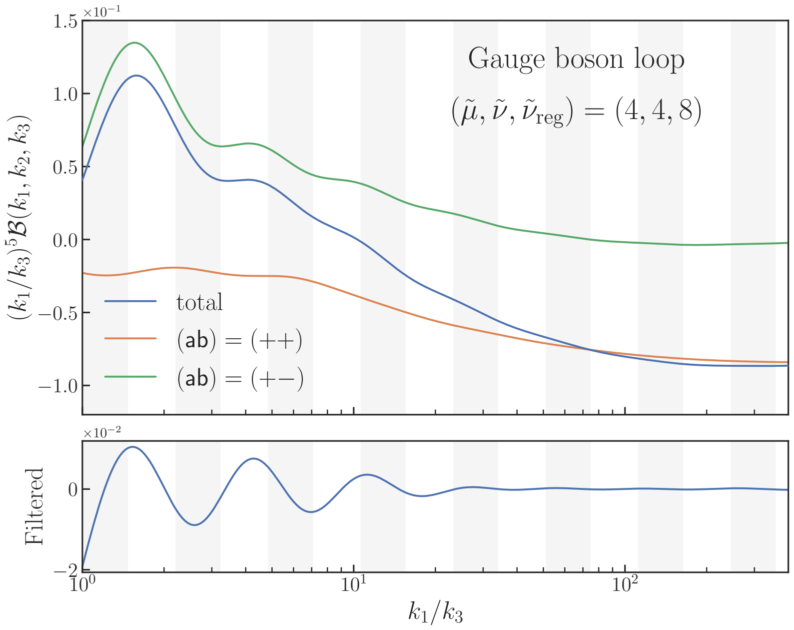
Feynman diagram in de Sitter space

Technical difficulties + conceptual problems

Many new developments recently

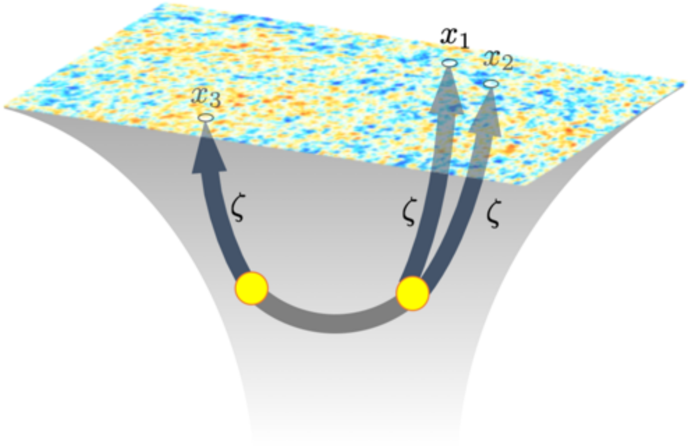
in both formal and phenomenological aspects





Lian-Tao Wang, ZZX, Yi-Ming Zhong, to appear

Outline



Background evolution

Perturbation theory

Quantum fields in dS

Schwinger-Keldysh formalism

Cosmic correlators

Phenomenology

Useful references

Cosmology in general

☐ Dodelson: *Modern cosmology* ☐ Weinberg: *Cosmology*
☐ Mukhanov: *Physical Foundations of Cosmology*

Inflation and non-Gaussianity

☐ Baumann & McAllister 1404.2601 ☐ Baumann: 0907.5424
☐ Maldacena: astro-ph/0210603 ☐ Chen: 1002.1416 ☐ Wang: 1303.1523

QFT in de Sitter space

☐ Spradlin, Strominger Volovich: hep-th/0110007 ☐ Akhmedov: 1309.2557

Schwinger-Keldysh formalism

☐ Jordan: PRD 33 (1986) 444 ☐ Chou, Su, Hao, Yu: Phys. Rept. 118 (1985) 1
☐ Chen, Wang, ZZX: 1703.10166

Cosmological collider physics

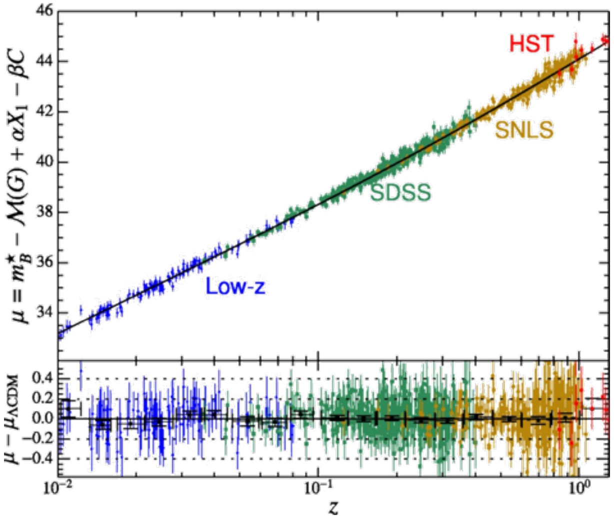
☐ Arkani-Hamed & Maldacena: 1503.08043
☐ Lee, Baumann, Pimentel: 1607.03735 ☐ Wang, ZZX: 1910.12876

The need for cosmic inflation

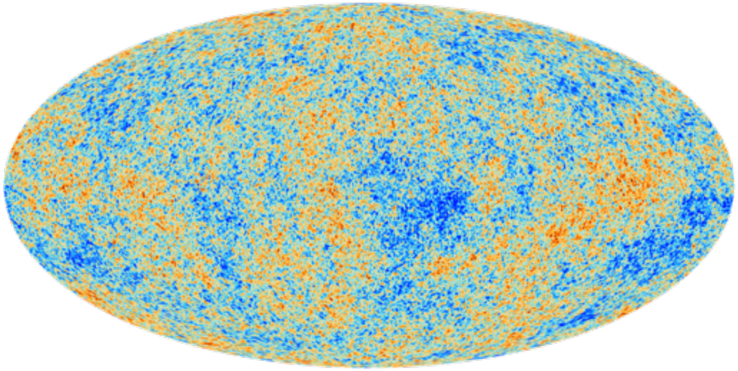
Two basic facts

1. The universe is expanding

2. The universe is homogenous & isotropic at large scales



Betoule M, et al. Astron Astrophys 568 (2014) 22



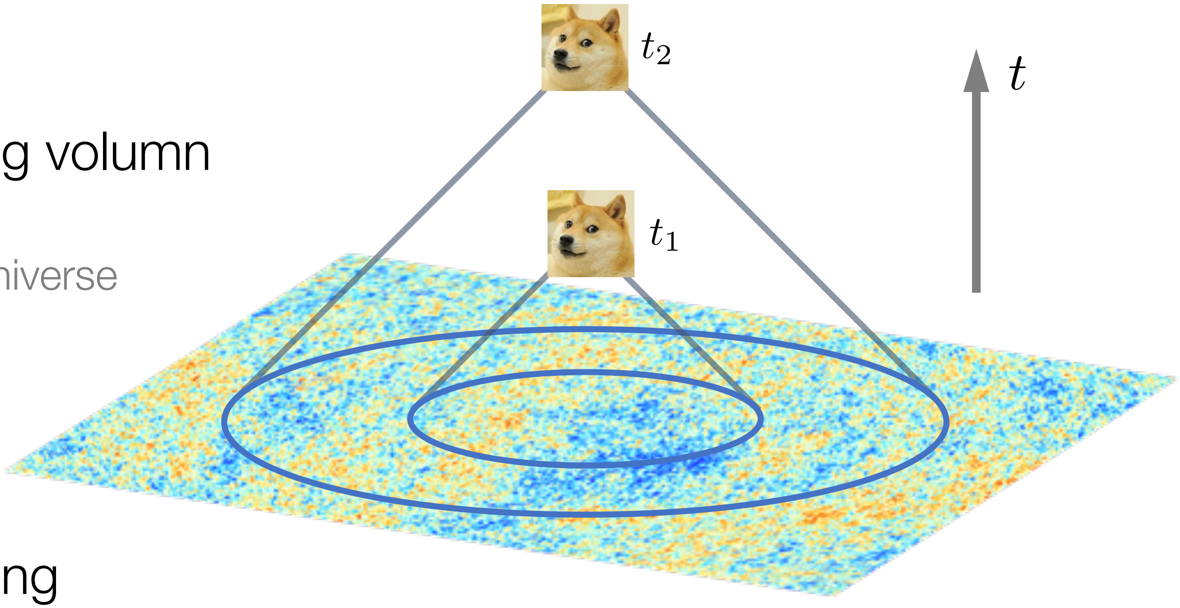
Planck

The need for cosmic inflation

Horizon problem

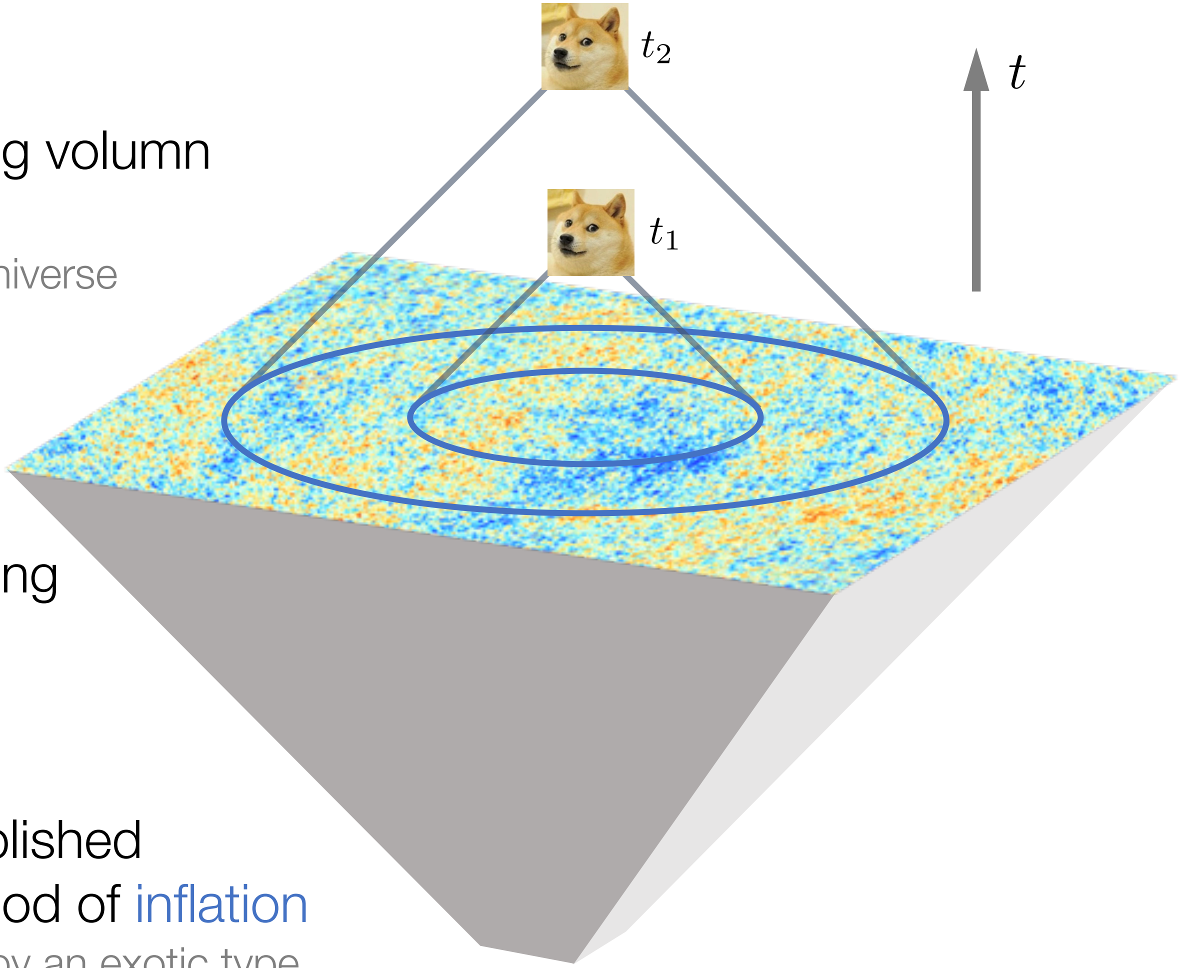
We see larger comoving volumn later than earlier

[Still true for an expanding universe filled with ordinary matter]



Why CMB sky is so uniform if there was no causal contact among different patches?

The need for cosmic inflation



Horizon problem

We see larger comoving volumn later than earlier

[Still true for an expanding universe filled with ordinary matter]

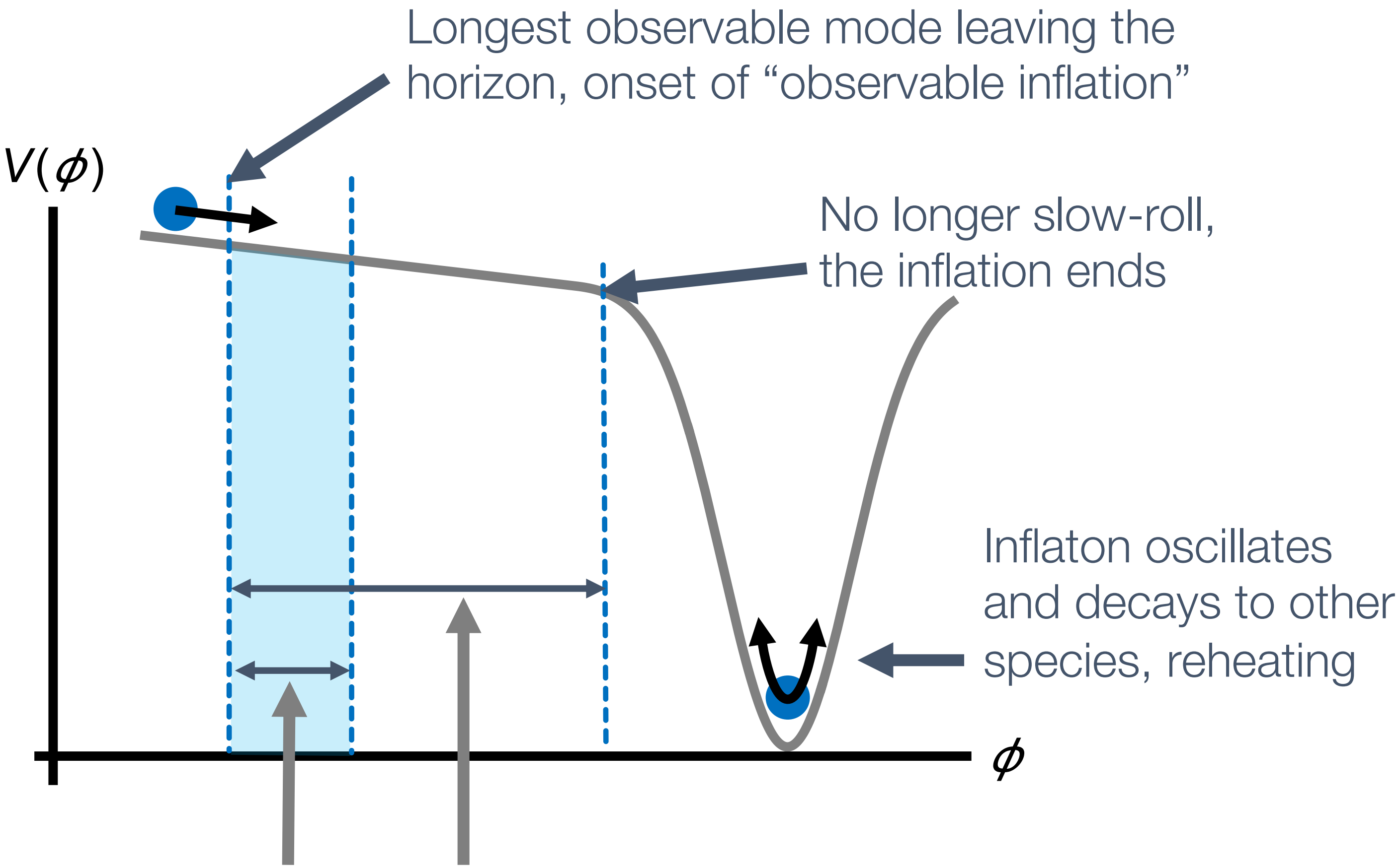
Why CMB sky is so uniform if there was no causal contact among different patches?

Solution

Causal contact reestablished if there has been a period of inflation

[fast expansion, dominated by an exotic type of energy, rather than ordinary matter]

Background dynamics



Longest observable mode leaving the horizon, onset of "observable inflation"

No longer slow-roll, the inflation ends

Inflaton oscillates and decays to other species, reheating

First O(10) e-folds that are actually resolved by CMB

Total amount of "observable" inflation, ~O(60) e-folds

Background dynamics

Einstein gravity + a scalar supporting inflation (inflaton)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right]$$

Homogeneity and isotropy dictate, at background level,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 \qquad \phi = \phi_0(t)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi(\phi) = 0$$

$$3M_{\mathrm{Pl}}^2H^2(t) = \rho(t) \qquad H \equiv \dot{a}/a$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$M_{\mathrm{Pl}} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}\mathrm{TeV}$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

The inflaton should roll slowly, described by slow-roll parameters

$$\epsilon = \frac{\dot{\phi}^2}{2M_{\mathrm{Pl}}^2H^2} \qquad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

$$\eta \sim \mathcal{O}(0.01)$$

$$\epsilon < \mathcal{O}(0.003)$$

Perturbation theory

Einstein gravity + a scalar supporting inflation (inflaton)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right]$$

$$\phi(t,\mathbf{x}) = \phi_0(t) + \varphi(t,\mathbf{x})$$

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

$$N = 1 + \alpha$$

$$N_i = \partial_i\psi + \beta_i$$

$$h_{ij} = a^2 e^{2\zeta}(\delta_{ij} + \gamma_{ij} + \partial_i\kappa_j + \partial_j\kappa_i + \partial_i\partial_j\lambda)$$

10 dofs in the metric, one in the inflaton

- 4 gauge dofs, 4 constrained variables

= 3 physical dofs

1 scalar & 2 tensor

Scalar perturbation action to 2nd order [try yourself if you wish]

$$S_2 = M_{\mathrm{Pl}}^2\int \mathrm{d}t\mathrm{d}^3x\,\epsilon\left[a^3\dot{\zeta}^2 - a(\partial_i\zeta)^2\right]$$

Perturbation theory

Using conformal time τ $\mathrm{d}t = a(\tau)\mathrm{d}\tau$

$$S_2 = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x\, z^2\Big[(\zeta')^2 - (\partial_i\zeta)^2\Big] \qquad \zeta' = \mathrm{d}\zeta/\mathrm{d}\tau \qquad z \equiv \sqrt{2\epsilon}M_{\mathrm{Pl}}a$$

Introducing a canonically normalized field $u = z\zeta$

$$S_2 = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x\left[(u')^2 - (\partial_i u)^2 + \frac{z''}{z}u^2\right]$$

Go to momentum space => "mode function"

$$u(\tau,\mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[u_k(\tau)e^{+\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \mathrm{c.c.}\right]$$

The general solution in a formal dS limit $\epsilon \to 0$ $a = e^{Ht} = -1/(H\tau)$

$$u_k = \frac{1}{\sqrt{2k}}\left[c_1\Big(1 - \frac{\mathrm{i}}{k\tau}\Big)e^{-\mathrm{i}k\tau} + c_2\Big(1 + \frac{\mathrm{i}}{k\tau}\Big)e^{+\mathrm{i}k\tau}\right]$$

Perturbation theory

Quantization

$$u(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[u_k(\tau)e^{+\mathrm{i}\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + u_k^*(\tau)e^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^\dagger\right]$$

Two conditions to determine c_1 and c_2 :

1. canonical commutator (Wronskian)

2. Vacuum at early time limit (Bunch-Davies vacuum)

$$u_k = e^{\mathrm{i}\theta}\frac{1}{\sqrt{2k}}\left(1 - \frac{\mathrm{i}}{k\tau}\right)e^{-\mathrm{i}k\tau}$$

In terms of the original fluctuation field

$$\zeta_k(\tau) = e^{\mathrm{i}\theta}\frac{H}{2M_{\mathrm{Pl}}\sqrt{\epsilon k^3}}(1 + \mathrm{i}k\tau)e^{-\mathrm{i}k\tau}$$

Perturbation theory

$$\zeta_k(\tau) = e^{i\theta}\frac{H}{2M_{\rm Pl}\sqrt{\epsilon k^3}}(1+ik\tau)e^{-ik\tau}$$

$$\tau \in (-\infty, 0)$$

Remarks:

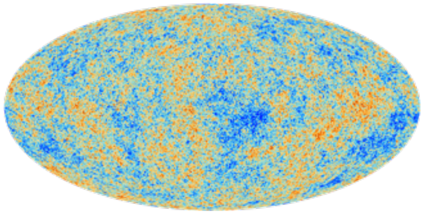
Early-time limit: plane waves of a massless field

[deep inside the horizon ~ flat space]

Late-time limit: reaches a constant

[conserved at superhorizon, irrespective to small scale physics]

A complex phase unfixed, schotastic quantum noise

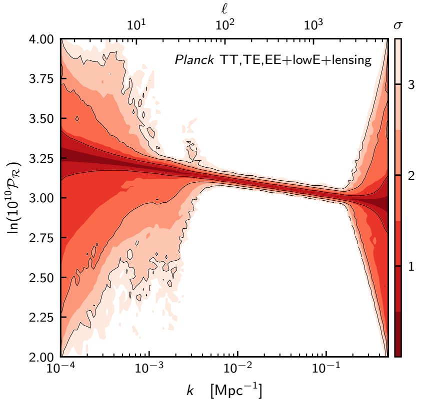


The complex phase cancels out in 2-point correlator [power spectrum]

$$\lim_{\tau\to 0}\langle 0|\zeta_{\mathbf{k}}(\tau)\zeta_{\mathbf{q}}(\tau)|0\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)$$

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon M_{\rm Pl}^2} = \frac{H^4}{(2\pi)^2\dot{\phi}_0^2} \quad \longleftarrow \text{scale invariance}$$

$$\mathcal{P}_\zeta(k) \simeq 2\times 10^{-9}$$



Planck 2018

Quantum fields in dS

Very much similar, now consider an arbitrary massive scalar field

$$(\Box - m^2)\sigma = 0 \quad \longrightarrow \quad \sigma''(\tau,\mathbf{x}) - \frac{2}{\tau}\sigma'(\tau,\mathbf{x}) - \partial_i^2\sigma(\tau,\mathbf{x}) + \frac{m^2}{H^2\tau^2}\sigma(\tau,\mathbf{x}) = 0$$

In momentum space, mode equation

$$\sigma_k''(\tau) - \frac{2}{\tau}\sigma_k'(\tau) + \left(k^2 + \frac{m^2}{H^2\tau^2}\right)\sigma_k(\tau) = 0$$

Unique normalized solution upon Bunch-Davies condition

$$\sigma_k(\tau) = \frac{\sqrt{\pi}}{2}e^{\mathrm{i}\nu\pi/2}H(-\tau)^{3/2}\underset{\uparrow}{\mathrm{H}_\nu^{(1)}}(-k\tau) \qquad \nu \equiv \sqrt{9/4 - (m/H)^2}$$

Hänkel function

You can already anticipate that $m < 3H/2$ and $m > 3H/2$
would behave very differently

Quantum fields in dS

$$\sigma_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\nu\pi/2} H(-\tau)^{3/2} \mathrm{H}_\nu^{(1)}(-k\tau) \qquad \nu \equiv \sqrt{9/4 - (m/H)^2}$$

Early-time limit: massless plain waves again [figure it out yourself]

Late-time limit:

$$\sigma_k(\tau) = -i\sqrt{\frac{2}{\pi k^3}} H \left[e^{-i\nu\pi/2} \Gamma(-\nu) \left(\frac{-k\tau}{2} \right)^{3/2+\nu} + (\nu \to -\nu) \right]$$

$m < 3H/2$ non-analytical scaling in kτ

$m > 3H/2$ oscillatory in kτ

In general: $\lim_{\tau \to 0} \sigma(\tau, \mathbf{x}) = \sigma_+(\mathbf{x})(-\tau)^{\Delta_+} + \sigma_-(\mathbf{x})(-\tau)^{\Delta_-}, \qquad \Delta_\pm = \frac{3}{2} \pm \nu$

The dS has isometry group SO(4,1), under which the late time coefficient $\sigma_\pm$ transforms like a scalar of conformal weight Δ

The isometry in the bulk ~ conformal group on the future boundary

Only a symmetry statement! Any dynamical realization? [dS/CFT]

Quantum fields in dS

Consider $m \gg H$ limit:

$$\nu = \mathrm{i}\widetilde{\nu}$$

$$\sigma_{\mathbf{k}}(t) = \frac{1}{\sqrt{2\widetilde{\nu}H}} e^{-3Ht/2}\left(e^{-\mathrm{i}\widetilde{\nu}Ht} b_{\mathbf{k}} + e^{+\mathrm{i}\widetilde{\nu}Ht} b^{\dagger}_{-\mathbf{k}}\right)$$

while at early times: $\sigma_{\mathbf{k}}(\tau) = \sigma_k a_{\mathbf{k}} + \sigma_k^* a^{\dagger}_{-\mathbf{k}}$

Two sets of operators linearly related $b_{\mathbf{k}} = \alpha_k a_{\mathbf{k}} + \beta_k a^{\dagger}_{-\mathbf{k}}$
[Bogoliubov transformation]

Computing the occupation number at late times:

$$n_{\mathbf{k}} = \langle 0 | b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} | 0 \rangle = |\beta_k|^2$$

$$|\beta_k|^2 = \frac{\widetilde{\nu}}{2\pi} |\Gamma(-\mathrm{i}\widetilde{\nu})|^2 e^{-\widetilde{\nu}\pi} = \frac{\coth \widetilde{\nu}\pi - 1}{2} \simeq e^{-2\pi m/H}$$

Nonzero β_k signals particle creation

An engine for the cosmological collider

looks like thermal distribution $e^{-m/T}$

A first hint that dS vacuum is thermal, with temperature $T = \dfrac{H}{2\pi}$

Quantum fields in dS

Classification of states

UIRs of dS isometry group SO(4,1)
classified by spin s under SO(3), and conformal weight Δ under SO(1,1)

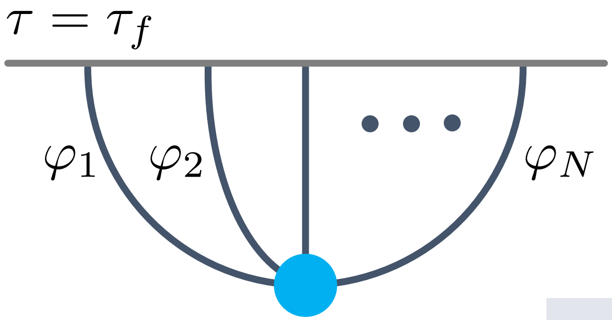
spin	$s=0$	$s\in\frac{1}{2}\mathbb{Z}_+$
weight	$\Delta=\frac{3}{2}\pm\sqrt{\frac{9}{4}-(m/H)^2}$	$\Delta=\frac{3}{2}\pm\sqrt{(s-\frac{1}{2})^2-(m/H)^2}$
principal series	$m>3H/2$	$(m/H)^2>(s-\frac{1}{2})^2$
complementary series	$0<m<3H/2$	$(m/H)^2<(s-\frac{1}{2})^2$
discrete eries		$(m/H)^2=s(s-1)-t(t+1)$ $t=0,1,\cdots,s-1$

Strickly speaking, massless scalar does not exist in dS

But both massless spin-1 and massless spin-2 are allowed

Schwinger-Keydish formalism

Goal: calculating equal-time correlators



$$\langle\Omega|\varphi^{A_1}(\tau,\mathbf{x}_1)\cdots\varphi^{A_N}(\tau,\mathbf{x}_N)|\Omega\rangle$$

$$1 = \sum |O_\alpha\rangle\langle O_\alpha|$$

Convert equal-time correlators into a pair of in-out amplitude, then apply standard path integral method

$$\begin{aligned}
&\langle\Omega|\varphi^{A_1}(\tau,\mathbf{x}_1)\cdots\varphi^{A_N}(\tau,\mathbf{x}_N)|\Omega\rangle \\
&= \sum_\alpha \langle\Omega|O_\alpha\rangle\langle O_\alpha|\varphi^{A_1}(\tau,\mathbf{x}_1)\cdots\varphi^{A_N}(\tau,\mathbf{x}_N)\big|\Omega\rangle \\
&= \int \mathcal{D}\varphi_+\mathcal{D}\varphi_-\,\varphi_+^{A_1}(\tau,\mathbf{x}_1)\cdots\varphi_+^{A_N}(\tau,\mathbf{x}_N) \\
&\quad\times \exp\left[\mathrm{i}\int_{\tau_0}^{\tau_f} \mathrm{d}\tau\mathrm{d}^3\mathbf{x}\left(\mathscr{L}_{\mathrm{cl}}[\varphi_+] - \mathscr{L}_{\mathrm{cl}}[\varphi_-]\right)\right] \\
&\quad\times \prod_{A,\mathbf{x}} \delta\Big(\varphi_+^A(\tau_f,\mathbf{x}) - \varphi_-^A(\tau_f,\mathbf{x})\Big)
\end{aligned}$$

Schwinger-Keydish formalism

2 fields => 4 types of bulk propagators

$$\begin{aligned} &= G_{++}(k;\tau_1,\tau_2) \\ &= u(\tau_1,k)u^*(\tau_2,k)\theta(\tau_1-\tau_2)+u^*(\tau_1,k)u(\tau_2,k)\theta(\tau_2-\tau_1) \end{aligned}$$

$$\begin{aligned} &= G_{--}(k;\tau_1,\tau_2) \\ &= u^*(\tau_1,k)u(\tau_2,k)\theta(\tau_1-\tau_2)+u(\tau_1,k)u^*(\tau_2,k)\theta(\tau_2-\tau_1) \end{aligned}$$

$$= G_{+-}(k;\tau_1,\tau_2) = u^*(\tau_1,k)u(\tau_2,k)$$

$$= G_{-+}(k;\tau_1,\tau_2) = u(\tau_1,k)u^*(\tau_2,k)$$

2 bulk-to-boundary propagators

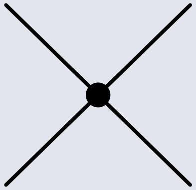
$$= G_{+}(k;\tau) \equiv G_{++}(k;\tau,\tau_f)$$

$$= G_{-}(k;\tau) \equiv G_{-+}(k;\tau,\tau_f)$$

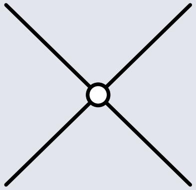
Schwinger-Keydish formalism

2 types of vertices

$$\mathscr{L}_{\text{int}} \supset -\frac{\lambda}{24} a^4(\tau)\varphi^4$$



$$= -\mathrm{i}\lambda \int_{\tau_0}^{\tau_f} \mathrm{d}\tau\, a^4(\tau) \cdots$$



$$= +\mathrm{i}\lambda \int_{\tau_0}^{\tau_f} \mathrm{d}\tau\, a^4(\tau) \cdots$$

Feynman rule

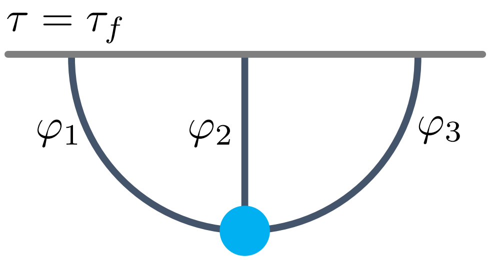
Draw square to each final endpoint; draw circle to each vertex
Color vertices in either white or black in all possible ways

Integrate over all unconstrained (loop) momenta
Integrate over the time variable at each vertex

Sum over all possible coloring

Cosmic correlators

Bispectrum



$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle' = \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{(k_1 k_2 k_3)^2} \mathcal{S}(k_1, k_2, k_3)$$

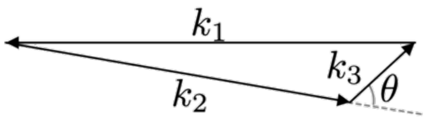
Momentum conservation δ stripped

Power spectrum
$P_\zeta \simeq 2 \times 10^{-9}$

Shape function
dimensionless and (usually) scale invariant

Conversion between ζ and φ

$$\zeta = -\frac{H}{\dot{\phi}_0} \varphi$$



The 3-point function depends only on the shape of the momentum triangle, [3d translation]
not on the orientation, [3d rotation]
not on the size [scale symmetry]

Cosmic correlators

3-point correlator from the inflaton: expanding the action to 3rd order

[don’t try yourself unless you really wish!]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right]$$

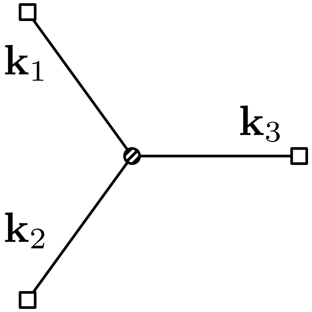
$$S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}t\mathrm{d}^3x\,\epsilon\left[a^3\dot{\zeta}^2 - a(\partial_i\zeta)^2\right]$$

$$S_3 = M_{\mathrm{Pl}}^2 \int \mathrm{d}\tau\mathrm{d}^3\mathbf{x}\left\{a^2\epsilon^2\left[\zeta(\zeta')^2 + \zeta(\partial_i\zeta)^2 - 2\zeta'\partial_i\zeta\partial_i^{-1}\zeta'\right] - \frac{1}{2}\partial_\tau\left(\epsilon\eta a^2\zeta^2\zeta'\right)\right\} + \cdots$$

Dots represent higher orders in slow-roll parameters

Cosmic correlators

$$S_3 = M_{\rm Pl}^2 \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left\{ a^2\epsilon^2 \left[\boxed{\zeta(\zeta')^2} + \zeta(\partial_i\zeta)^2 - 2\zeta'\partial_i\zeta\partial_i^{-1}\zeta' \right] - \frac{1}{2}\partial_\tau\left(\epsilon\eta a^2\zeta^2\zeta'\right)\right\} + \cdots$$



$$\begin{aligned}
&\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle'_{(1)} \\
&= 2\mathrm{i}M_{\rm Pl}^2\epsilon^2 \sum_{\mathsf{a}=\pm} \mathsf{a} \int_{-\infty}^{\tau_f} \frac{\mathrm{d}\tau}{(H\tau)^2} G_{\mathsf{a}}(k_1,\tau)\partial_\tau G_{\mathsf{a}}(k_2,\tau)\partial_\tau G_{\mathsf{a}}(k_3,\tau) + 2\text{ perms} \\
&= -\,4M_{\rm Pl}^2\epsilon^2\,\mathrm{Im}\left[\prod_{i=1}^{3}\zeta_{k_i}(\tau_f)\int_{-\infty}^{\tau_f}\frac{\mathrm{d}\tau}{(H\tau)^2}\zeta^*_{k_1}(\tau)\zeta'^*_{k_2}(\tau)\zeta'^*_{k_3}(\tau)\right] + 2\text{ perms} \\
&= -\frac{H^4}{16M_{\rm Pl}^4\epsilon k_1^3k_2k_3}\mathrm{Im}\int_{-\infty}^{\tau_f}\mathrm{d}\tau\,(1-\mathrm{i}k_1\tau)e^{+\mathrm{i}k_t\tau} + 2\text{ perms} \\
&= \frac{H^4}{4M_{\rm Pl}^4\epsilon k_1^3k_2k_3k_t} + 2\text{ perms}
\end{aligned}$$

$$k_t \equiv k_1 + k_2 + k_3$$

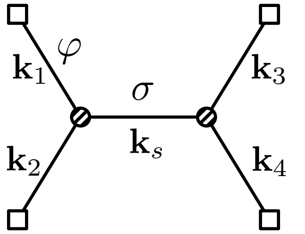
$$\mathcal{S} = \epsilon\left(\frac{k_1k_2}{k_3k_t} + 2\text{ perms}\right) + \frac{\epsilon}{8}\left(\frac{k_1}{k_2} + 5\text{ perms}\right) + \frac{\eta-\epsilon}{8}\left(\frac{k_1^2}{k_2k_3} + 2\text{ perms}\right)$$

The bispectrum from the inflaton+gravity is slow-roll suppressed

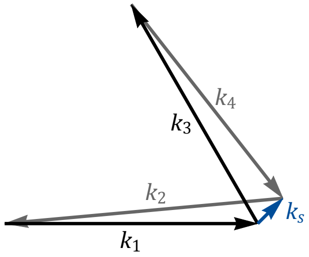
Cosmic correlators

Consider now a cosmological collider process:
a 4-point function mediated by a massive scalar σ

$$a^2(\varphi')^2\sigma/\Lambda \subset \sqrt{-g}(\partial_\mu\phi)^2\sigma/\Lambda \subset \mathscr{L}$$



$$\begin{aligned}
&\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle' \\
&= \frac{1}{\Lambda^2}\sum_{\mathsf{a},\mathsf{b}=\pm}\mathsf{ab}\int_{-\infty}^{0}\frac{\mathrm{d}\tau_1}{(-H\tau_1)^2}\frac{\mathrm{d}\tau_2}{(-H\tau_2)^2} \\
&\quad\times G'_{\mathsf{a}}(k_1,\tau_1)G'_{\mathsf{a}}(k_2,\tau_1)G'_{\mathsf{b}}(k_3,\tau_2)\overset{\overset{\varphi}{\downarrow}}{G'_{\mathsf{b}}(k_4,\tau_2)}\overset{\overset{\sigma}{\downarrow}}{D_{\mathsf{ab}}(k_s,\tau_1,\tau_2)} \\
&= \frac{1}{\Lambda^2}\frac{1}{16k_1k_2k_3k_4}\sum_{\mathsf{a},\mathsf{b}=\pm}\mathsf{ab}\int_{-\infty}^{0}\mathrm{d}\tau_1\mathrm{d}\tau_2\, e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2)
\end{aligned}$$



In the collapsed limit, the late-time expansion is valid

The intermediate soft mode well outside the horizon

$$D_>(k;\tau_1,\tau_2) = \sigma_k(\tau_1)\sigma_k^*(\tau_2) \sim D_{\mathrm{l}}(k;\tau_1,\tau_2) + D_{\mathrm{nl}}(k;\tau_1,\tau_2)$$

$$D_{\mathrm{l}}(k;\tau_1,\tau_2) = \frac{H^2}{4\pi}(\tau_1\tau_2)^{3/2}\Gamma(-\mathrm{i}\widetilde{\nu})\Gamma(\mathrm{i}\widetilde{\nu})\left[e^{\widetilde{\nu}\pi}(\tau_1/\tau_2)^{\mathrm{i}\widetilde{\nu}} + e^{-\widetilde{\nu}\pi}(\tau_1/\tau_2)^{-\mathrm{i}\widetilde{\nu}}\right]$$

$$D_{\mathrm{nl}}(k;\tau_1,\tau_2) = \frac{H^2}{4\pi}(\tau_1\tau_2)^{3/2}\left[\Gamma^2(-\mathrm{i}\widetilde{\nu})(k^2\tau_1\tau_2)^{+\mathrm{i}\widetilde{\nu}} + \Gamma^2(+\mathrm{i}\widetilde{\nu})(k^2\tau_1\tau_2)^{-\mathrm{i}\widetilde{\nu}}\right]$$

Cosmic correlators

$$\int_{-\infty}^{0} \mathrm{d}\tau_1\,(-\tau_1)^{3/2} e^{\mathrm{i}ak_{12}\tau_1}(-k_s\tau_1)^{\mathrm{i}c\widetilde{\nu}} = \frac{1}{k_{12}^{5/2}}\left(\frac{k_s}{k_{12}}\right)^{\mathrm{i}c\widetilde{\nu}} \int_0^{\infty} \mathrm{d}z\, z^{3/2+\mathrm{i}c\widetilde{\nu}} e^{-\mathrm{i}az}$$

$$= e^{-\mathrm{i}a\pi/4+ac\pi\widetilde{\nu}/2}\,\Gamma\left(\frac{5}{2}+\mathrm{i}c\widetilde{\nu}\right)\frac{1}{k_{12}^{5/2}}\left(\frac{k_s}{k_{12}}\right)^{\mathrm{i}c\widetilde{\nu}}$$

$$\int_{-\infty}^{0} \mathrm{d}\tau_2\,(-\tau_2)^{3/2} e^{\mathrm{i}bk_{34}\tau_2}(-k_s\tau_2)^{\mathrm{i}c\widetilde{\nu}} = e^{-\mathrm{i}b\pi/4+bc\pi\widetilde{\nu}/2}\,\Gamma\left(\frac{5}{2}+\mathrm{i}c\widetilde{\nu}\right)\frac{1}{k_{34}^{5/2}}\left(\frac{k_s}{k_{34}}\right)^{\mathrm{i}c\widetilde{\nu}}$$

$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'_{\mathrm{nl}} \simeq$$

$$-\frac{H^2}{32\pi\Lambda^2}\frac{1}{k_1k_2k_3k_4}\frac{1}{(k_{12}k_{34})^{5/2}}\,\mathrm{Re}\left[\Gamma^2(-\mathrm{i}\widetilde{\nu})\Gamma^2(\frac{5}{2}+\mathrm{i}\widetilde{\nu})(1+2\mathrm{i}\sinh\pi\widetilde{\nu})\left(\frac{k_s^2}{k_{12}k_{34}}\right)^{\mathrm{i}\widetilde{\nu}}\right]$$

$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'_{\mathrm{nl}} \simeq \mathcal{A}(m,\Lambda)\frac{1}{k_1k_2k_3k_4(k_{12}k_{34})^{5/2}}\sin\left[\widetilde{\nu}\log\left(\frac{k_s^2}{k_{12}k_{34}}\right)+\theta(\widetilde{\nu})\right]$$

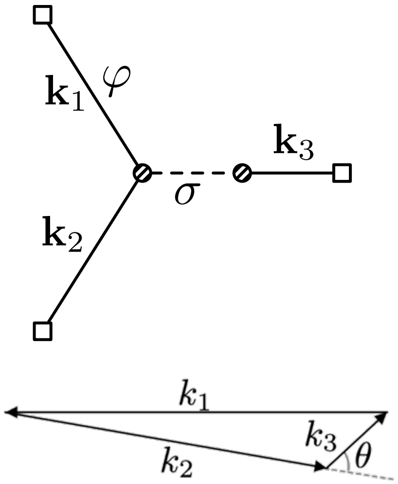
Oscillatory dependence on momentum ratio

$$\mathcal{A} = \frac{H^2}{32\pi\Lambda^2}\left|\Gamma^2(-\mathrm{i}\widetilde{\nu})\Gamma^2(\frac{5}{2}+\mathrm{i}\widetilde{\nu})(1+2\mathrm{i}\sinh\pi\widetilde{\nu})\right|$$

$$\mathcal{A} \simeq \frac{\pi H^2}{8\Lambda^2}\left(\frac{m}{H}\right)^3 e^{-\pi m/H}$$

Boltzmann suppressed for large mass

Cosmic correlators

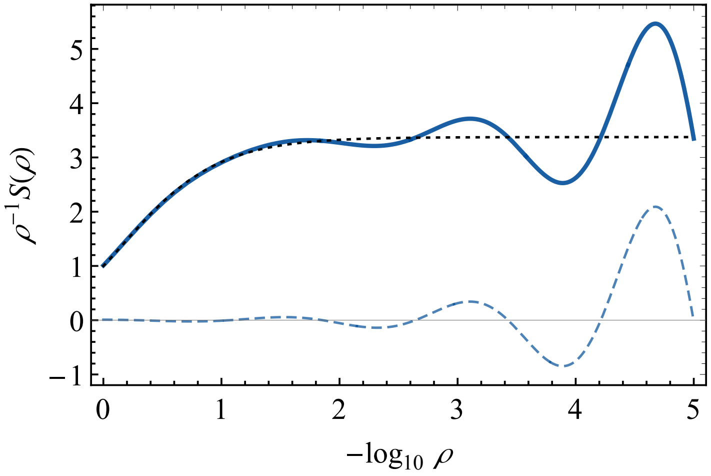


Bispectrum: similar but more difficult to calculate
In the squeezed limit, the signal part is

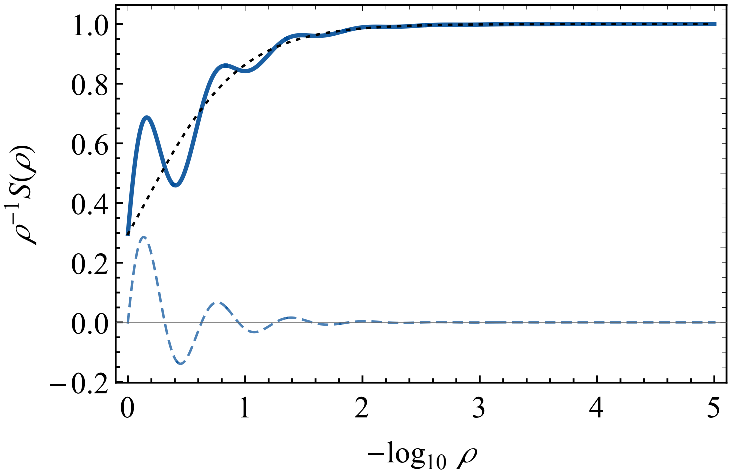
$$\lim_{k_3/k_1\to 0} \mathcal{S}(k_1,k_2,k_3)$$
$$\propto \frac{\dot{\phi}_0}{\Lambda^2}\left(\frac{m}{H}\right)^{3/2} e^{-\pi m/H}\left(\frac{k_1}{k_3}\right)^{-1/2} \sin\left(\widetilde{\nu}\log\frac{k_1}{k_3}+\varphi\right)$$

Can generalize to loop processes

$\rho \equiv k_3/k_1$

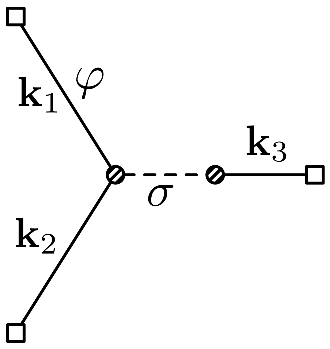


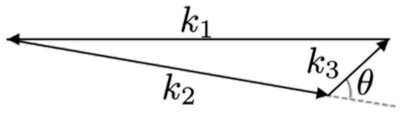
Tree process



Loop process

Cosmic correlators





Angular dependence in the squeezed limit

For intermediate states with mass m and spin s

described by a symmetry tensor of rank s

$$\mathcal{O}_2 \sim \psi_{i_1 \cdots i_s} \partial^{i_1} \cdots \partial^{i_s} \varphi$$

$$\mathcal{O}_3 \sim \psi_{i_1 \cdots i_s} (\partial^{i_1} \cdots \partial^{i_n} \varphi)(\partial^{i_{n+1}} \cdots \partial^{i_s} \varphi)$$

The 3-point vertex generates, in the squeezed limit,

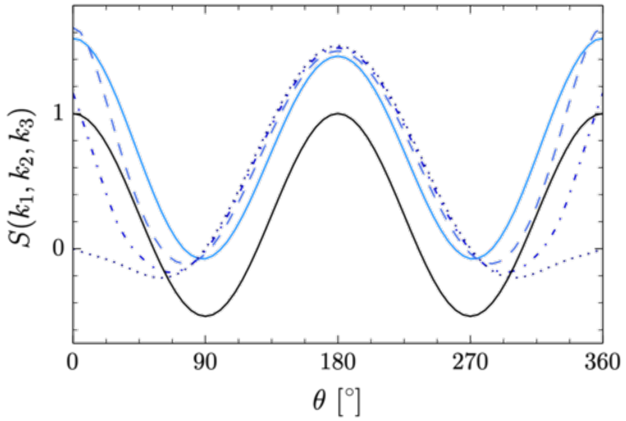
$$\epsilon^0_{i_1 \cdots i_s}(\mathbf{k}_3) k_1^{i_1} \cdots k_1^{i_s} \longrightarrow \epsilon^0_{i_1 \cdots i_s}(\mathbf{k}_3) \epsilon^{0*}_{i_1 \cdots i_s}(\mathbf{k}_1)$$

$\mathbf{k}_1 \simeq -\mathbf{k}_2$

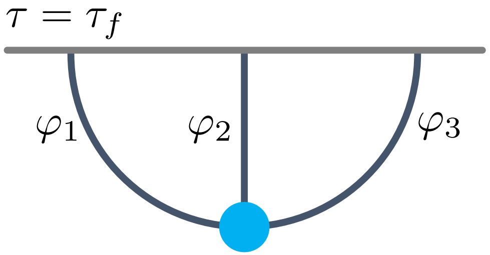
Longitudinal polarization tensor symmetric and traceless

Matrix element of 3d rotation bringing **k**$_1$ to **k**$_3$ between two states of spin-s and helicity-0

$$\epsilon^0_{i_1 \cdots i_s}(\mathbf{k}_3) k_1^{i_1} \cdots k_1^{i_s} \propto P_s(\widehat{\mathbf{k}}_1 \cdot \widehat{\mathbf{k}}_3)$$



Phenomenology

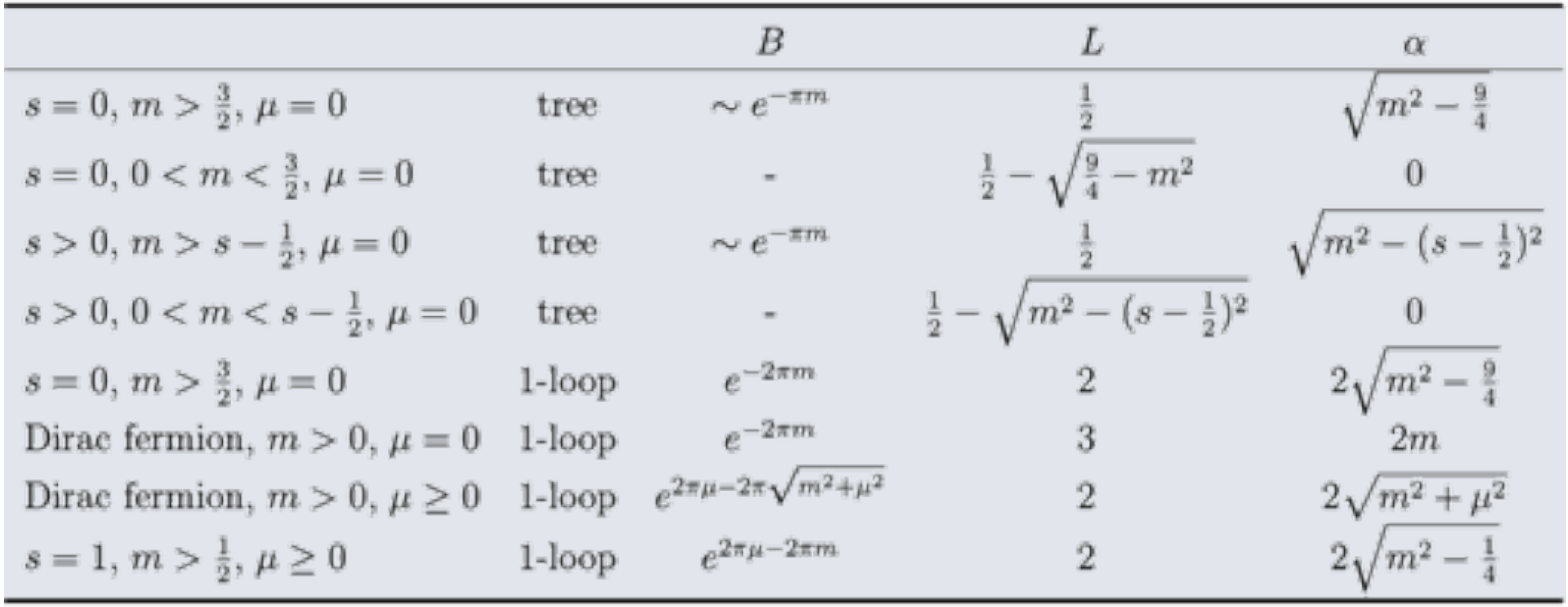


Cosmological collider observables @ 3pt level

Angular dependence not shown

$$\rho \equiv k_3/k_1$$

$$\lim_{\varrho\to 0} \mathcal{S}(\varrho) = \underbrace{A\varrho^N\left[1+\mathcal{O}(\varrho)\right]}_{\text{analytic}} + \underbrace{B\varrho^L\left[\sin\left(\alpha\log\varrho+\varphi\right)+\mathcal{O}(\varrho)\right]}_{\text{nonanalytic}}$$



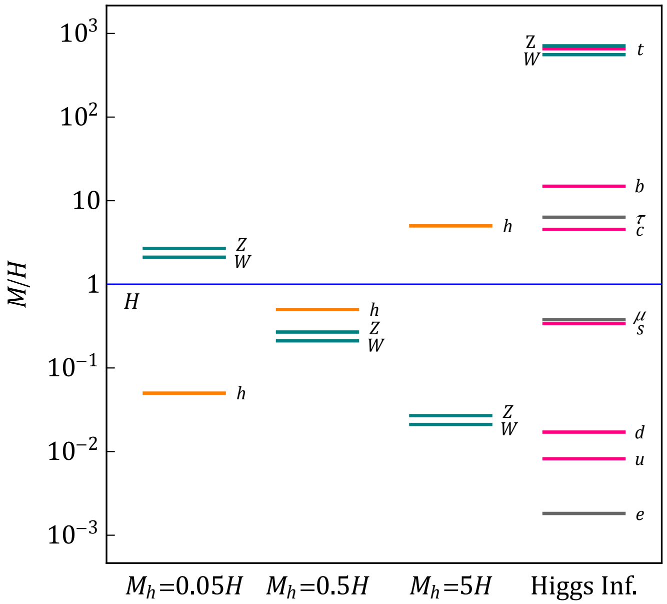
		B	L	α
$s=0,\ m>\frac{3}{2},\ \mu=0$	tree	$\sim e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-\frac{9}{4}}$
$s=0,\ 0<m<\frac{3}{2},\ \mu=0$	tree	-	$\frac{1}{2}-\sqrt{\frac{9}{4}-m^2}$	0
$s>0,\ m>s-\frac{1}{2},\ \mu=0$	tree	$\sim e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-(s-\frac{1}{2})^2}$
$s>0,\ 0<m<s-\frac{1}{2},\ \mu=0$	tree	-	$\frac{1}{2}-\sqrt{m^2-(s-\frac{1}{2})^2}$	0
$s=0,\ m>\frac{3}{2},\ \mu=0$	1-loop	$e^{-2\pi m}$	2	$2\sqrt{m^2-\frac{9}{4}}$
Dirac fermion, $m>0,\ \mu=0$	1-loop	$e^{-2\pi m}$	3	$2m$
Dirac fermion, $m>0,\ \mu\geq 0$	1-loop	$e^{2\pi\mu-2\pi\sqrt{m^2+\mu^2}}$	2	$2\sqrt{m^2+\mu^2}$
$s=1,\ m>\frac{1}{2},\ \mu\geq 0$	1-loop	$e^{2\pi\mu-2\pi m}$	2	$2\sqrt{m^2-\frac{1}{4}}$

Phenomenology

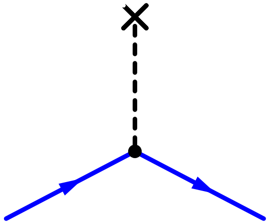
Quantum fields during inflation can receive various mass corrections

What you measured at the cosmological collider is the dressed mass

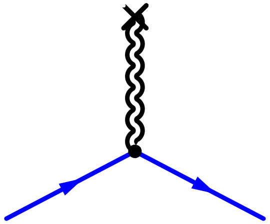
An example: The SM spectrum in inflation



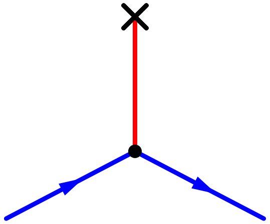
Chen, Wang, ZZX, 1604.07841, 1610.06597, 1612.08122



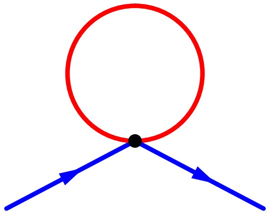
inflaton bg.



gravity bg.



Higgs bg.
EWSB



“thermal” bg.

Phenomenology

Signal size usually tiny in minimal models

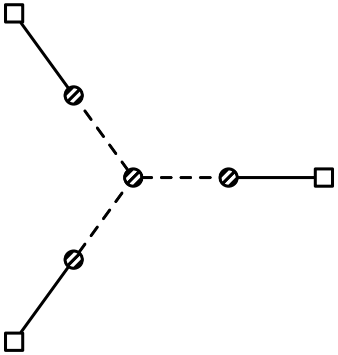
Slow-roll inflaton; Scale invariance; O(1) coupling; No tuning

$$\frac{1}{\Lambda^2}(\partial_\mu\phi)^2\sigma^2 \longrightarrow \frac{\dot{\phi}_0^2}{\Lambda^2} \sim H^2 \longrightarrow \Lambda \simeq 3600H$$

No Boltzmann supression

$\dot{\phi}_0 \simeq (60H)^2$

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \lambda\sigma^4 + \frac{1}{\Lambda^2}(\partial_\mu\phi)^2\sigma^2$$



$$f_{\mathrm{NL}} \sim 3600 \cdot \left(\frac{\dot{\phi}_0}{\Lambda^2}\langle\sigma\rangle\right)^3 \cdot \lambda\langle\sigma\rangle \sim 10^{-7} \cdot \lambda\langle\sigma\rangle^4$$

QSFI: a very shallow potential is needed

$$\langle\sigma\rangle^2 \sim H^2/\lambda \longrightarrow \lambda\langle\sigma\rangle^4 \sim 1/\lambda \longrightarrow \lambda \lesssim 10^{-7}$$

$f_{\mathrm{NL}} \gtrsim 1$

Phenomenology

$$\frac{1}{\Lambda}(\partial_\mu\phi)\mathcal{J}^\mu \longrightarrow \frac{1}{\Lambda}\dot{\phi}_0\mathcal{N}$$

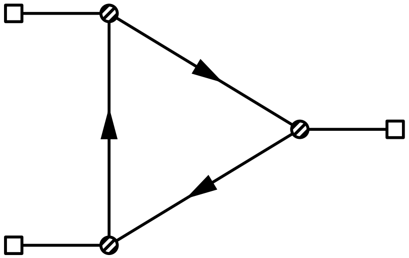
A new source of particle production

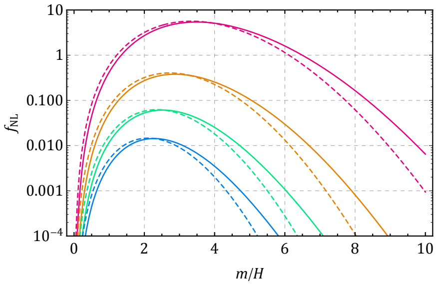
Lian-Tao Wang, ZZX, 1910.12876

Fermion $(\partial_\mu\phi)\overline{\Psi}\gamma^\mu\gamma^5\Psi$

Probing heavy neutrinos

Chen, Wang, ZZX, 1805.02656



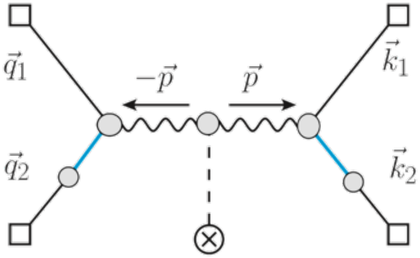


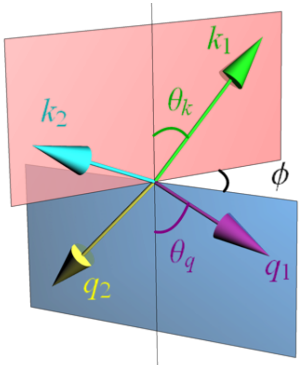
Gauge boson $\phi F\widetilde{F}$

Lian-Tao Wang, ZZX, 2004.02887

CP-breaking in trispectrum

Liu, Tong, Wang, ZZX, 1909.01819





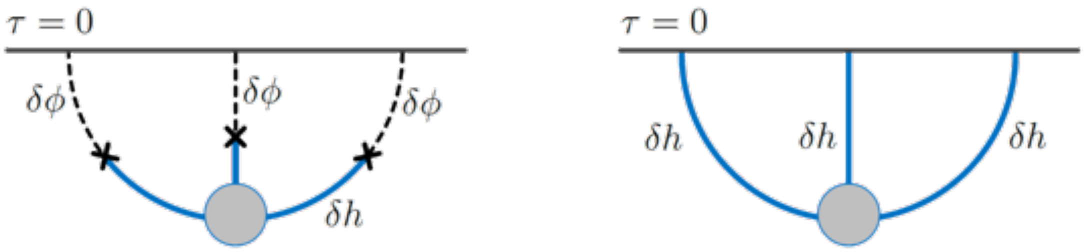
Phenomenology

Beyond slow-roll inflation

$$\zeta(t_2,\mathbf{x}) = \underset{\substack{\uparrow \\ \text{Standard inflation}}}{\zeta(t_1,\mathbf{x})} + \underset{\substack{\uparrow \\ \text{Modulated reheating}}}{\delta N(t_1,t_2,\mathbf{x})}$$

$$\boxed{\delta N = \frac{1}{6}\frac{\delta t_{\rm reh}}{t_{\rm reh}} = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma}}$$

Modulated reheating: scalar perturbation generated not by the inflaton, but by a light field that modulates the inflaton decay



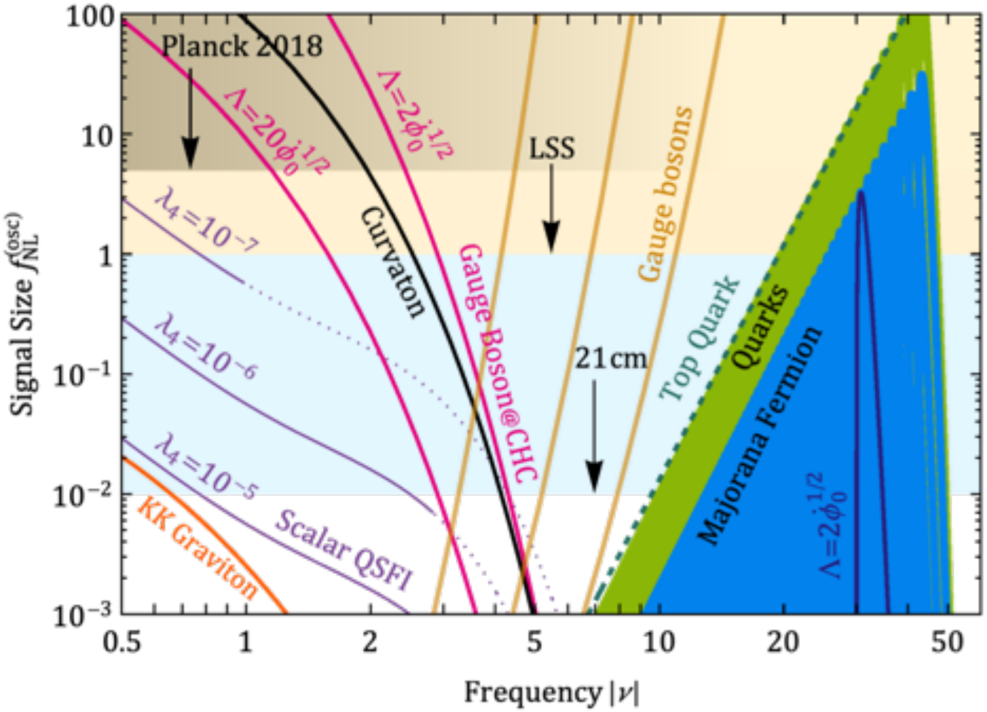
SM Higgs can do it → A cosmological Higgs collider

Large couplings / large signal / much less free parameters

Lu, Wang, ZZX, 1907.07390

Phenomenology

A status summary: known particle models producing observably large signals without tuning parameters



Lian-Tao Wang, ZZX, 1910.12876, 2004.02887

Final remarks

The non-G is from interactions
It must be there since fields must interact, at least gravitationally
A guaranteed signal at 0.01 level

The oscillatory non-G is largely a kinematic property
The signal is quite inflation-model-independent
but are sensitive to how particles couple to long-lived scalar mode

We are still at a very preliminary stage in understanding
& calculating cosmic correlators
A theory challenge

Observation data ahead!
But we are still very far from exhausting interesting
phenomenological possibilities @ cosmological collider
More efforts from particle physics are called for!