Introduction to String Compactifications

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Pre-SUSY Summer School, Beijing, 9 Aug, 2021

Outline

The Road to Unification I

Physics, in Greek, meaning nature, is the study of matter and its motion through spacetime and all that derives from these, such as energy and force.

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Experiment Aspects:

Epoch of precise measurement at a very high energy

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How to describe the nature?

Experiment Aspects:

Epoch of precise measurement at a very high energy

- Particle physics: LHC (14 TeV), SuperKEKB, BEPCII ...
- Cosmology: PLANCK, South Pole Telescope, FAST ...

The Road to Unification II

High Energy Theoretical Aspects:

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    - Electric force, Weak force, Strong force.

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- Dark Matter and Dark Energy?
- UV complete of particle theory?
- Cosmological constant?

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Unification?

- Quantum Gravity $\Leftrightarrow$ Algebraic Geometry

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- Remarkable way to describe cosmology
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However, a consistent superstring theory is 10 dimension.
From string to the real wold: 10D $\rightarrow$ 4D

$\Rightarrow$ String Compactification

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General Properties

Properties of Bosonic String

- Spacetime dimension $D = 26$ (Conformal anomaly free)
- Massless spectrum including graviton
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Properties of Supersting

- Include Fermion
- Spacetime dimension $D = 10$
- To remove tachyon, one need GSO-projection $\Rightarrow$ SUSY

Worldsheet action

String is a field of $1+1$ dimensional world-sheet:

$$X^{\mu}(\tau,\sigma) : \text{world sheet } \Sigma \longrightarrow \text{spacetime } \mathcal{M} \text{ with } \mu = 1,\ldots,D$$

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From the simplest relativistic free particle action, we can end up with a Lorentz covariance manifested, square-root free, called Polyakov action (Bose):

$$S_P = -\frac{T}{2} \int_{\Sigma} d\sigma d\tau \sqrt{-h} h^{\alpha\beta} g_{\mu\nu}(X) \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu}$$

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- Poincare invariant: $X'^{\mu} = \Lambda^{\mu}{}_{\nu} X^{\nu} + a^{\mu}$
- World sheet reparameterization invariant: $X'^{\mu}(\tau', \sigma') = X^{\mu}(\tau, \sigma)$ with word sheet metric $h_{ab} = \frac{\partial \tau'^{d}}{\partial \tau^{b}} \frac{\partial \sigma'^{c}}{\partial \sigma^{a}} h'_{cd}$
- Weyl Invariance in 2D: $X'^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma)$ with $h'_{ab} = e^{2\omega} h_{cd} \Rightarrow$ $(\tau, \sigma) \rightarrow (z, \bar{z}) \Rightarrow$ CFT with $X(z, \bar{z})$
- 26 scalars X^{μ} cancels the conformal anomaly $\Rightarrow D = 26$

Quantization I-Boundary Condition

$$S_P = -T\int_\Sigma d\sigma d\tau\ g_{\mu\nu}(X)\left(\partial_\alpha X^\mu \partial^\alpha X^\nu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi^\nu\right)$$

- Superconformal anomaly is canceled in the presenc of 10 fields and thier superpartners $\Rightarrow$ $D = 10$ Ramond-Neveu-Schwarz formulism

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General boundary condition

- Neumann: $\partial X^r|_{\sigma=0,2\pi} = 0$ for $r = 0, \ldots, p$
- Dirichlet: $\delta X^s|_{\sigma=0,2\pi} = 0$ for $s = p+1, \ldots, 9$
- Ramond: $\psi^\mu(\sigma + 2\pi) = +\psi^\mu(\sigma)$
- Neveu-Schwarz: $\psi^\mu(\sigma + 2\pi) = -\psi^\mu(\sigma)$

Diff b.c $\Leftrightarrow$ Diff modes expansion. ($z = e^{\tau + i\sigma}$)

- Neumann: $X_L^\mu(z) = \frac{x_0^\mu}{2} - i\frac{\alpha'}{2} p_{0,L}^\mu \ln(z) + i\sqrt{\frac{\alpha'}{2}} \sum_{0\neq n\in\mathbb{Z}} \frac{\alpha_n^\mu}{n} z^{-n}$
- Ramond: $\psi_L^\mu(z) = \sum_{n\in\mathbb{Z}} d_n^\mu z^{-n-\frac{1}{2}}$
- Neveu-Schwarz: $\psi_L^\mu(z) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_n^\mu z^{-r-\frac{1}{2}}$

Quantization II-Spectrum

Skipping the standard steps of canonical quantisation, display the commutation relations of the oscillator modes for closed string

- Bose: $[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}, \quad [x_0^\mu, p_0^\nu] = i\eta^{\mu\nu}$
- Ramond: $\{d_m^\mu, d_n^\nu\} = \{\tilde{d}_m^\mu, \tilde{d}_n^\nu\} = \delta_{m+n,0}\eta^{\mu\nu}$
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Spectrum

- Dynamical degrees of freedom $\Rightarrow$ 8 transverse directions $\mu = 2, \ldots, 9$
  Light-cone gauge
- NS ground state is tachyonic, R ground state is degenerate $\Rightarrow$ GSO-projection
- GSO-Projection: Consistent with Modular Invariant of Scattering Amplitude, Superconformal anomaly free in critical dimension $10D$

10D Type II Spectrum

Gluing left-moves and right-moves together: R-R, R-NS, NS-R, NS-NS sectors

NS-NS	$\tilde{b}^{\mu}_{-\frac{1}{2}}\vert 0\rangle_{NS} \otimes b^{\nu}_{-\frac{1}{2}}\vert 0\rangle_{NS}$	dilaton ϕ, B-field $B_{\mu\nu}$, Graviton $g_{\mu\nu}$
	$8_v \otimes 8_v$	$1 + 28 + 35$
R-R	$\vert a\rangle \otimes \vert b\rangle$	IIB: C_0, C_2, C_4
	$8_s \otimes 8_s$	$1 + 28 + 35_+$ C_4 self-dual
	$\vert a\rangle \otimes \vert \dot{a}\rangle$	IIA: C_1, C_3
	$8_s \otimes 8_c$	$8 + 56$
NS-R	$\tilde{b}^{\mu}_{-\frac{1}{2}}\vert 0\rangle_{NS} \otimes \vert a\rangle$	dilatino, gravitino
	$8_v \otimes 8_s$	$8 + 56$

10D Type IIB Effective Action

In Einstein frame ($g^E_{\mu\nu} = e^{-\phi/2} g^s_{\mu\nu}$)

$$S^{(10D)}_{IIB} = -\int \left(\frac{1}{2} R * 1 + \frac{1}{4} d\phi \wedge * d\phi + \frac{1}{4} e^{-\phi} H_3 \wedge * H_3 \right) - \frac{1}{4} \int \left(e^{2\phi} dC_0 \wedge * dC_0 + e^{\phi} F_3 \wedge * F_3 + \frac{1}{2} F_5 \wedge * F_5 \right) - \frac{1}{4} \int C_4 \wedge H_3 \wedge F_3$$

$$H_3 = d\hat{B}_2, \qquad F_3 = dC_2 - C_0 dB_2$$
$$F_5 = dC_4 - \frac{1}{2} dB_2 \wedge C_2 + \frac{1}{2} B_2 \wedge dC_2.$$

where the self-dual condition $*F_5 = F_5$ is inposed at EOM level.

Five perturbative Superstring Theory

	SUSY	# of Real Supercharge
Heterotic $SO(32)$	$\mathcal{N} = 1$ SUSY in 10D	16
Heterotic $E_8 \times E_8$	$\mathcal{N} = 1$ SUSY in 10D	16
Type I string	$\mathcal{N} = 1$ SUSY in 10D	16
Type IIA /IIB	$\mathcal{N} = 2$ SUSY in 10D	32

Outline

Reference II

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Two Aspects of String Phenomenology

Two longstanding problems of realistic string model building

- Global Issues: Complete compact Calabi-Yau compactification with, moduli stabilization, tadpole cancellation and Freed-Witten anomaly cancellation, SUSY breaking, realizing dS solution and cosmology . . .
- Local Issues: Local sets of lower dimensional D-branes, which are localised in some area of the Calabi-Yau and reproduce chiral spectrum, tree-level Yukawa couplings, gauge couplings . . .

However, it is fair to say that models developed so far are still far from being realistic.

Flux Compactification I

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What we want: $\mathcal{N} = 1$ Supersymmetry with chiral spectrum

Best under control: $\mathcal{N} = 1$ Flux Compactification

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Background Flux (in Type II):

- *Neveu-Schwarz flux:* $H_3 = dB_2, \; dH_3 = 0.$
- *Ramond flux:* $F_{p+1} = dC_p, \; dF_{p+1} = 0.$
- *Metric flux:* ${F_{ij}}^k$ from T-dual of H_{ijk}.
- *Non-geometric flux:* T-duality with Buscher rules.

$$H_{ijk} \xleftrightarrow{T_k} {F_{ij}}^k \xleftrightarrow{T_j} {Q_i}^{jk} \xleftrightarrow{T_i} R^{ijk} \, .$$

Flux Compactification II

$\mathcal{M}_4 \times X$:

$$ds^2 = e^{A(y)}\,\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + \bar{g}_{mn}(y)dy^m dy^n$$

where $\mu,\,\nu = 1,\ldots,4$. $m,\,n$ run away the inner coordinates

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1. Find a compactified space X, such as $\mathcal{M}_4$ satisfy:
   - Maximal Symmetry, i.e. $\mathcal{M}_4 = \{dS_4, AdS_4, Minks\}$
   - Chiral $\mathcal{N} = 1$ SUSY in 4D, i.e. 4 real supercharge

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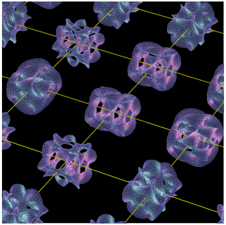
3. Get the effective theory of the chiral spectrum and moduli
4. Based on some concrete model study the particle phenomenology and cosmology

Flux Compactification III

Four dimensional $\mathcal{N} = 1$ supersymmetry flux compactification:

- Het string on CY_3
- Type IIA/B on CY_3 with orientifold (include Type I $\cong$ Type IIB orientifold with $O9-$plane) ✓
- F-theory on CY_4
- M-theory on $CY_3 \times S^1/\mathbb{Z}_2$ or on $\mathcal{M}^7$ with G_2 holonomy

$\Rightarrow$ Calabi-Yau threefold CY_3 or fourfold CY_4.



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Calabi-Yau Space

Q: What is Calabi-Yau and Why it appears?

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Calabi-Yau n-folds is a complex n-dimentional compacted Kähler Manifold satisfied:

- Its first chern class vanish, i.e $c_1(M) = 0 \in H^2(M, \mathbb{Z})$.
- The normal bundle $K_M = \wedge^n T^*(1,0)(M)$ is trivial since $c_1(K_M) = -c_1(M)$
- There exist a unique no where vanishing holomophic n-form, $\Omega_n \in \Omega^{n,0}(M), d\Omega_n = 0$
- The Ricci tensor vanish, i.e. $R_{mn} = 0$
- The holonomy group of M is $SU(n)$

The Original of Calabi-Yau I

- Under $\mathcal{M}_4 \times X_6$, the Lorentz group $SO(1,9)$ splits into

$$SO(1,9) \longrightarrow SO(1,3) \times SO(6)$$

- The corresponding spinor representation $\mathbf{16} \in SO(1,9)$ splits into:

$$\mathbf{16} \longrightarrow (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}})$$

where 4 and $\bar{4}$ is the weyl spinor of $SO(6)$ while 2, $\bar{2} \in \mathrm{SO}(1,3)$

- Q: What is the consequence if we require SUSY?

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$$\begin{aligned} < \delta_\epsilon \mathrm{Fermion} > &= < \nabla_M \epsilon > + < \ldots, \mathrm{Bosonic}, H_3, \cdots >= 0. \\ < \nabla_M \epsilon > &\equiv \bar{\nabla}_M \epsilon = 0 \end{aligned}$$

$\Rightarrow$ Global well defined Killing spinor, s.t. $\bar{\nabla}_M \epsilon = 0$

The Original of Calabi-Yau II

- The spinor $\epsilon(x,y) \in SO(1,9)$ can be spllited as:

$$\epsilon(x,y) = \xi(x) \otimes \eta(y)$$

- $\nabla_\mu \xi = 0 \quad \Rightarrow \quad [\nabla_\mu, \nabla_\nu]\xi = \frac{1}{4} R_{\mu\nu\rho\sigma}\Gamma^{\rho\sigma}\xi = 0$
  By the requirement of Maximal Symmetry, $R_{\mu\nu\rho\sigma} = \frac{R}{12}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\rho\nu})$
  $\Rightarrow$ scalar curvature $R = 0$, i.e. 4D Minkowski spacetime
- $\nabla_m \eta = 0 \quad \Rightarrow \quad [\nabla_m, \nabla_n]\eta = \frac{1}{4} R_{mnpq}\Gamma^{pq}\eta = 0$
  Multiply by Γ^p get $R_{mn}\Gamma^n\eta = 0 \Rightarrow R_{mn} = 0$, i.e. 6D Ricci Flat Space

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- If η is irreducible on X, define (1,1)-form J and (3,0)-form Ω through η:

$$\eta_\pm^\dagger \gamma^{mn} \eta_\pm = \pm\frac{i}{2} J^{mn}, \quad \eta_-^\dagger \gamma^{mnp} \eta_+ = \frac{i}{2}\Omega^{mnp}, \quad \eta_+^\dagger \gamma^{mnp} \eta_- = \pm\frac{i}{2}\bar{\Omega}^{mnp},$$

$\Rightarrow dJ = 0$, $d\Omega = 0 \Rightarrow J$ is Kähler form and X is Calabi-Yau threefold

The Original of Calabi-Yau III

- From representations, it equivalent to find a inner spacetime with holonomy group $SU(3) \subset SO(6) \cong SU(4)$
$\Rightarrow$ $\mathbf{4} \in SO(6) \cong SU(4)$ split under $SU(3)$ as

$$\mathbf{4} \longrightarrow \mathbf{3} \oplus \mathbf{1}$$

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Similarly

- Compactified with holonomy $SU(2)$,
  $$\mathbf{4} \longrightarrow \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1} \Rightarrow \mathcal{N} = 2 \text{ in 4D}$$
- Compactified on torus $T^6 \Rightarrow \mathcal{N} = 4$ in 4D
- Start from $\mathcal{N} = 2$, compactified on $T^6 \Rightarrow \mathcal{N} = 8$ in 4D
- Start from $\mathcal{N} = 2$, compactified on $K_3 \times T^2 \Rightarrow \mathcal{N} = 4$ in 4D, compactified on $CY_3 \Rightarrow \mathcal{N} = 2$ in 4D

Moduli

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- The size and shape of the internal manifold is dynamically determined by the vacuum expectation values of moduli.

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Moduli of Calabi-Yau X:

$$g \to g + \delta g \quad s.t. \quad R_{m\bar{n}}(g + \delta g) = 0.$$

For Kähler manifold, under proper gauge $\nabla(\delta g) = 0$, it decouples

- Kähler moduli: $\delta g_{m\bar{n}} = i v^i (\hat{D}_i)_{m\bar{n}}, \quad i = 1, \ldots, h^{11}(X)$
- Complex moduli: $\delta g_{mn} = \frac{i}{||\Omega||^2} \bar{U}^a (\bar{\chi}_a)_{m\bar{p}\bar{q}} \Omega_n^{\bar{p}\bar{q}}, \;\; a = 1, \ldots, h^{12}(X)$
- Moduli gives the spectrum in 4D.

Spectrum in 4D

- Under $\mathbb{R}^{3,1} \times X_6$, the EOM of scalar ϕ satisfied

$$\Delta_{10}\phi = (\Delta_4 + \Delta_6)\,\phi = \left(\Delta_4 + m^2\right)\phi = 0.$$

  $\Rightarrow$ the number of 4D massless field is determined by the harmonic form of X_6, i.e. the Zero modes of Δ_6.

- In fact, all the massless field in 4D is determined by harmonic (p, q)-form, which in CY_3 case, is $H^{p,q}(X)$ with Hodge number $h^{p,q} = \dim(H^{p,q}(X))$.

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  - Hodge $\star$-duality $\quad H^{p,q}(X) \cong H^{3-p,3-q}(X)$
  - Complex conjugate $\quad H^{p,q} \cong H^{q,p}(M)$

$$\begin{array}{ccccccc}
 & & & h^{0,0} & & & \\
 & & h^{1,0} & & h^{0,1} & & \\
 & h^{2,0} & & h^{1,1} & & h^{0,2} & \\
h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\
 & h^{3,1} & & h^{2,2} & & h^{1,3} & \\
 & & h^{3,2} & & h^{2,3} & & \\
 & & & h^{3,3} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{1,2} & & h^{1,2} & & 1 \\
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\end{array}$$

Moduli Space

Moduli of Calabi-Yau X: $g \to g + \delta g \quad s.t. \quad R_{m\bar{n}}(g + \delta g) = 0.$

- Kähler moduli: $\delta g_{m\bar{n}} = i v^i (\hat{D}_i)_{m\bar{n}}, \quad i = 1, \dots, h^{1,1}(X)$
  - In Type II theory, complexfied Kähler form $J_c = J + iB_2 = t^i \hat{D}_i$
    Kähler moduli space $\{t^i\}$ $\Rightarrow$ special Kähler manifold $\mathcal{M}^K_{h^{1,1}}$
  - Kähler form can be written as $J = -ig_{m\bar{n}} dy^m \wedge d\bar{y}^{\bar{n}}$.
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- Complex moduli:
  $\delta g_{mn} = \frac{i}{||\Omega||^2} \bar{U}^a (\bar{\chi}_a)_{m\bar{p}\bar{q}} \Omega_n^{\bar{p}\bar{q}}, \ a = 1, \dots, h^{12}(X)$
  Complex moduli space $\Rightarrow$ special Kähler manifold $\mathcal{M}^{cs}_{h^{1,2}}$

At tree level, we have $\mathcal{M} = \mathcal{M}^{cs}_{h_{1,2}} \times \mathcal{M}^K_{h_{1,1}}$

Outline

Orientifold

Q: Why orientifold?

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$$\mathcal{O}=\begin{cases}\Omega_p\,\sigma & \text{with} \quad \sigma^*(J)=J\,, \quad \sigma^*(\Omega_3)=\Omega_3\,, \quad O5/O9 \\ (-)^{F_L}\,\Omega_p\,\sigma & \text{with} \quad \sigma^*(J)=J\,, \quad \sigma^*(\Omega_3)=-\Omega_3, \quad O3/O7\end{cases}$$

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each σ defines a new CY in the orbifold limit unless it is free action.

- In Type IIB orientifold, Complex,dilaton moduli decoupled with Kähler moduli.
  - Complex and dilaton moduli can be stabilized by background fluxes at tree level. Gukov/Vafa/Witten
  - Kähler moduli can be stabilized by quantum correction (KKLT, Large Volume scenario). Kachru/Kallosh/Linde/Trivedi, Balasubramanian/Berglund/Colon/Quevedo

Orientifold II

Type of orientifold	Dim of fixed point locus in X	O-plane	D-brane
$\sigma^*(\Omega_3) = \Omega_3$	2	O5-plane	D5-brane
	6	O9-plane	D9-brane
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ϕ, B_2, $g_{\mu\nu}$, C_0, C_2, C_4, two dilatino, two gravitino

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Q: What is the spectrum left under oreintifold?

- $(-)^{F_L}$: NS-NS (+), NS-R (+), R-NS (-), R-R (-)
- Ω : NS-NS, only sym part of tensor product is even under parity transformation, $\phi, g_{\mu\nu}$
  R-R, two dilatino, two gravitino combined to one dilatino and one gravitino, provide $56 + 8 = 64$ fermion d.o.f. $\phi, g_{\mu\nu}$ provide $35 + 1$ Bose d.o.f $\Rightarrow C_2$ left, 28 d.o.f

Orientifold III

Four dimensional orientifold invariant closed string fields.

	$(-)^{F_L}$	Ω_p	σ^*
ϕ	$+$	$+$	$+$
C_0	$-$	$-$	$+$
$g_{\mu\nu}$	$+$	$+$	$+$
B_2	$+$	$-$	$-$
C_2	$-$	$+$	$-$
C_4	$-$	$-$	$+$

$$J = t^{\alpha}\,\hat{D}_{\alpha}, \qquad \alpha = 1, \ldots h_{+}^{1,1}(X)$$

$$C_2 = c^{a}\,\hat{D}_{a}\,, \qquad B_2 = b^{a}\,\hat{D}_{a} \qquad a = 1, \ldots h_{-}^{1,1}(X)$$

$$C_4 = Q_2^{\alpha} \wedge \hat{D}_{\alpha} + V^{\tilde{\alpha}} \wedge \alpha_{\tilde{\alpha}} + V_{\tilde{\alpha}} \wedge \beta^{\tilde{\alpha}} + \rho_{\alpha}\,\tilde{D}^{\alpha}$$

$\tilde{D}^{\alpha}$ and $\tilde{D}^{a}$ is a basis of $H_{+}^{2,2}(X)$ and $H_{-}^{2,2}(X)$.

$(\alpha_{\tilde{\alpha}}, \beta^{\tilde{\alpha}})$ is a real symplectic basis of $H_{+}^{3}(X)$.

Closed Spectrum of Type IIB on CY_3/σ

chiral multiplets	$h_-^{2,1}$ $h_+^{1,1}$ $h_-^{1,1}$ 1	$\bar{U}^{\tilde{a}}$ (t^α, ρ_α) (b^a, c^a) (ϕ, C_0)
vector multiplet	$h_+^{2,1}$	$V^{\tilde{\alpha}}$
gravity multiplet	1	$g_{\mu\nu}$

$\mathcal{N} = 1$ massless bosonic spectrum of Type IIB Calabi Yau orientifold with $O3/O7-$plane.

Low Energy Effective Action I

Supergravity form: Kähler potential K , Holomorphic superpotential W, holomorphic gauge-kinetic coupling function f.

- $\mathcal{N}=1$ F-term and D-term gives the scalar potential:

$$V = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2) + \frac{1}{2}(\mathrm{Re}\, f)^{-1\,ab} D_a D_b$$

with the tree-level superpotential, Gukov, Vafa, Witten ...

$$W = \int_X G_3 \wedge \Omega_3 .$$

- $K^{I\bar{J}}$ is the inverse of Kähler metric $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K(\Phi, \bar{\Phi})$,
- Covariant derivative $D_I W = \partial_I W + W \partial_I K$.
- $D_a = \left[K_I + \frac{W_I}{W}\right] (T_a)_{IJ} \Phi_J$ with T_a here the gauge generator.

Low Energy Effective Action II

Moduli spectrum from dimension reduction are not necessarily the good Kähler coordinate appearing in SUGRA formalism.

$$\tau = C_0 + ie^{-\phi}\,, \; U^{\tilde{a}} = u^{\tilde{a}} + iv^{\tilde{a}}, \; G^a = c^a - \tau\, b^a\,,$$

$$T_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma}\, t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab}\, c^a b^b\right) - \frac{1}{4}\, e^\phi\, \kappa_{\alpha ab}\, \bar{G}^a (G + \bar{G})^b\,.$$

Becker, Jockers, Louis, Grimm . . .

$$k_{\alpha ab} = \int_X \hat{D}_\alpha \wedge \hat{D}_a \wedge \hat{D}_b, \qquad k_{\alpha\beta\gamma} = \int_X \hat{D}_\alpha \wedge \hat{D}_\beta \wedge \hat{D}_\gamma.$$

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To leading order, the low-energy tree-level Kähler potential is:

$$K = -\ln\left(-i\int_X \Omega_3(U)\wedge\bar{\Omega}_3(\bar{U})\right) - \ln\left(-i(\tau-\bar{\tau})\right) - 2\ln\left(\mathcal{V}\left(T_\alpha\right)\right)$$

Complex structure deformations do not mix with the other scalars

$$\mathcal{M} = \mathcal{M}^{cs}_{h^{12}_-} \times \mathcal{M}^{K}_{h^{11}+1}$$

Tadpole Cancellation in Type IIB orientifolds

- D7-brane tadpole cancellation: $\sum_a N_a(\hat{D}_a + \hat{D}'_a) = 8\hat{O}7$
- D3-brane tadpole cancellation:

$$N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{12} + \sum_a N_a \frac{\chi_o(D_a) + \chi_o(D'_a)}{48}$$

with $N_{\text{flux}} = \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3, \; N_{\text{gauge}} = -\sum_a \frac{1}{8\pi^2} \int_{D_a} \text{tr}\mathcal{F}_a^2$

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- D5-brane tadpole cancellation: If $H^2_-(X) \neq 0$ with some non-trivial gauge-flux turned on, for all $\omega \in H^2_-(\mathcal{M})$,

$$\sum_a \int_{\mathcal{M}} \omega \wedge (\text{tr}\mathcal{F}_a \wedge D_a + \text{tr}\mathcal{F}_{a'} \wedge D_{a'}) = 0$$

- Freed-Witten anomaly: $c_1(L) - i^* B + \frac{1}{2} c_1(K_{D_a}) \in H^2(D_a, \mathbb{Z})$
  When the divisor D_a wrapped by a $D7-$brane is non-spin,i.e, $c_1(K_{D_a}) \neq 0 \bmod 2$, K_D is the canonical bundle of D_a.

Outline

Moduli Stabilisation

- Compactifications $\Rightarrow$ Extra massless spectrum in four dimensions.
- The problem of giving masses to these unwanted scalars is called Moduli Stabilisation.

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- How to describe the cosmology?

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⇒ Best understood in Type IIB orientifold with KKLT and LARGE Volume Scenario (LVS). Kachru, Kallosh, Linde, Trivedi, Becker, Becker, Haack, Louis, Balasubramanian, Berglund, Conlon, Quevedo

Procedure of Moduli Stabilisation

1. Stabilize complex moduli and dilaton modulus by perturbative contribution, i.e, flux generated superpotential. Gukov, Vafa, Witten
2. The scalar potentail generated by GVW-superpotential for the Kähler moduli is still flat due to the no-scale structure.
3. Stabilize Kähler moduli by all possible perturbative and non-perturbative corrections.

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   - KKLT: Non-perturbative correction to superpotential $\Rightarrow$ Fine-tune tree level superpotential W_0 to be very small s.t. $\delta V_{\alpha'} \ll \delta V_{np}$

     Kachru, Kallosh, Linde, Trivedi
   - LVS: Perturbative α'^3 correction to Kähler potential
     Non-perturbative contribution to superpotentail s.t. $\delta V_{\alpha'} \sim \delta V_{np}$
     $\Rightarrow W_0 \sim \mathcal{O}(1)$ naturely. For $\tau = \mathrm{Re}\, T$, $\mathcal{V} \sim e^{a\tau}$ with $\tau \geq 1$.

$$\delta V_{\alpha'} \sim \delta V_{np} \sim \mathcal{O}(\frac{1}{\mathcal{V}^3})$$

Becker, Becker, Haack, Louis, Balasubramanian, Berglund, Conlon, Quevedo

Dilaton and Complex Moduil Stabilisation I

Start with Gukov-Vafa-Witten Fluxed-superpotential

$$W_{\tau,U} = \int_X G_3 \wedge \Omega, \qquad G_3 = F_3 - \tau H_3$$

$\Rightarrow$ A scalar potential :

$$V = e^K \left\{ K^{\tau\bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}} \bar{W} + K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3 |W|^2 \right\}$$

$$\begin{aligned} D_\alpha W &= \frac{\partial W}{\partial T_\alpha} + W \frac{\partial K}{\partial T_\alpha} \equiv W_\alpha + W K_\alpha, \\ D_{\bar{\beta}} \bar{W} &= \frac{\partial \bar{W}}{\partial \bar{T}_{\bar{\beta}}} + \bar{W} \frac{\partial K}{\partial \bar{T}_{\bar{\beta}}} \equiv \bar{W}_{\bar{\beta}} + \bar{W} K_{\bar{\beta}} \end{aligned}$$

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D-term ($V_D \sim \mathcal{O}(\frac{1}{\mathcal{V}^2})$) will set to be zero if $h^{1,1}_{-}(X) = 0$ or by choosing proper orientifold odd field b_2.

Dilaton and Complex Moduil Stabilisation II

$$V = e^{K}\left\{K^{\tau\bar{\tau}}D_{\tau}WD_{\bar{\tau}}\bar{W} + K^{U\bar{U}}D_{U}WD_{\bar{U}}\bar{W} + \left(K^{i\bar{j}}K_{i}K_{\bar{j}} - 3\right)|W|^{2}\right\}$$

- No-scale structure: $\left(\frac{\partial^2 K_{tree}}{\partial T_i \partial \bar{T}j}\right)^{-1} \frac{\partial K_{tree}}{\partial T_i}\frac{\partial K_{tree}}{\partial \bar{T}j} = 3$

$\Rightarrow$ A scaler potential dose not dependent on Kähler moduli.

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After integrate out these heavy modes, the no-scale structure still hold. The direction of Kähler moduli is still flat at tree level.

Quantum Correction I

$\mathcal{N}=1$ non-renormalisable theorem, superpotential only get non-perturbative correction.

$$K = K_{tree} + K_p + K_{np},$$
$$W = W_{tree} + W_{np}.$$

For scalar potential:

$$\delta V = \delta V_{\alpha'} + \delta V_{\mathrm{np}}.$$

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$$W = \int_X G_3 \wedge \Omega + \sum_E \mathcal{A}_E(U^{\tilde{a}}, G^a, \mathcal{F}_E, \ldots)\, e^{-a_E^{\alpha} T_\alpha}$$
$$= W_0 + W_{np}.$$

Quantum Correction II

NOT all D-brane instanton and gaugino condensation can contribute superpotential. (Zero modes)

- The instanton must have exact two zero modes ($\mathcal{O}(1)$-instanton). Mathematically, the divisor D_a wrapped by the D-brane should be rigid.
  - Necessary condition: $1 = \chi_0(D_a) := \sum_{p=0}^{3}(-1)^p \, h^{p,0}(D_a)$
  - Sufficient condition: $h^{0,0}(D_a) = 1\,,\; h^{0,p}(D_a) = 0 \quad p \geq 1$
- The same condition for gaugino condensation

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Get meta-stable dS vacua in 3-steps:

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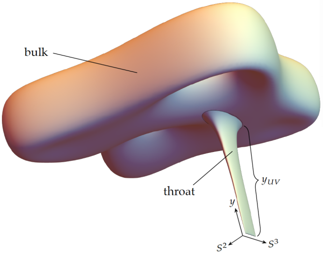
The geometry with strongly warped throat in Type IIB is locally described by Klebanov-Strassler (KS) solution.

Klebanov/Strassler, Giddings/Karchru/Polchinski

The fluxes number is given by fluxes warpping on two 3-cycles at the conifold :

$$K = \int_A H_3, \quad M = \int_B F_3,$$

The throat carries $N = K \cdot M$ units of D3-brane charge (tadpole).



from Ralph's paper

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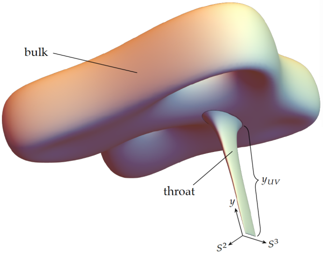


from Ralph's paper

- Stabilize complex and dilaton moudli:

  CY with all complex-structure moduli fixed by fluxes, leading to a non-SUSY Minkowski minimum ($W = W_0 \neq 0, V = 0$). Gukov/Vafa/Witten

KKLT Scenario II

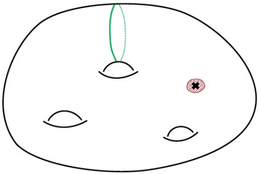
- Stabilize Kähler moduli:

  Non-perturbative effects (E3-instanton (E3 on 4-cycle Σ)/gaugino condensation (D7)) stabilize the Kähler moduli T, leading to an AdS minimum V_{AdS}.

$$K = -3\ln(T+\overline{T}), \quad W = W_0 + e^{-T}$$

$$V = e^K (K^{T\overline{T}} |\partial_T + K_T W|^2 - 3|W|^2)$$

$$V_{AdS} \sim -e^{-\mathrm{Re}(T)}$$

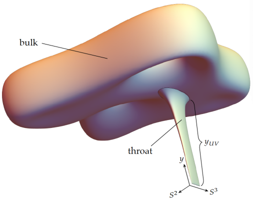


- Uplift to dS:

  Uplift to dS by palcing $\overline{D3}$ in the throat tip, contribute $V_{\text{uplift}} \sim e^{-K/g_s M}$.

  Meta-stable if uplift energy is not too large:

$$V_{\text{uplift}} \sim |V_{AdS}| \;\Rightarrow\; \mathrm{Re}(T) \sim \frac{N}{g_s M^2}$$



Large Volume Senario I

- The leading correction to Kähler potential is α'^3 correction, which comes from $\mathcal{O}(\alpha'^3)R^4$-term in supergravity.

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) = -2\ln\mathcal{V} - \frac{\xi}{g_s^{3/2}\mathcal{V}} + \mathcal{O}\left(1/\mathcal{V}^2\right)$$

with $\xi = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3}$, $\chi(X) = 2\left(h_{1,1} - h_{2,1}\right)$, $\zeta(3) \equiv \sum_{k=1}^{\infty} 1/k^3 \simeq 1.2$.

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- $e^K \sim 1/\mathcal{V}^2$, in large volume limit, the expansion of α' is equivalent to large volume expansion, i.e. $\delta V_{\alpha'} \sim \delta V_{\mathrm{np}}, \quad \alpha'^3 \sim \frac{1}{\mathcal{V}^3}$.

Large Volume Senario II

e.g. $h^{1,1}_{-} = 0$,

$$W_{np} = \sum_i A_i e^{-a_i T_i}$$

$\tau_i = \mathrm{Re}\, T_i$. From $\mathcal{N} = 1$ supergravity $V = V_F + V_D$:

$$V_F \quad = \quad e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right),$$

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$$V_F \sim \left(\frac{K^{\tau\bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}} \bar{W}}{\mathcal{V}^2} \right) + \left(\frac{A e^{-2a\tau_i}}{\mathcal{V}} - \frac{B e^{-a\tau_i} W_0}{\mathcal{V}^2} + \frac{C|W_0|^2}{\mathcal{V}^3} \right).$$

- $D_\tau W = D_U W = 0 \Rightarrow \tau$ and U stabilised at $\mathcal{O}(\frac{1}{\mathcal{V}^2})$
- $0.1 \le |W_0| \le 100$ in general parameter space
- $C \sim \alpha'^3$. If $C > 0$, the minimal condition for the second term:

$$\mathcal{V} \sim e^{a\tau_i} \quad \text{with} \quad \tau_i \ge 1$$

- We get the AdS minimal and uplift it in the same way as KKLT.

Moduli Inflations

Requirements and subsequent attempts for a (semi)realistic inflationary model in string theoretic framework

- Moduli stabilization (and *AdS* to *dS* uplifting)
- Looking for available flat-directions
- UV sensitivity: η problem (protecting flatness against higher order operators)
- Consistent realization of cosmological observables from the point of view of present/future experimental constraints;
  - No. of e-foldings, $N_e \sim \mathcal{O}(60)$
  - Almost scale invariant power spectrum, $n_s \sim 1$
  - Signatures of non-Gaussianities, $f_{NL\,local}$
  - tensor-to-scalar ratio, r

Outlook

- There are many things we didn't mentions. Some of them are crucial.
- Particle Physics from intersection D-branes is not discussed here.
- Here it is the starting point to study string cosmology in Type IIB framework. How about other string theory?
- Non-geometric flux case?
- More mathematical issues.
- ...

Thanks for your attention!